

Digital Signatures

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And Putting It All Together

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Lecture 12

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 - This can then be used to build a full-fledged signature scheme starting from one-time signatures (skipped)

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- Diffie-Hellman suggestion (heuristic): $\text{Sign}(M) = f^{-1}(M)$ where $(SK, VK) = (f^{-1}, f)$, a Trapdoor OWP pair. $\text{Verify}(M, \sigma) = 1$ iff $f(\sigma) = M$.

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 - All probabilities also over the initialization of the RO



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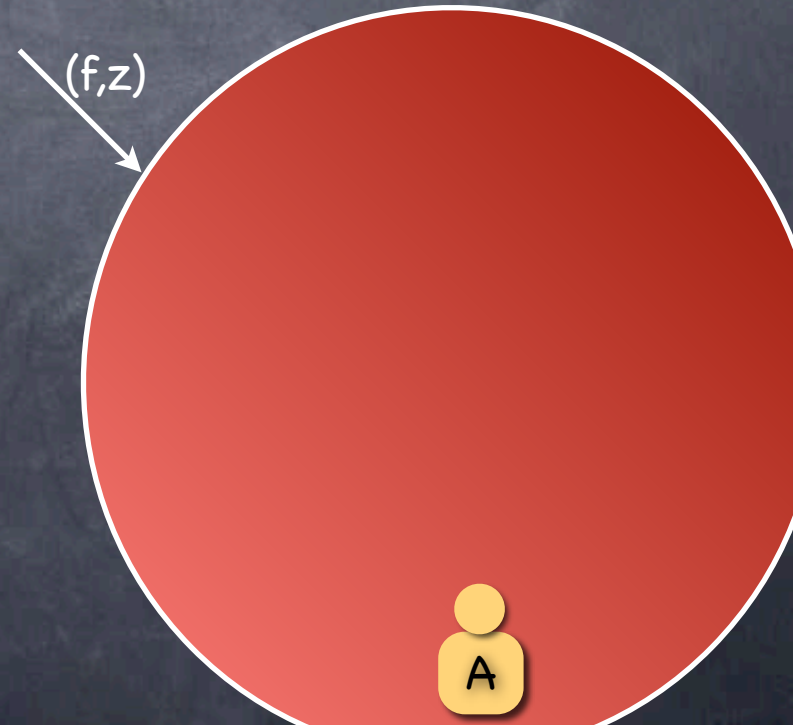
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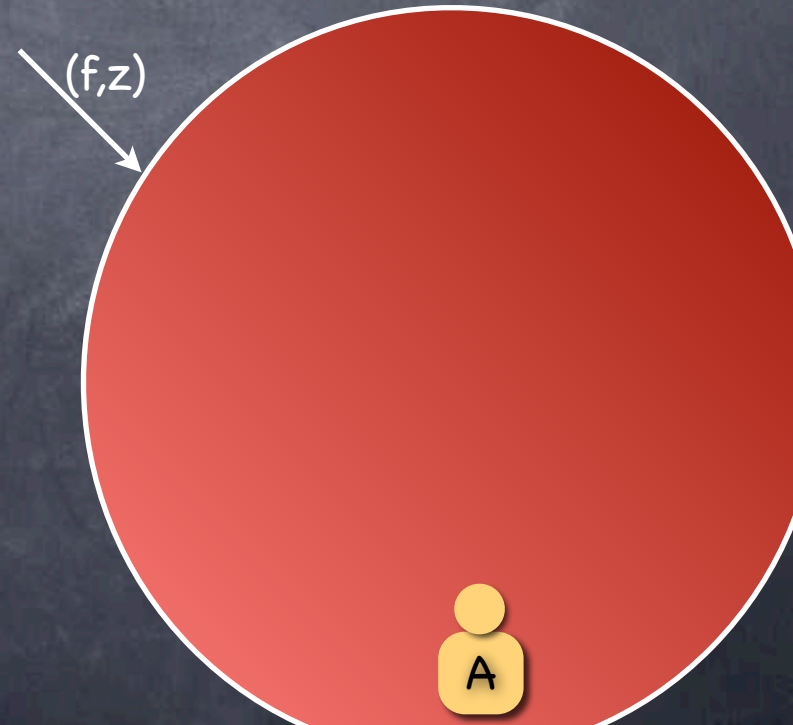
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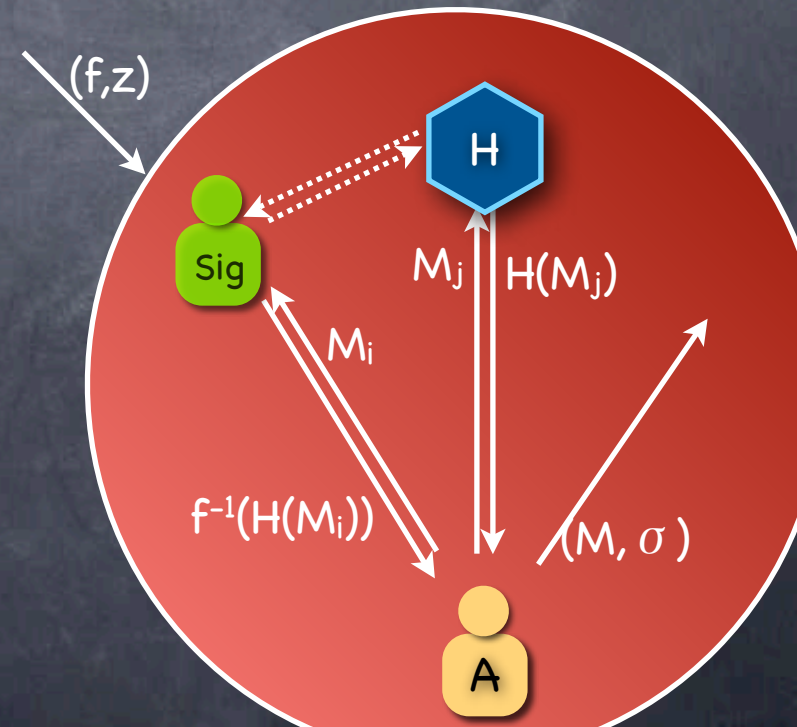
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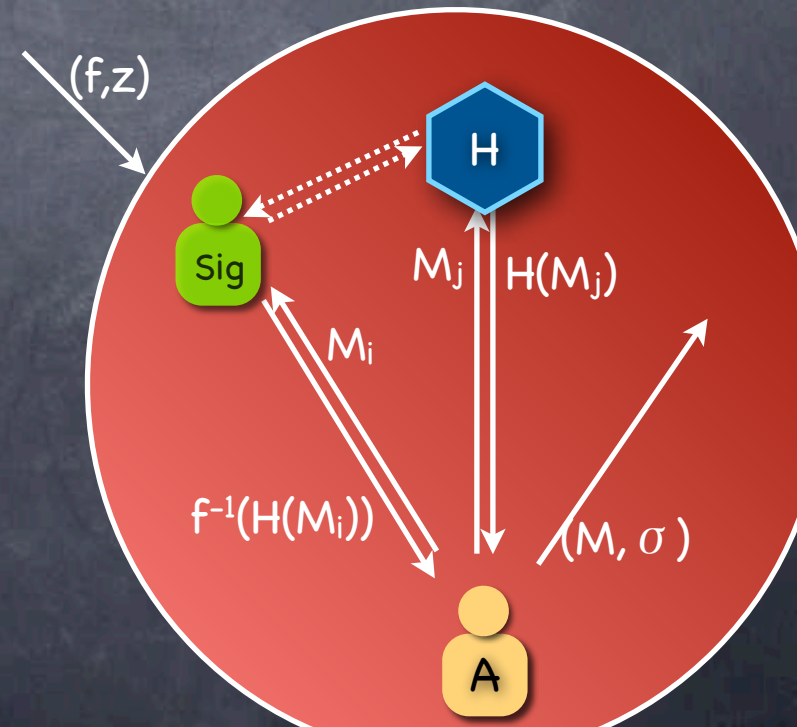
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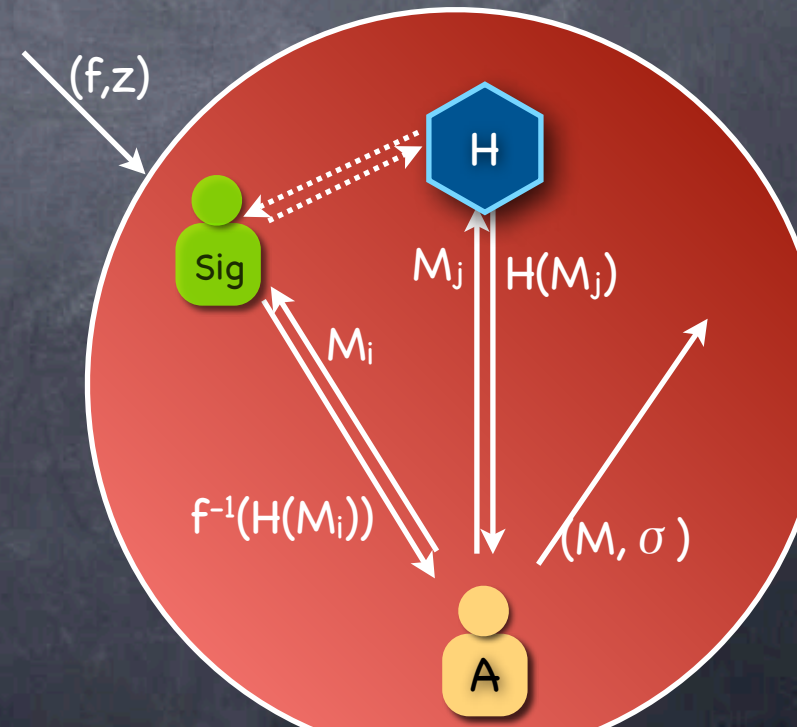
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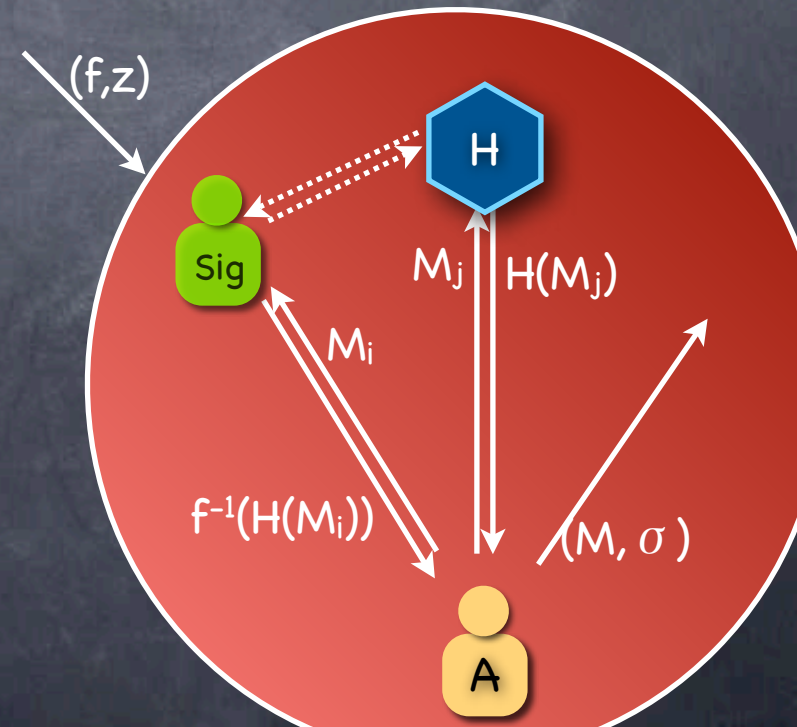
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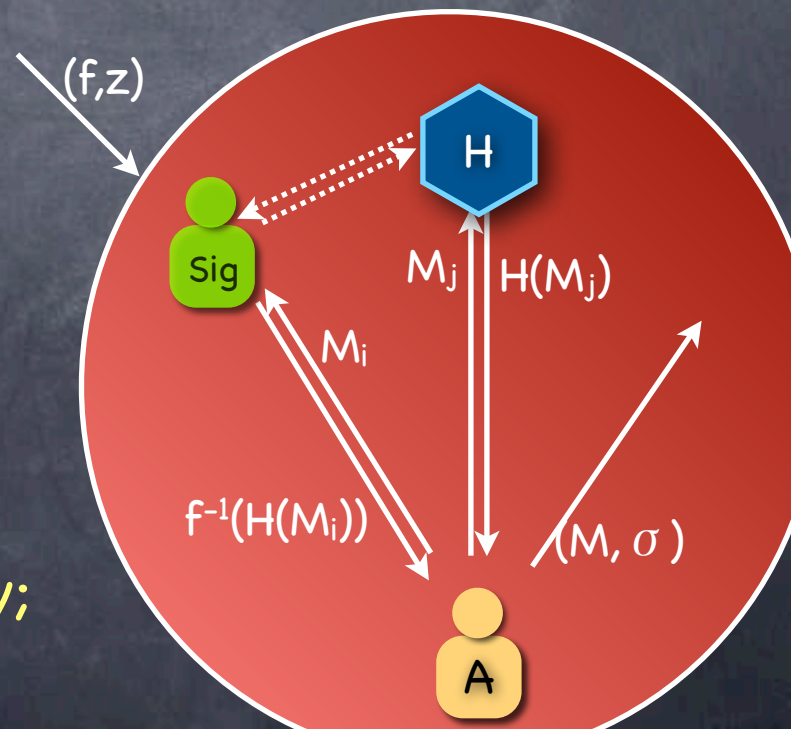
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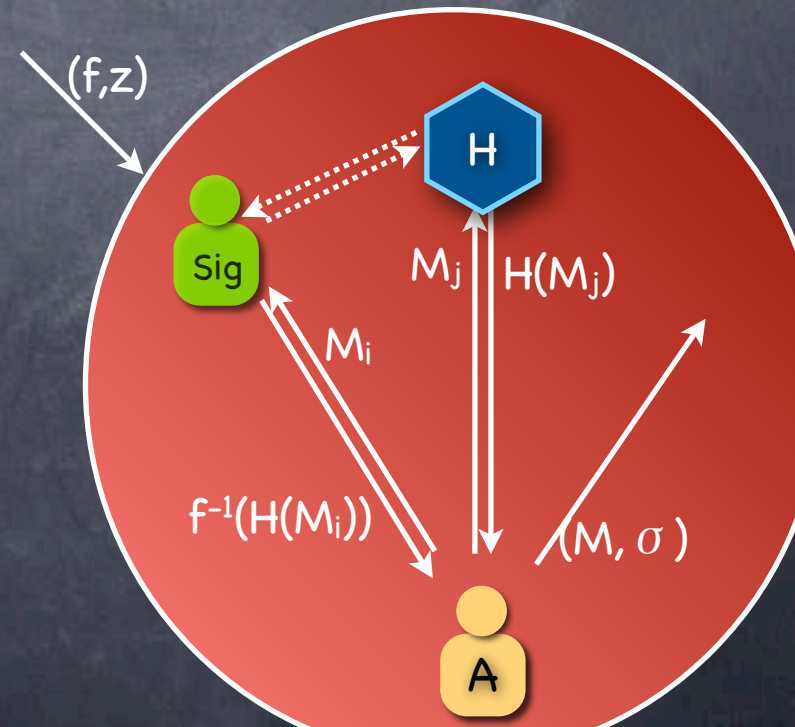
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 - A^* picks $H(M)$ as $x = f(y)$ for random y ;**
then $\text{Sign}(M) = f^{-1}(x) = y$

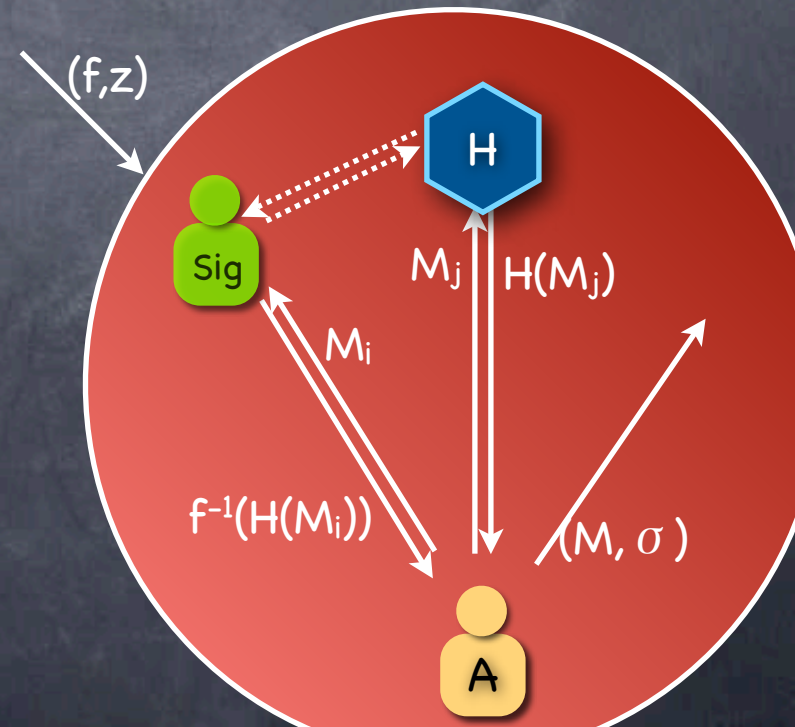


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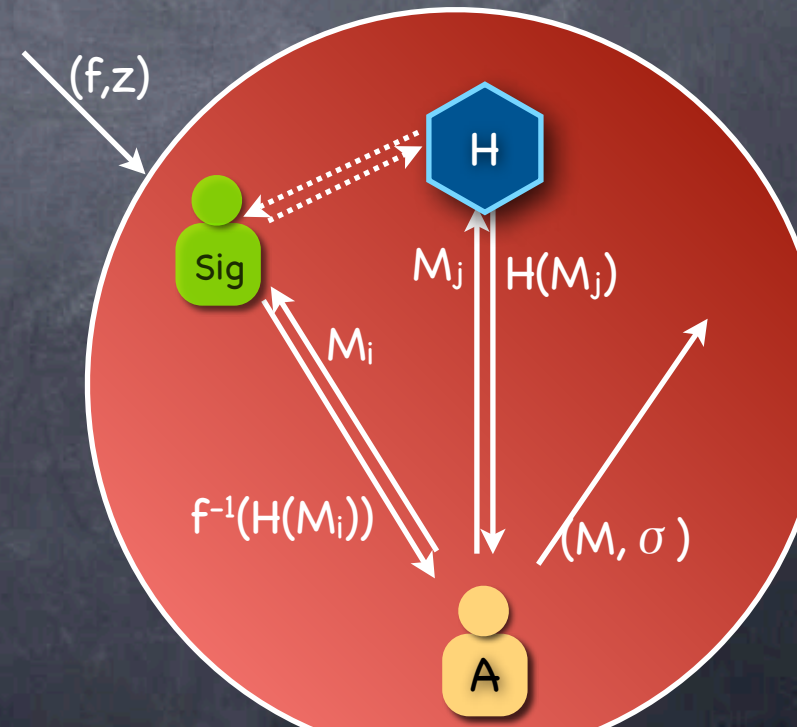
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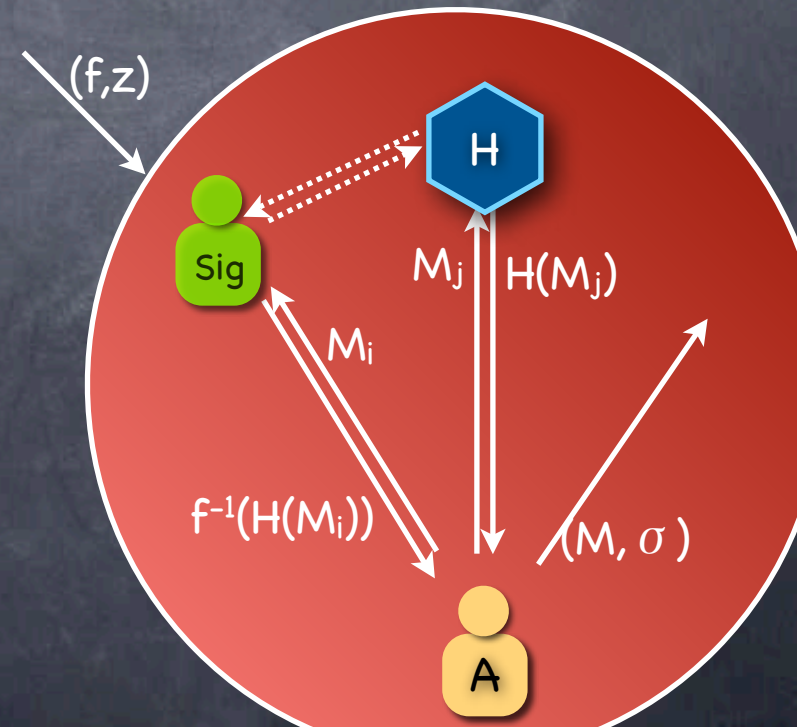
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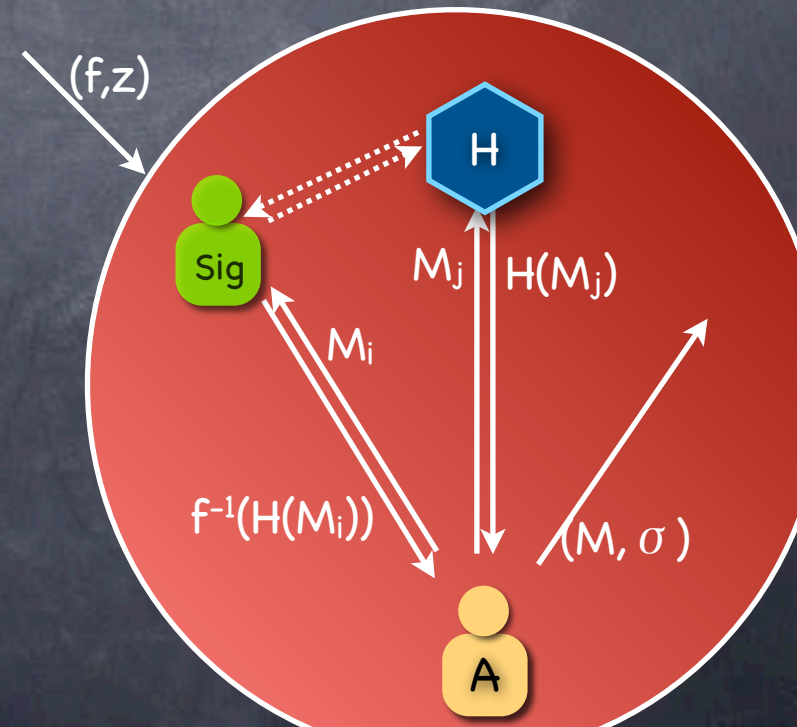
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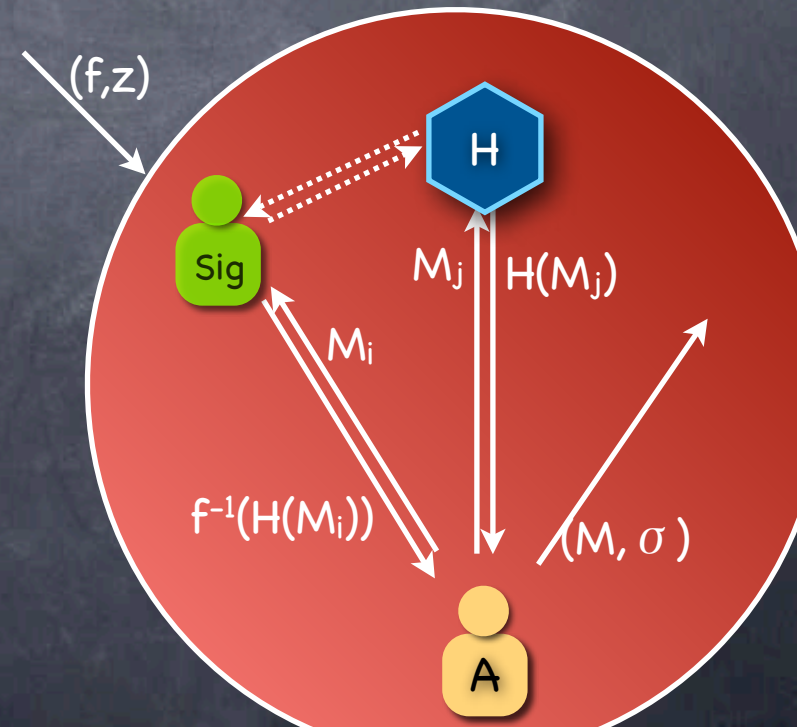
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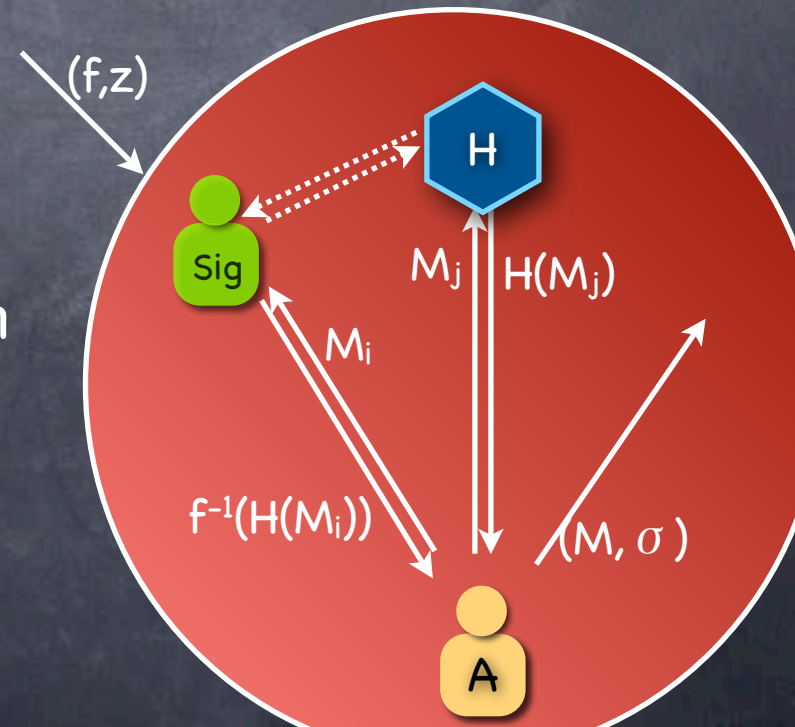
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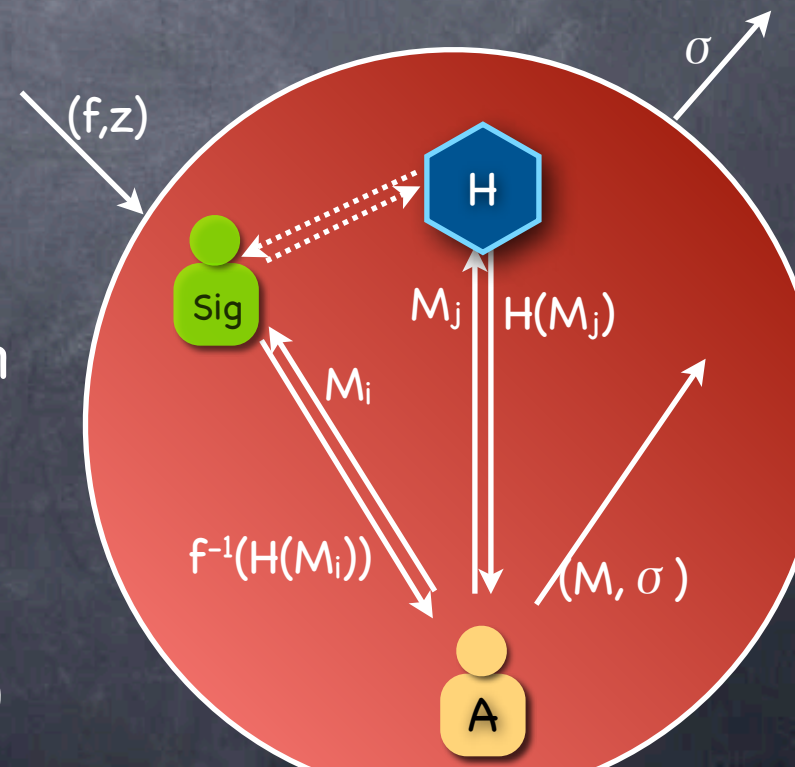
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 - In that case forgery $\Rightarrow \sigma = f^{-1}(z)$



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- A general tool for purifying randomness: Randomness Extractor

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 - Statistical guarantee, if compression function/block-cipher is a random function/random permutation (not random oracle)

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 - Key derivation: Alice and Bob extract a new key, which is pseudorandom (i.e., indistinguishable from a uniform bit string)

Secure Communication: Wrap-Up

We saw...

- Symmetric-Key Components
 - SKE, MAC
- Public-Key Components
 - PKE, Digital Signatures
- Building blocks: Block-ciphers (AES), Hash-functions (SHA-3), Trapdoor PRG/OWP for PKE (e.g., DDH, RSA) and Random Oracle heuristics (in RSA-OAEP, RSA-PSS)
- Symmetric-Key primitives much faster than Public-Key ones
 - Hybrid Encryption gets best of both worlds

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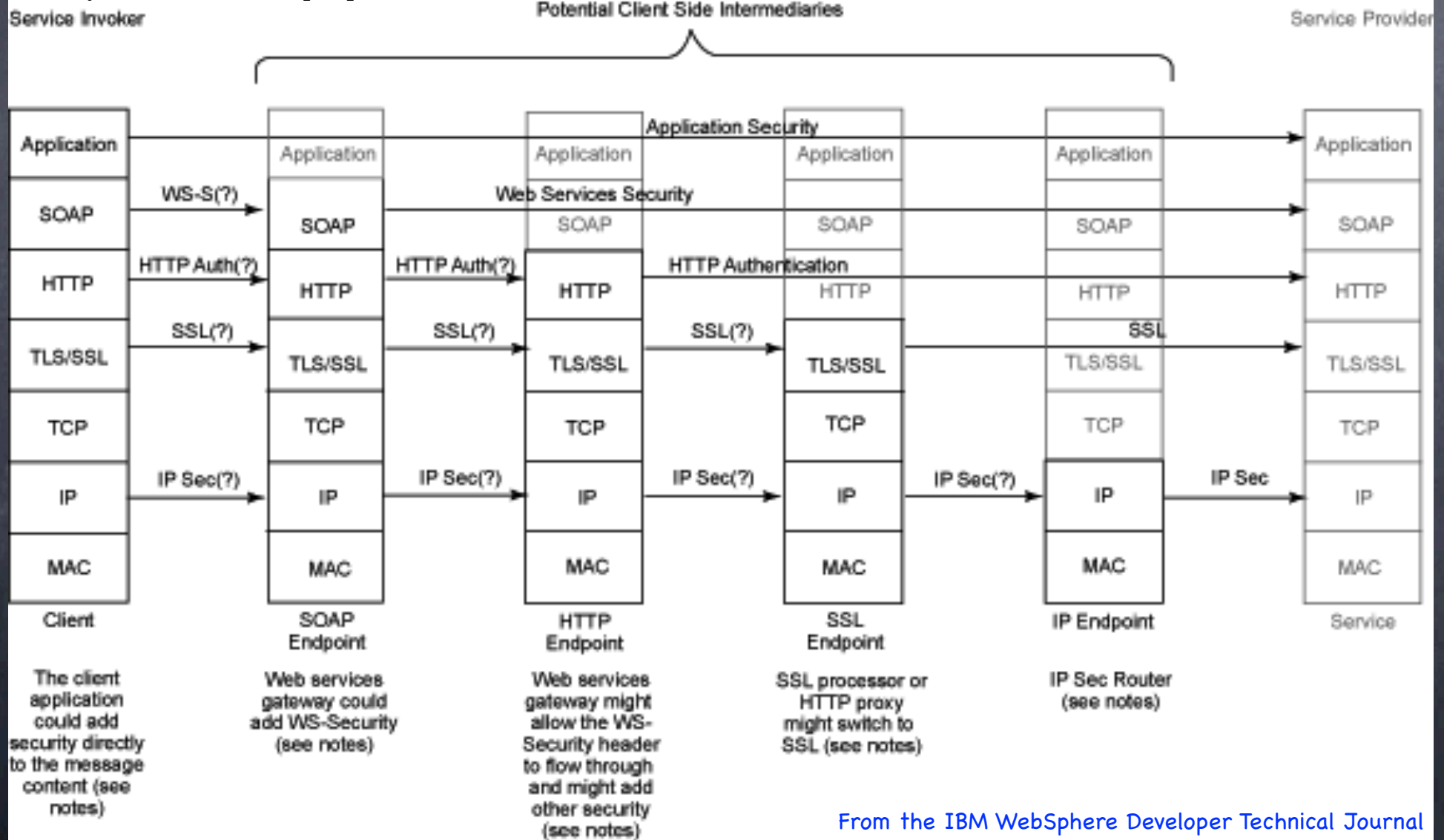
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 - e.g.: SSL/TLS (used in https), IPSec (in the "network layer")

Security Architectures

(An example)

Security architecture (client perspective)



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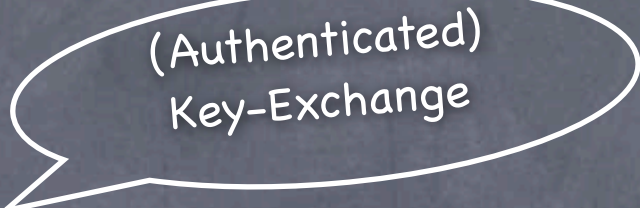
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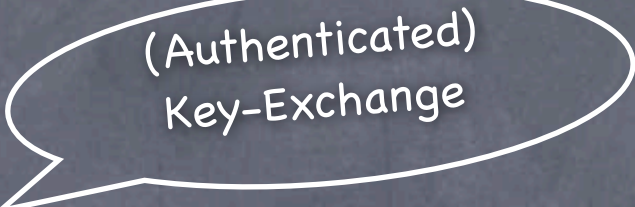
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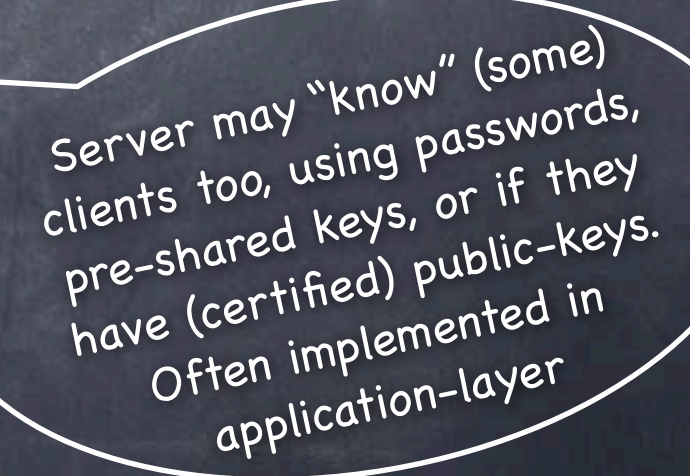
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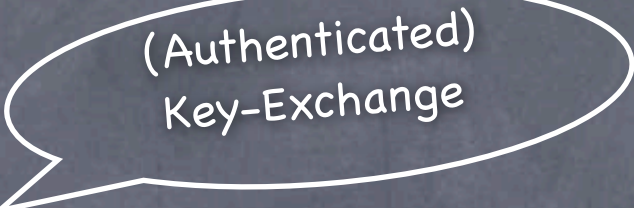
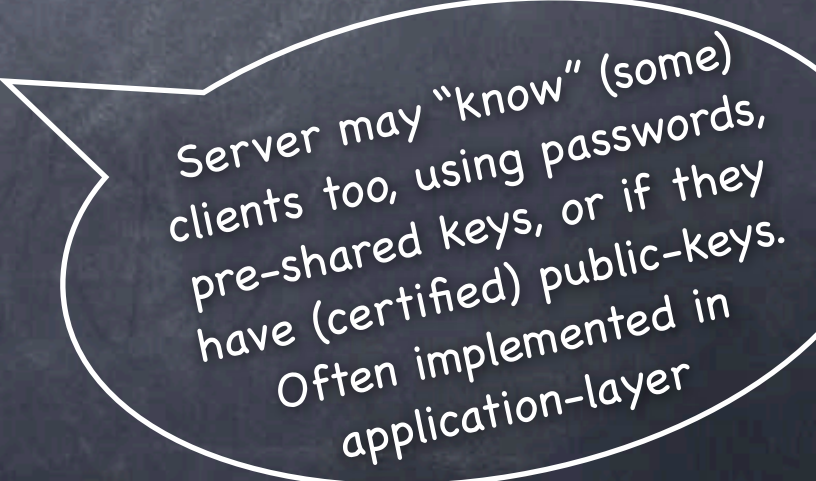
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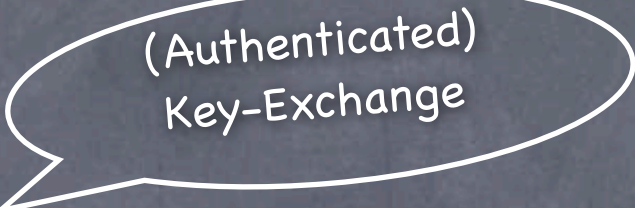


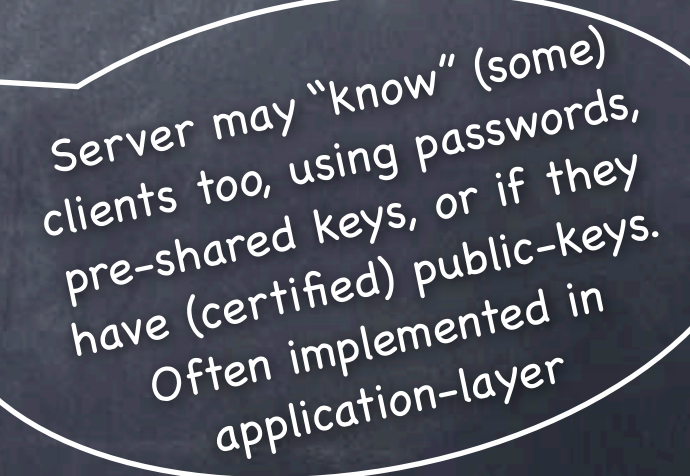
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Authentication for free: MAC serves dual purposes!

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