Public-Key Cryptography

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Lecture 8

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Public-Key Encryption from Trapdoor OWP

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CCA Security

El Gamal Encryption

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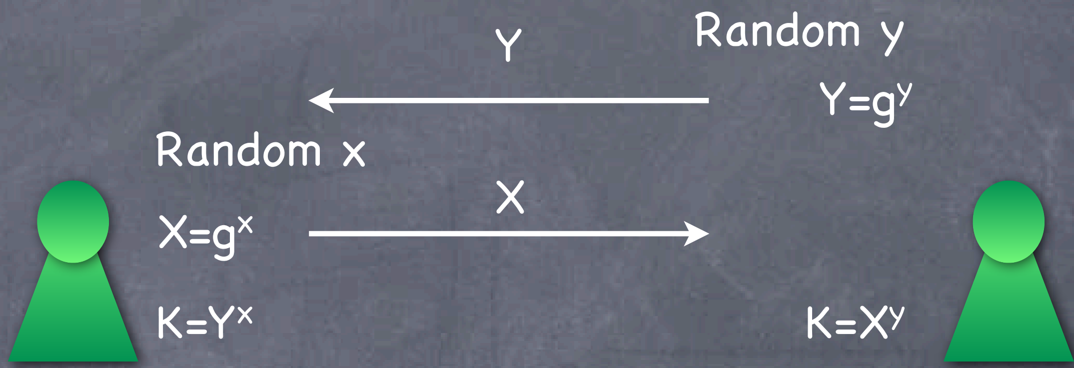

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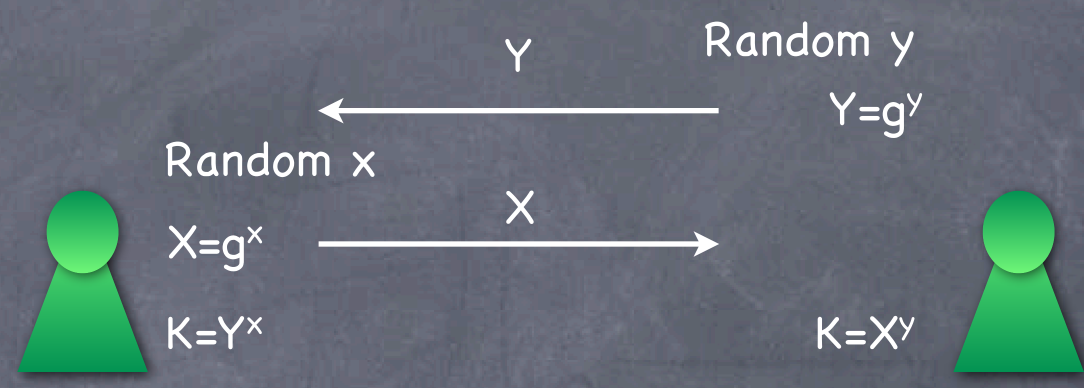

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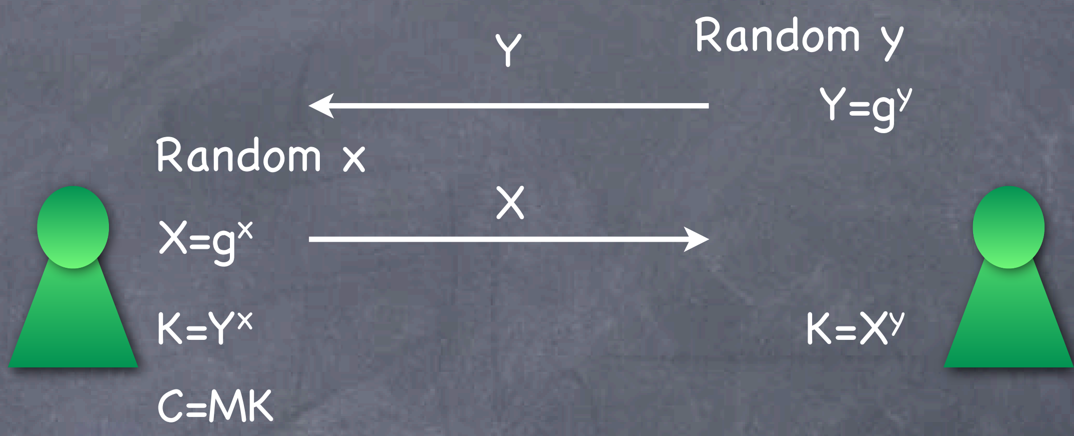

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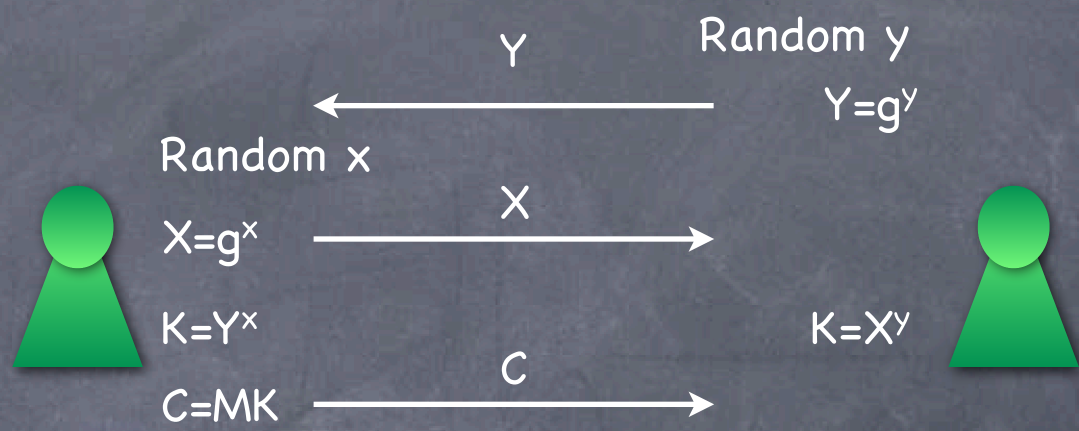

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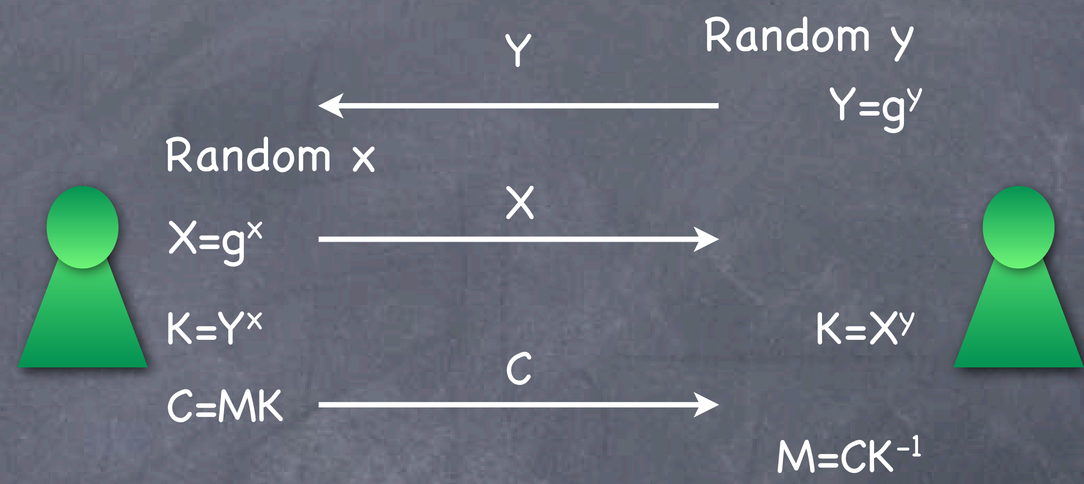

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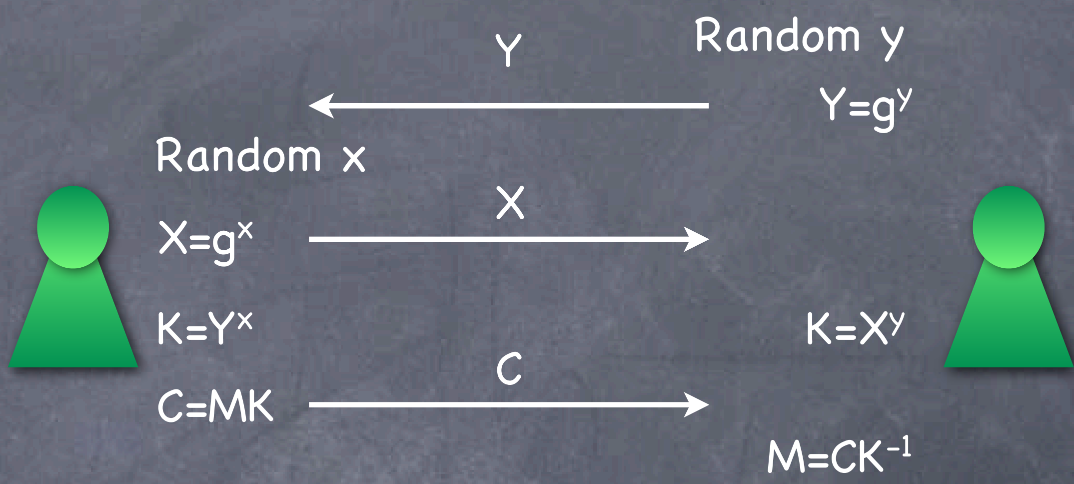

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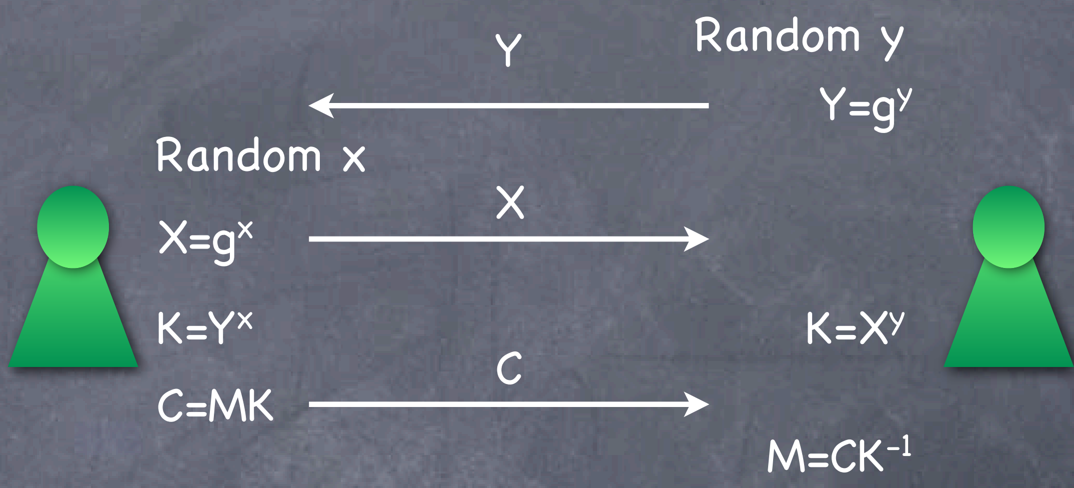

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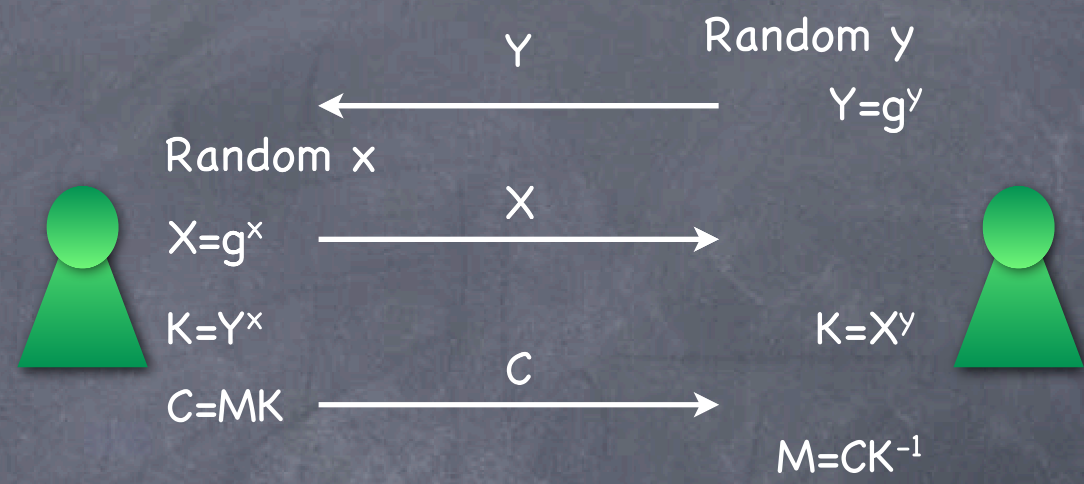

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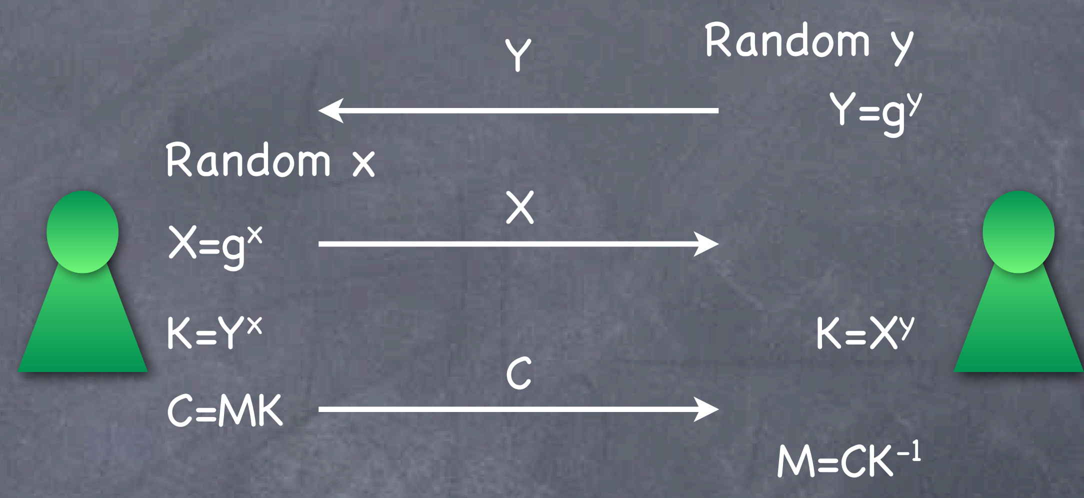

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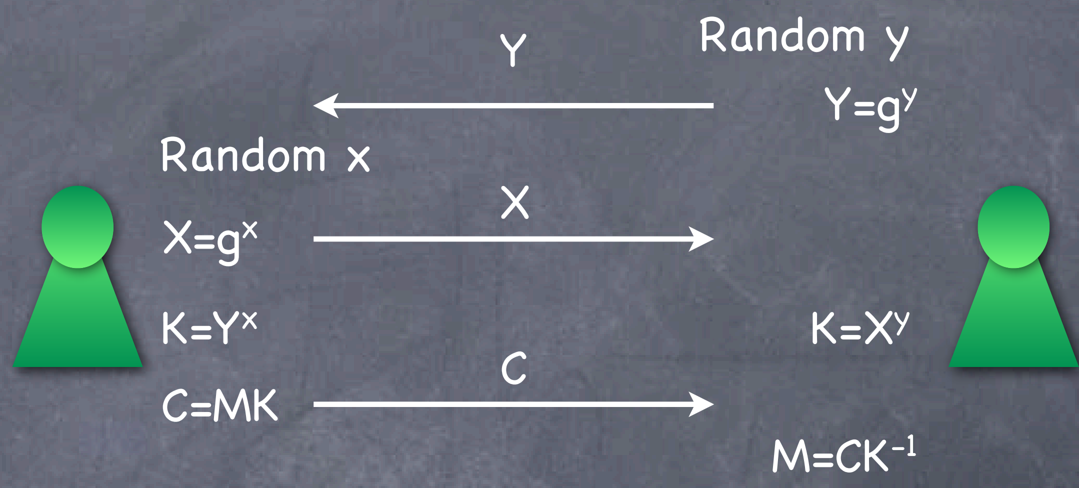

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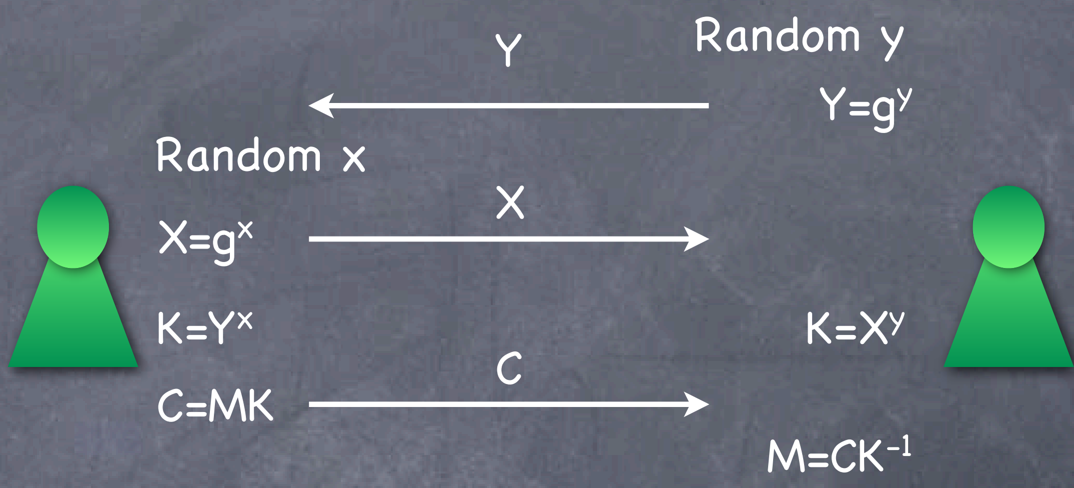

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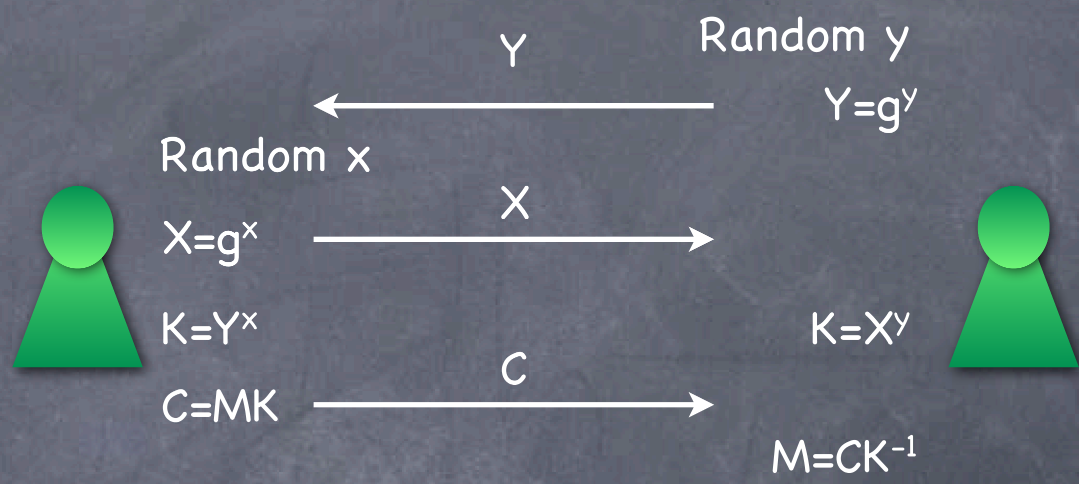

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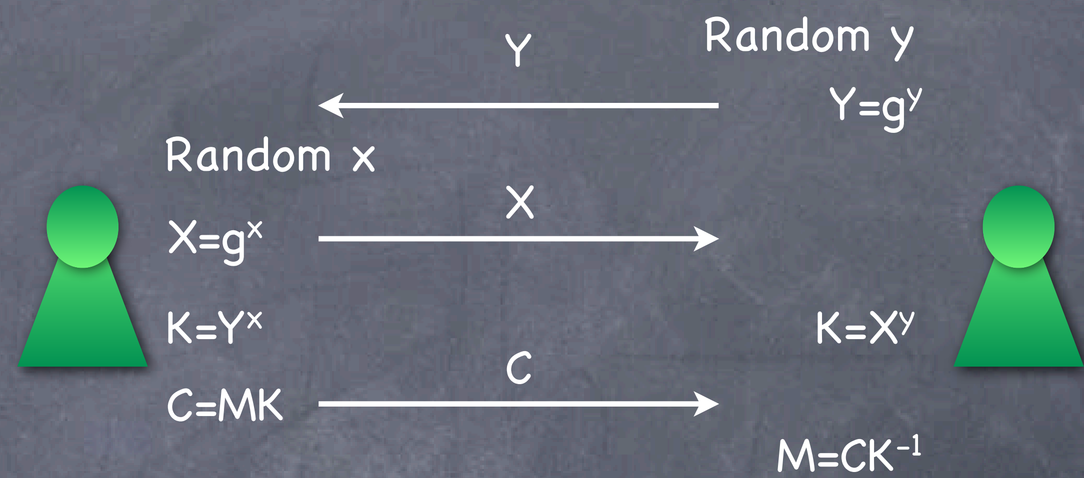

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- KeyGen uses GroupGen to get (G,g)
- x, y uniform from [|G|]
- Message encoded into group element, and decoded

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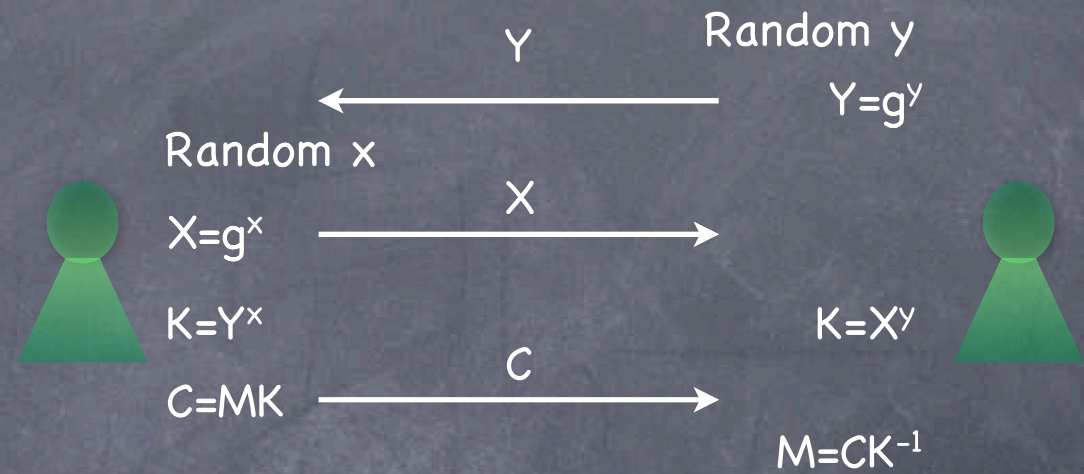
  - When z=random, A* outputs 1 with probability = 1/2
  - When z=xy, exactly IND-CPA experiment: A* outputs 1 with probability = 1/2 + advantage of A.

Abstracting El Gamal



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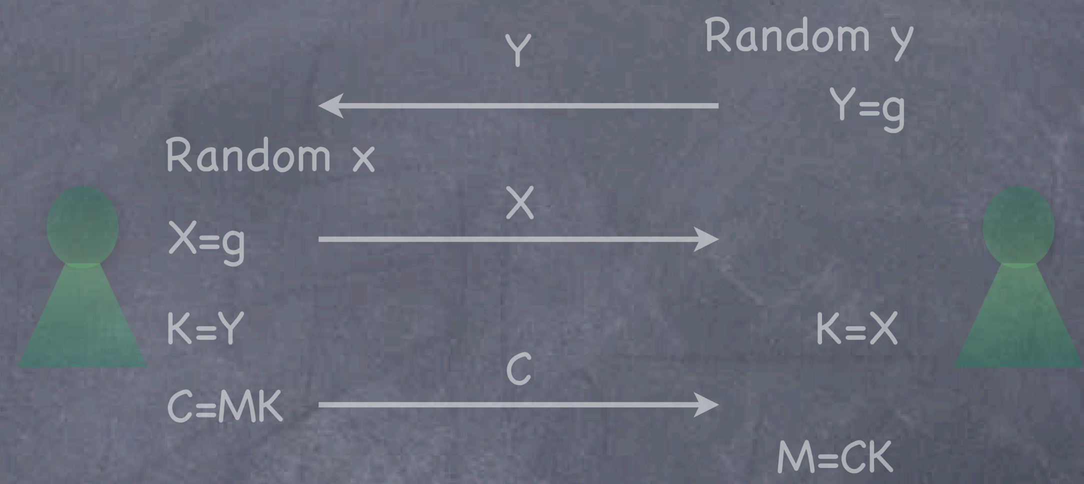

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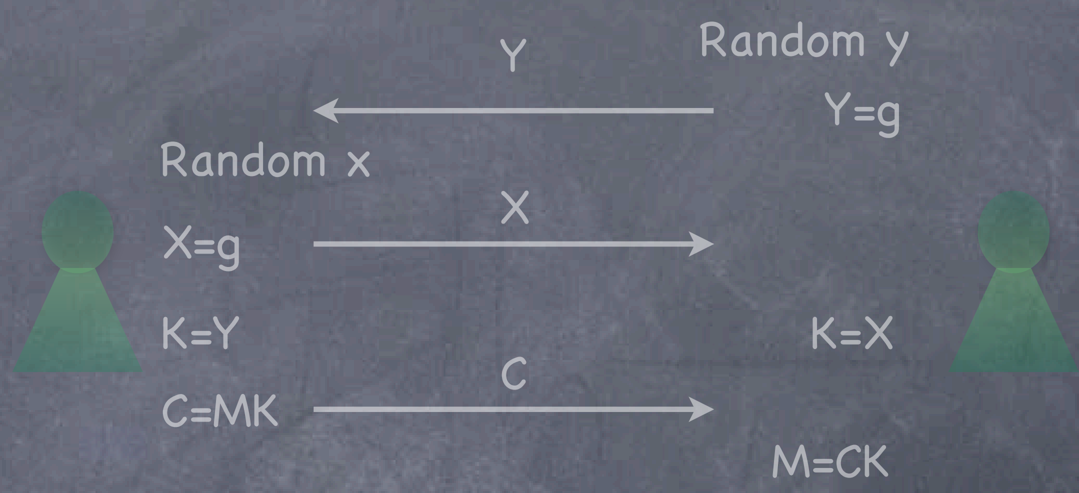
Abstracting El Gamal

- Trapdoor PRG:



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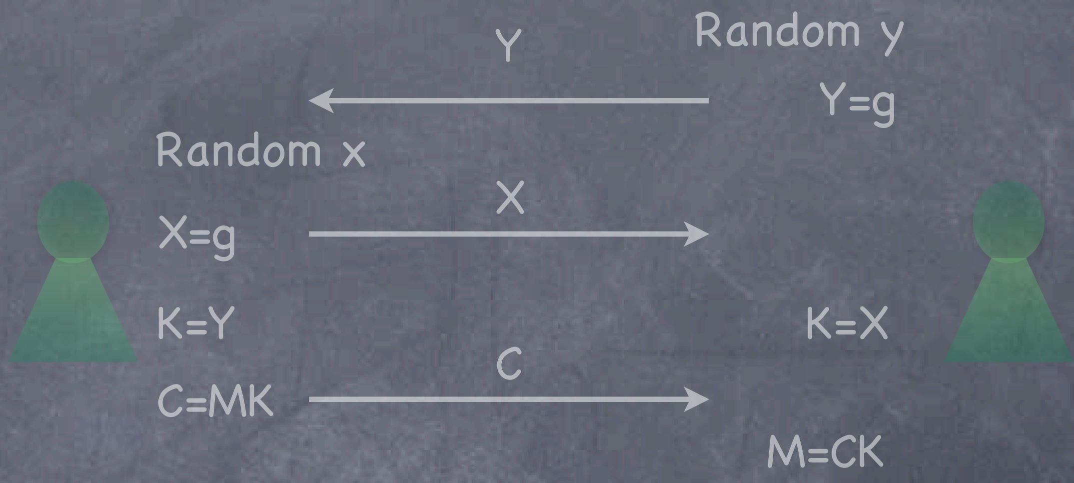

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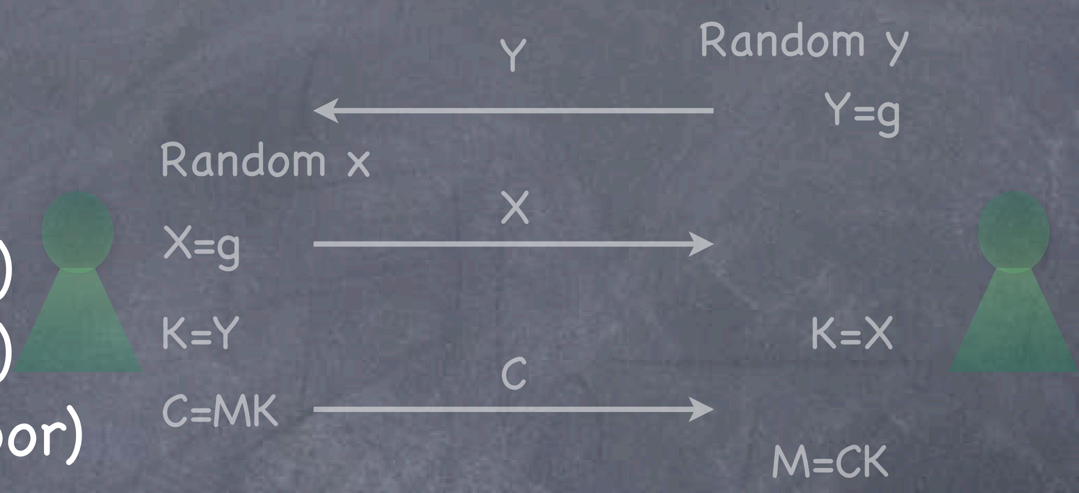

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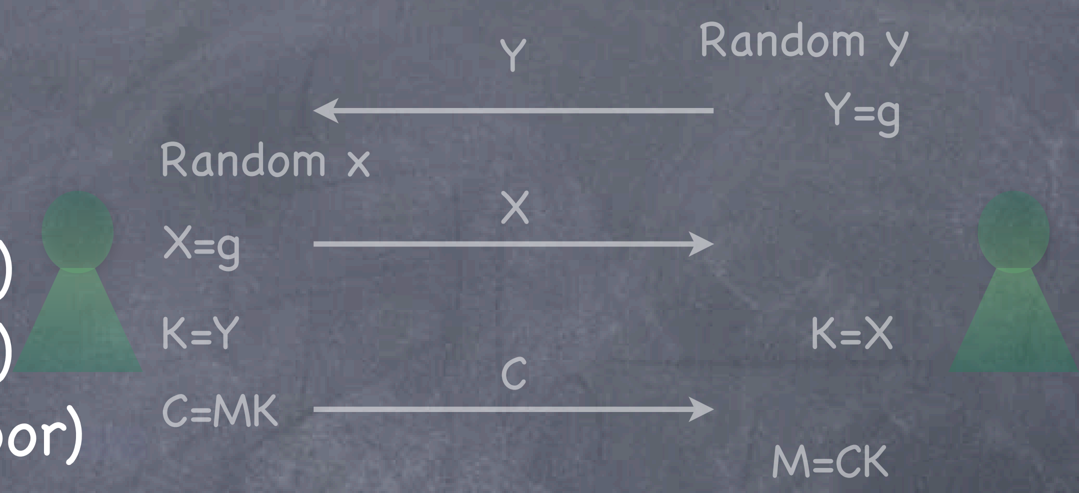

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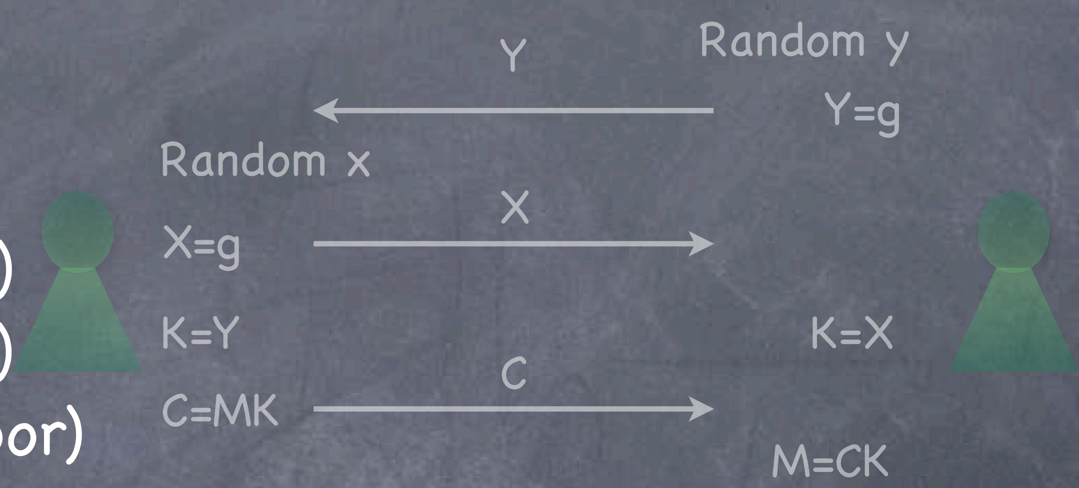

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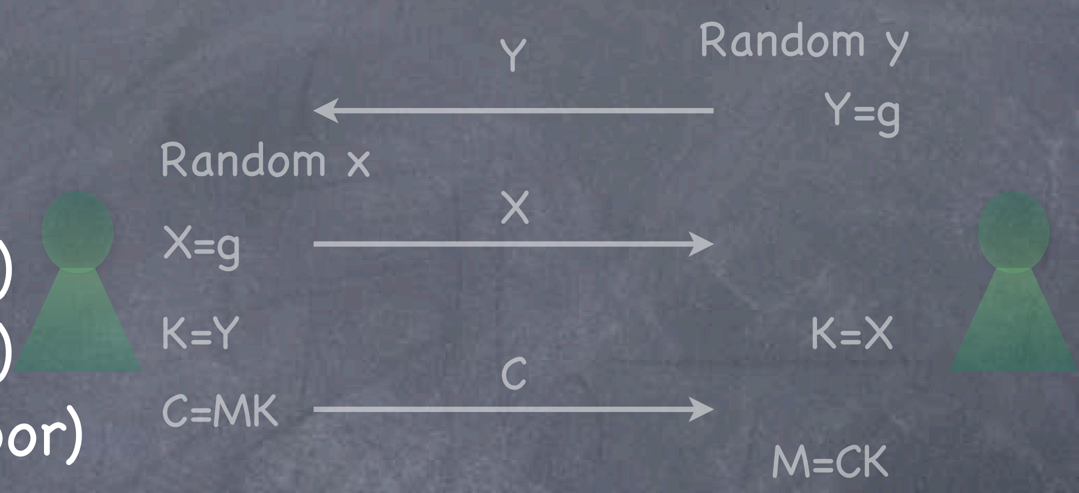

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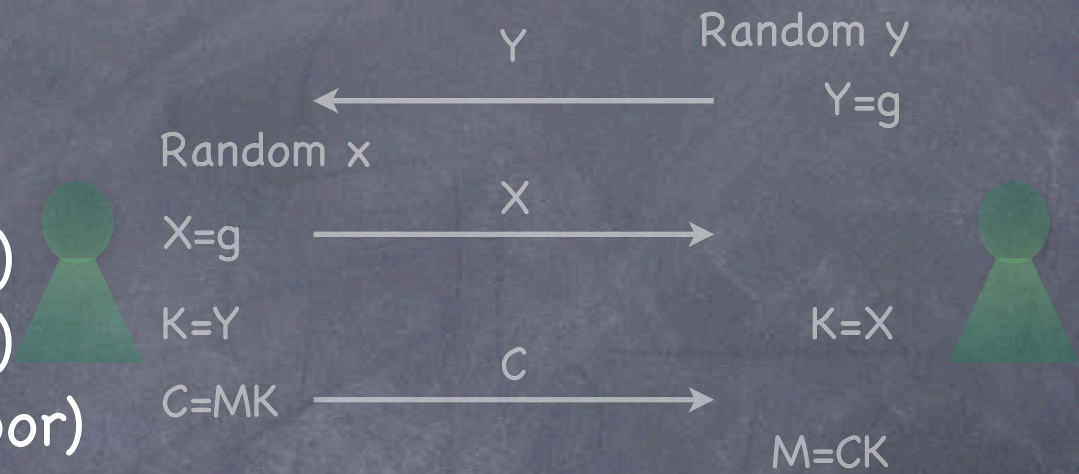

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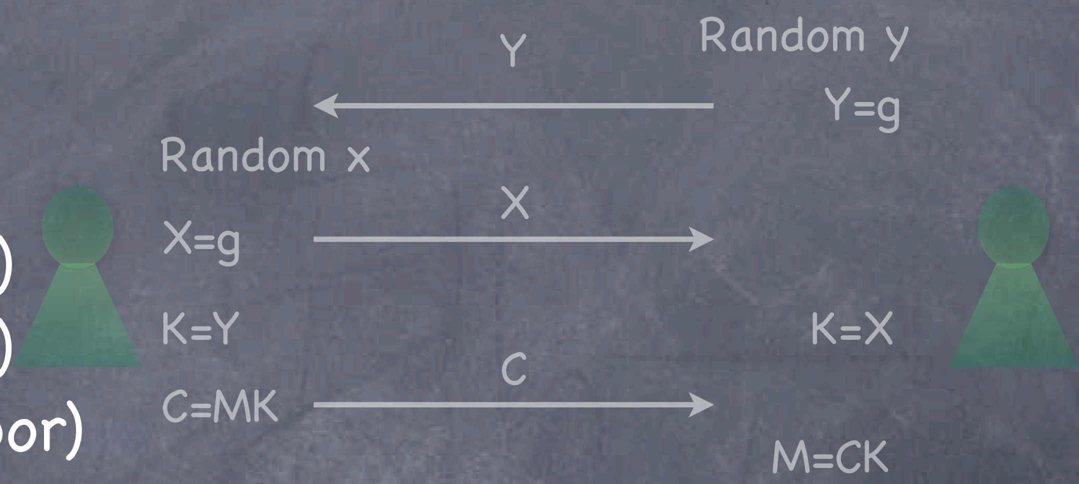

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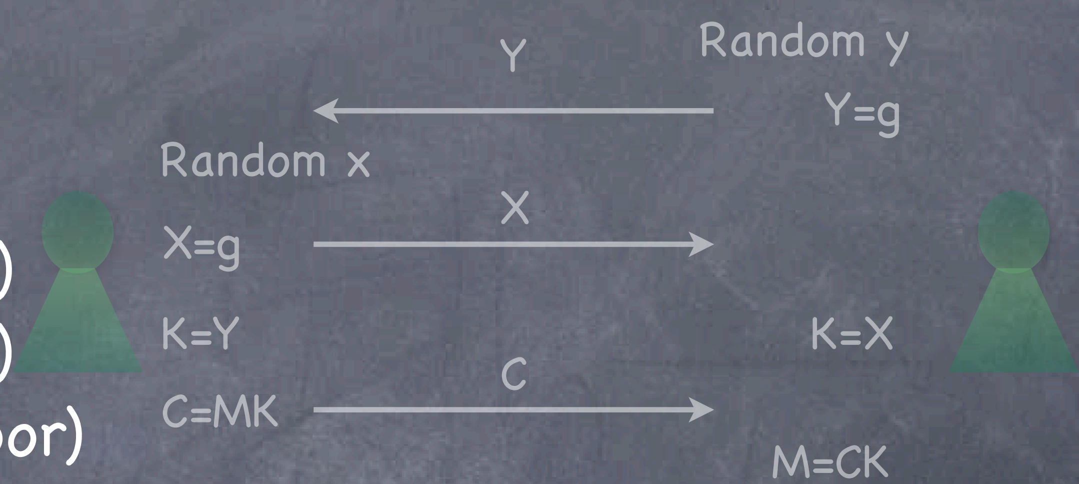

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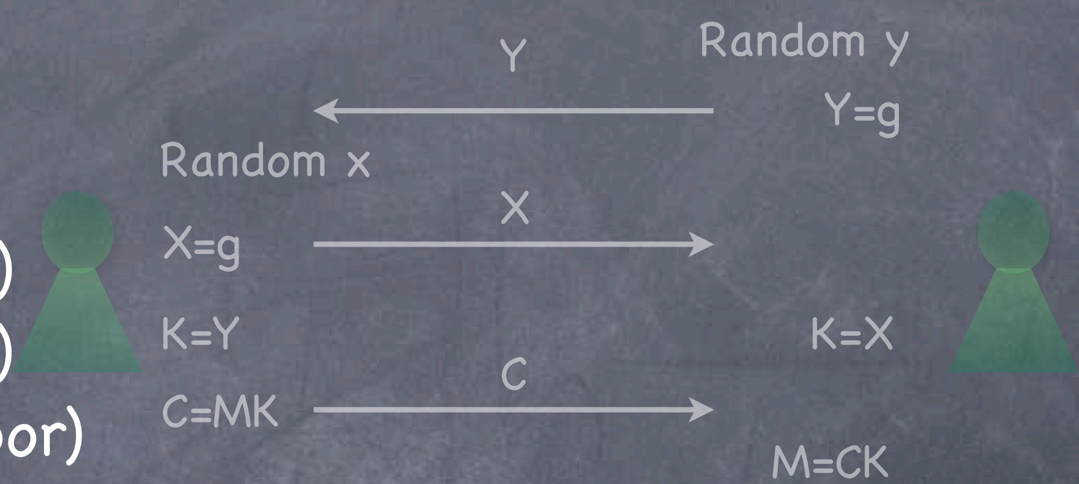

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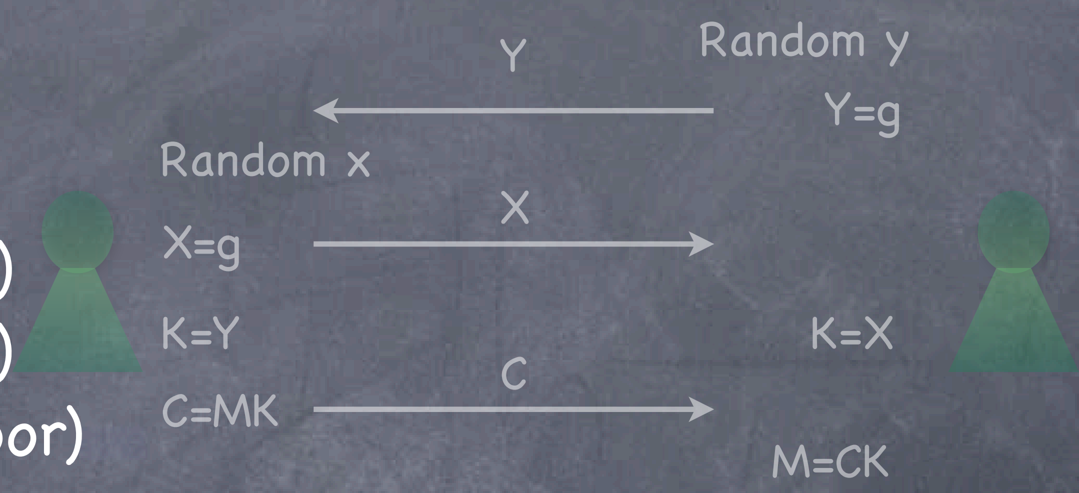
- Enough for an IND-CPA secure PKE scheme (cf. Security of El Gamal)



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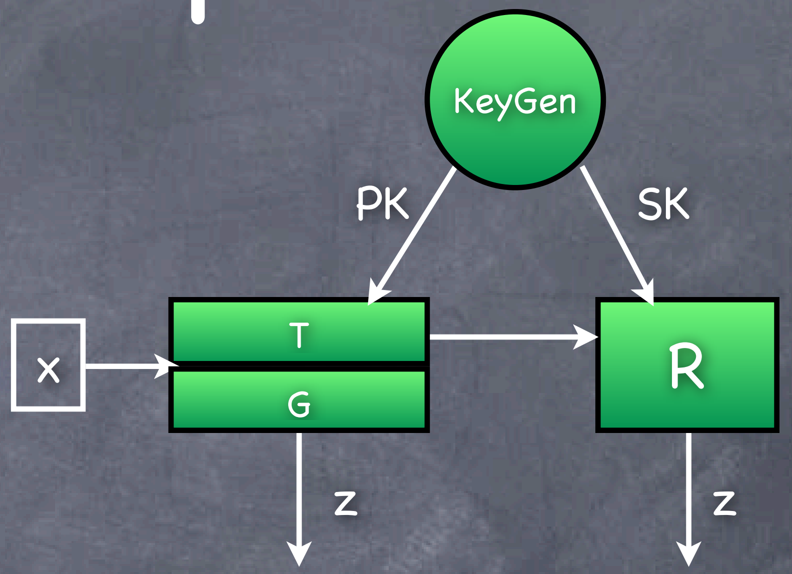
Trapdoor PRG from Generic Assumption?



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Trapdoor PRG from Generic Assumption?

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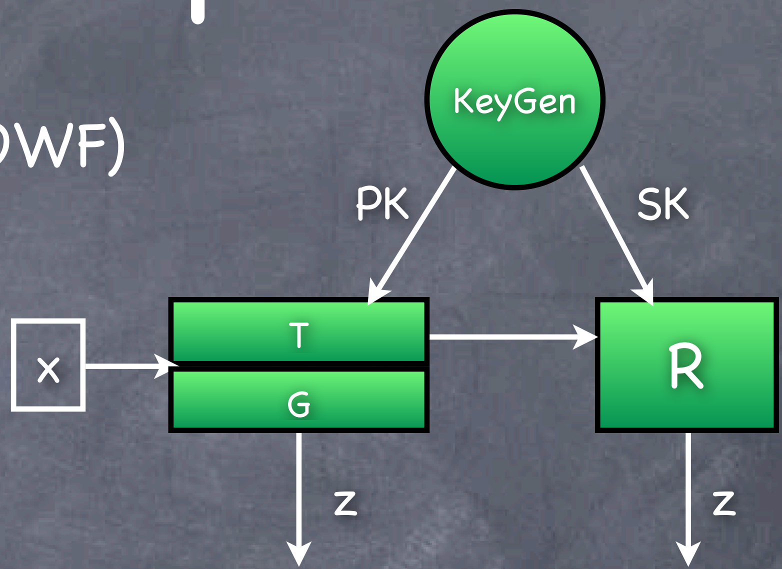

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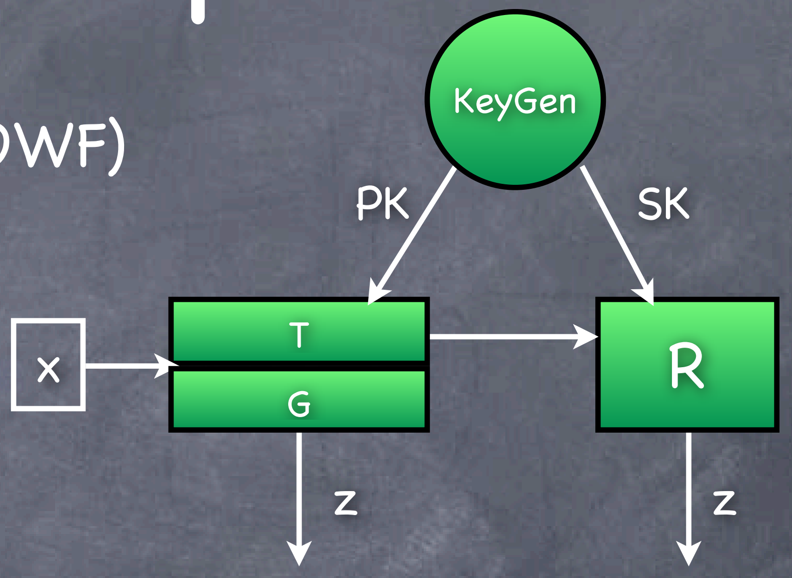

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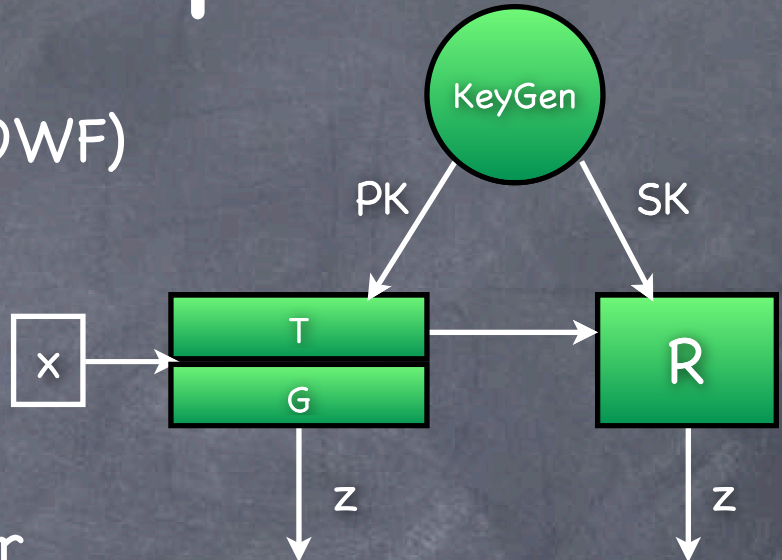

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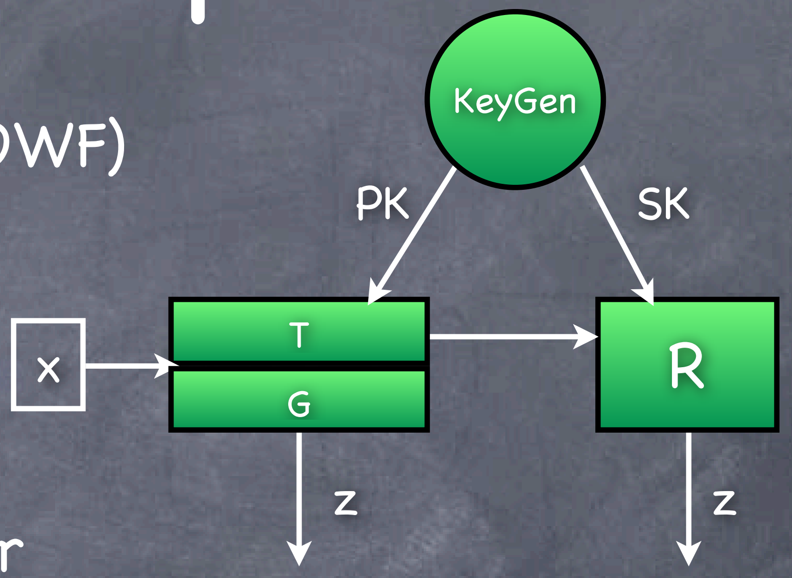

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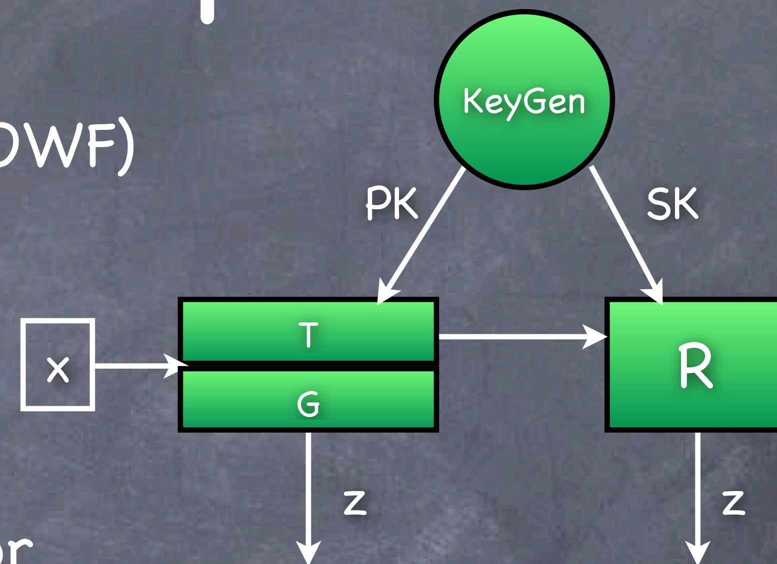
  - Will start with "Trapdoor OWP"



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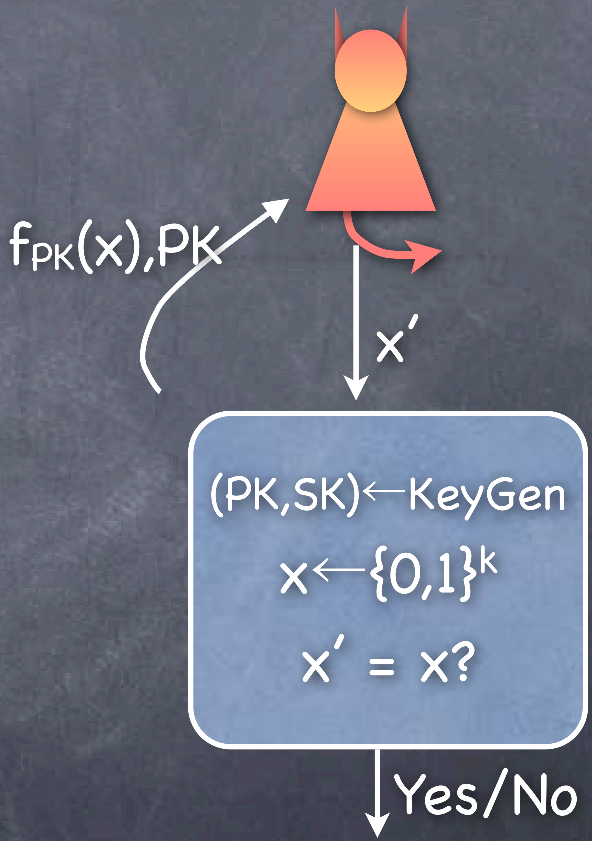

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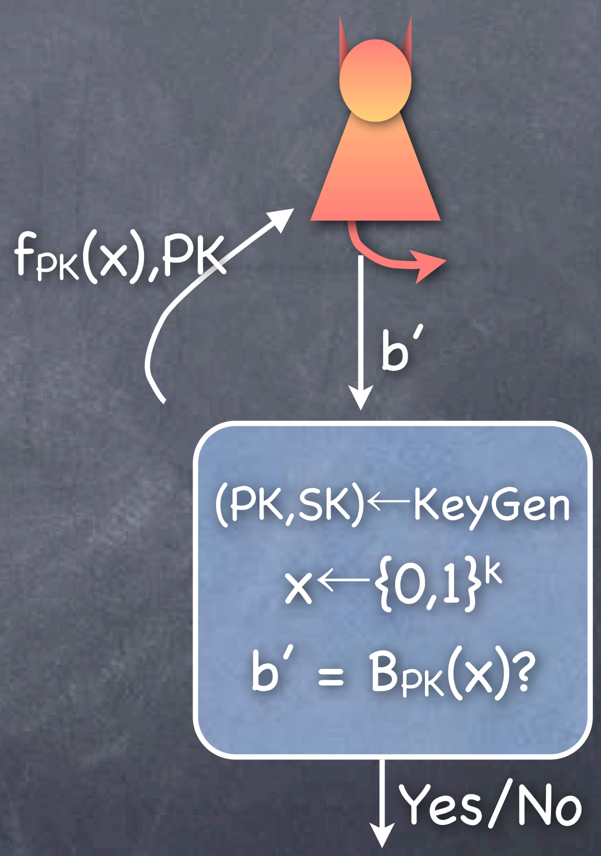

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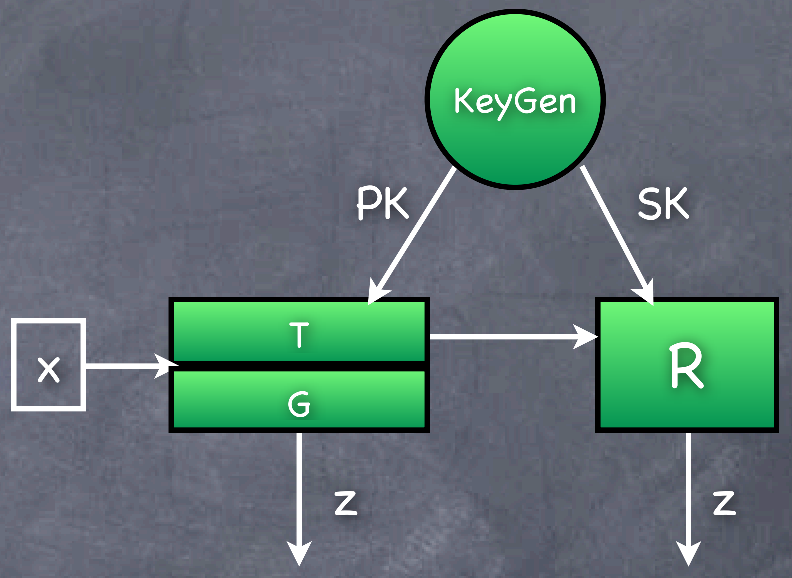

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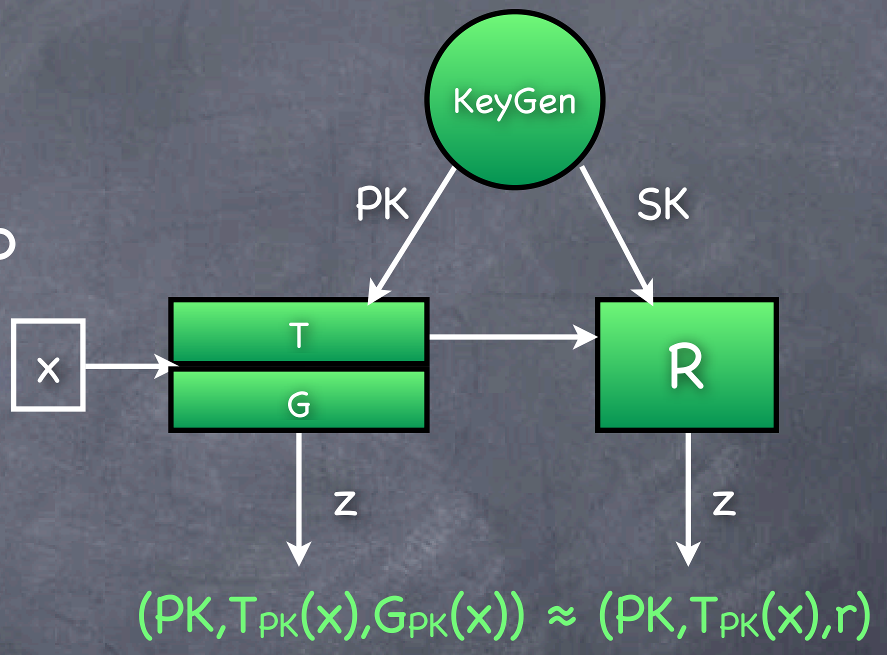

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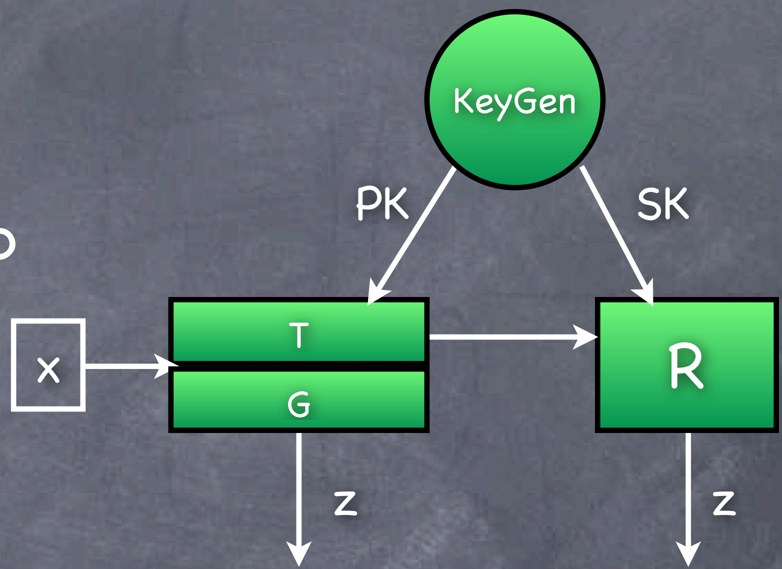

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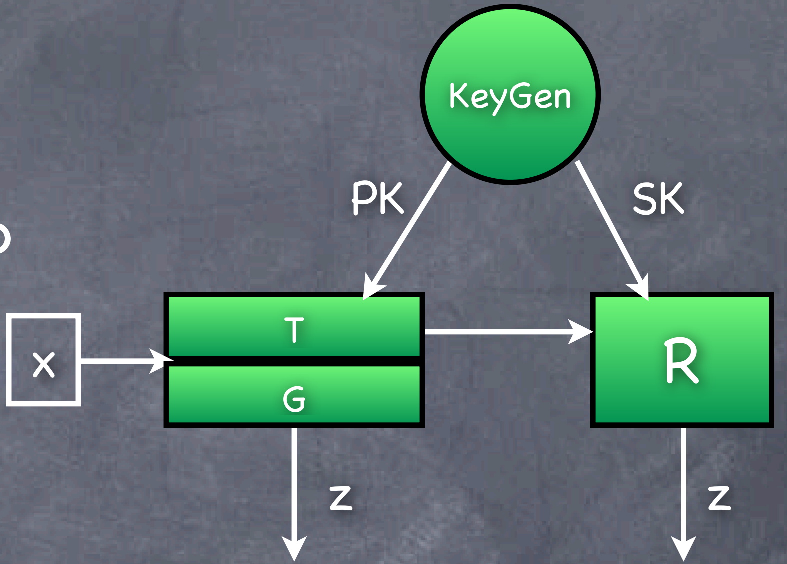

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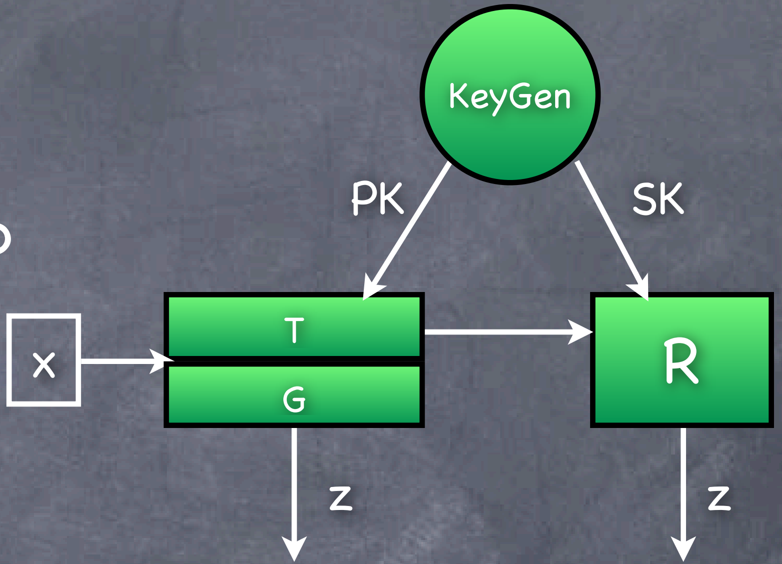

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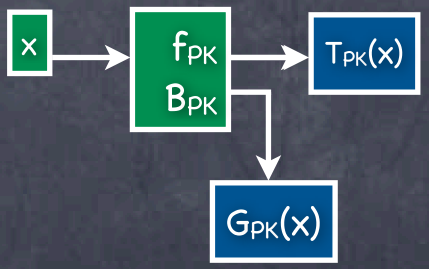

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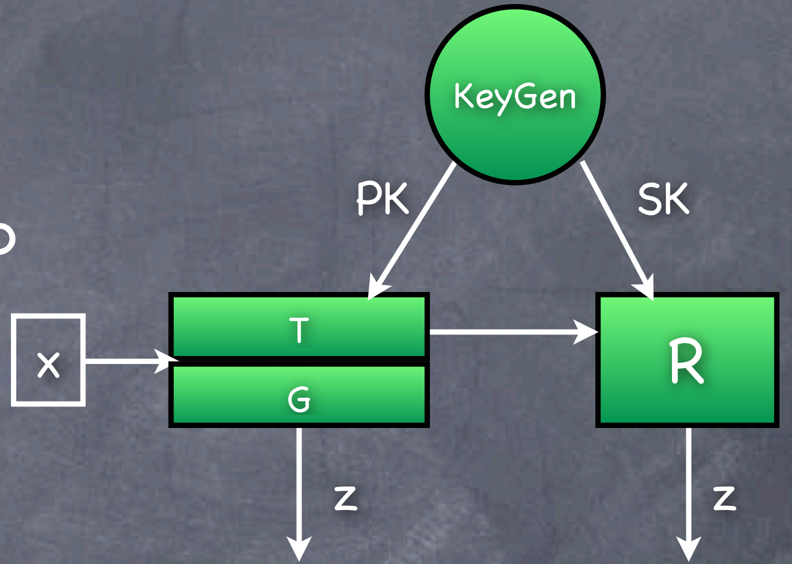

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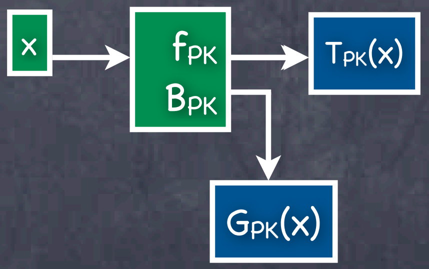

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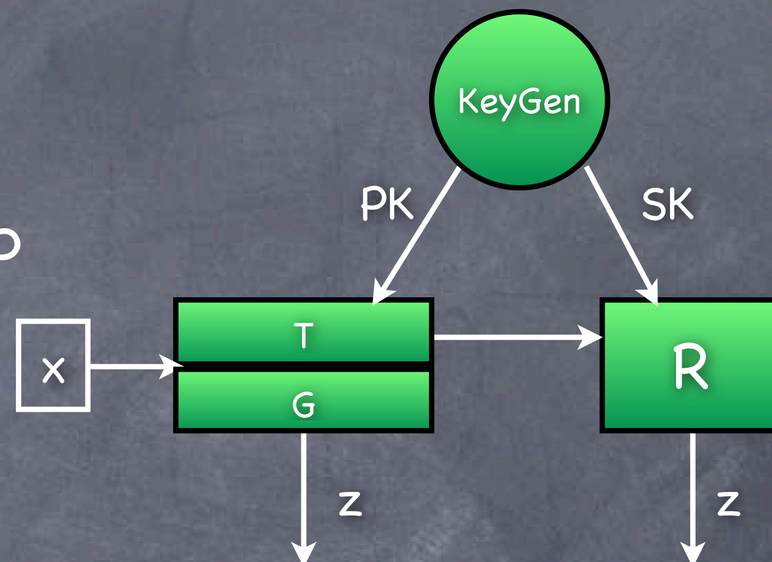

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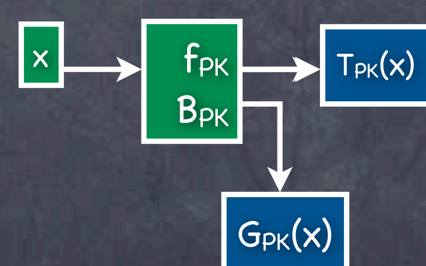

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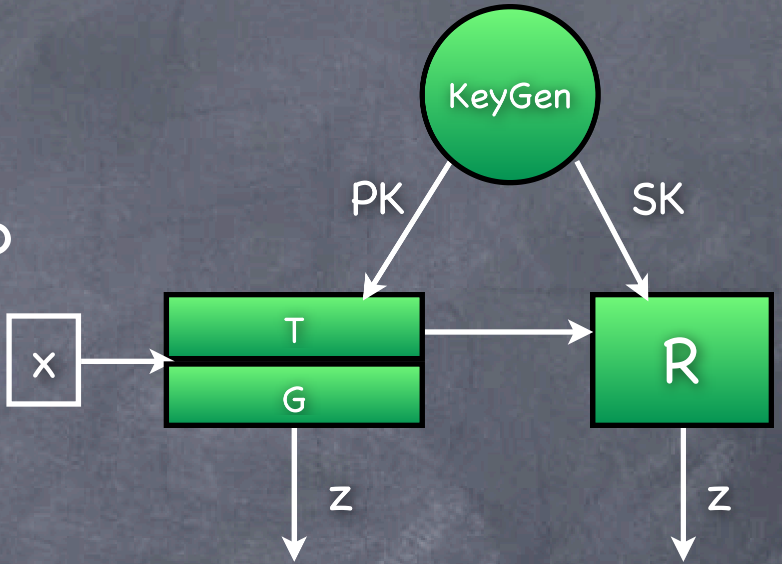

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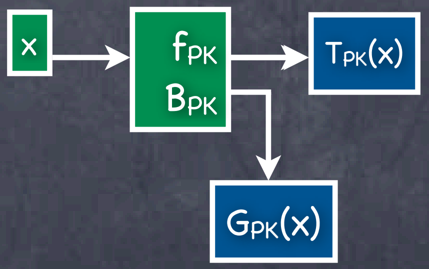

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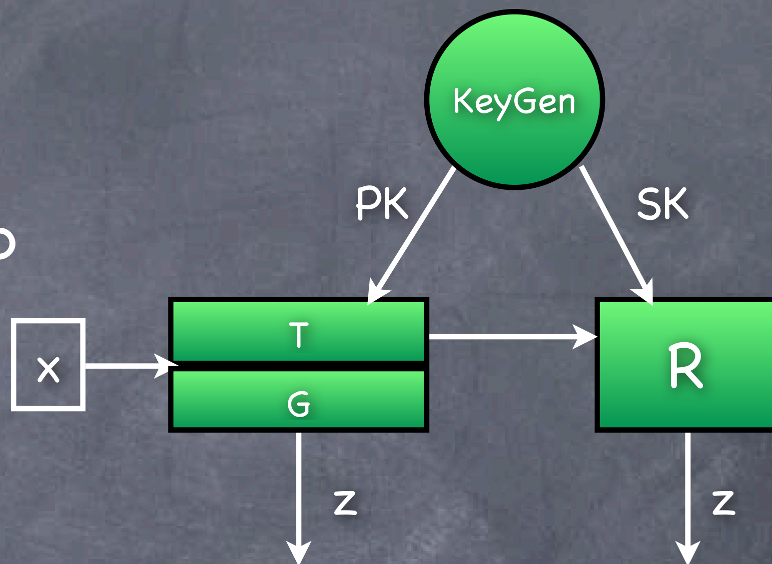

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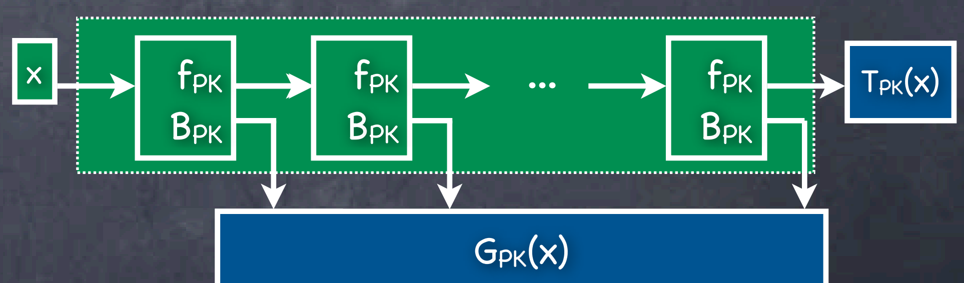

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see handout

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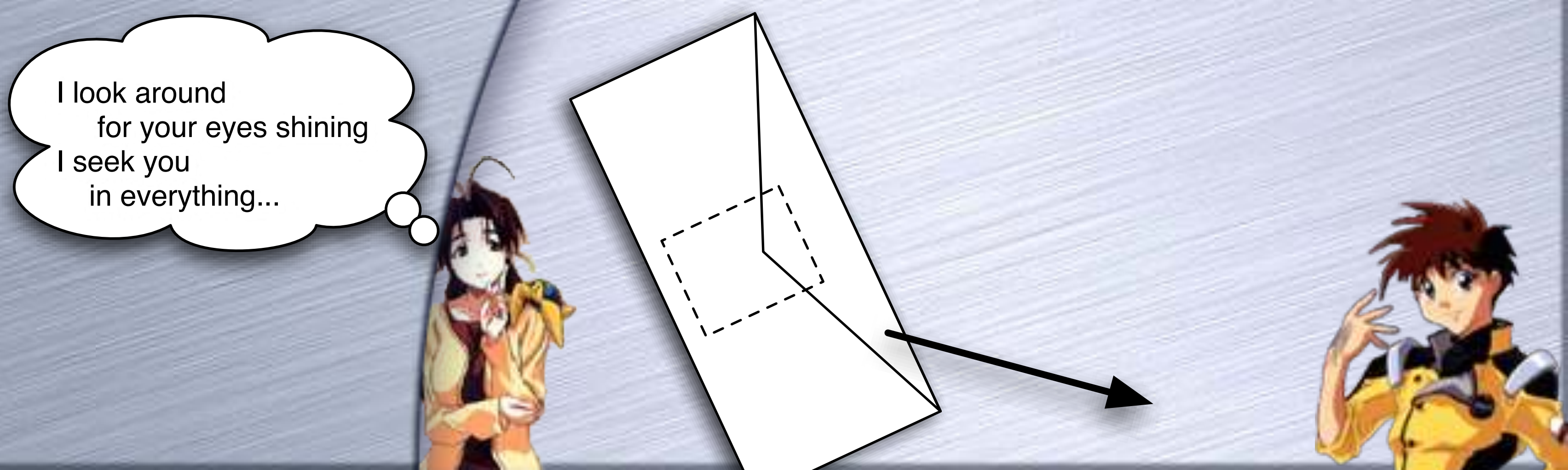

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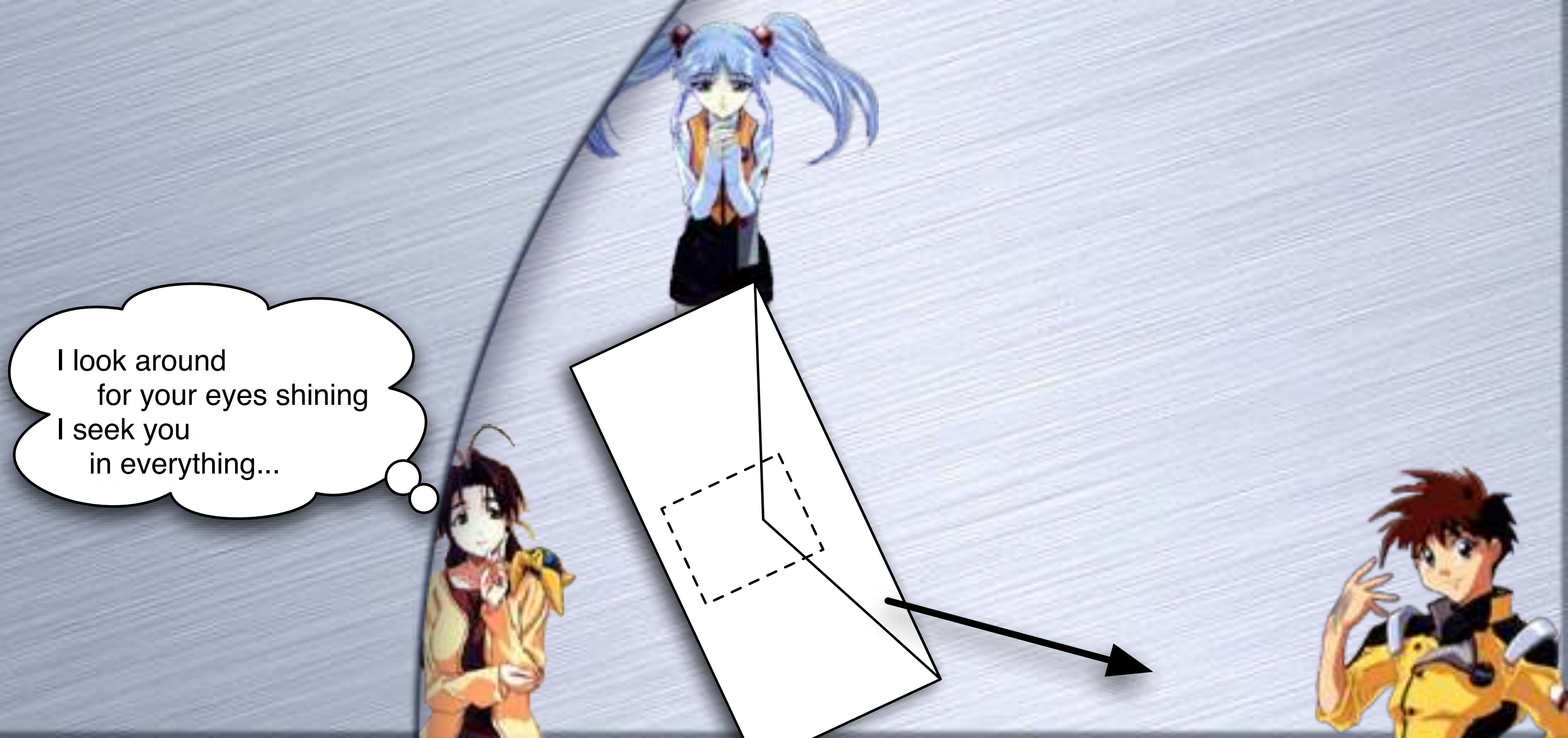

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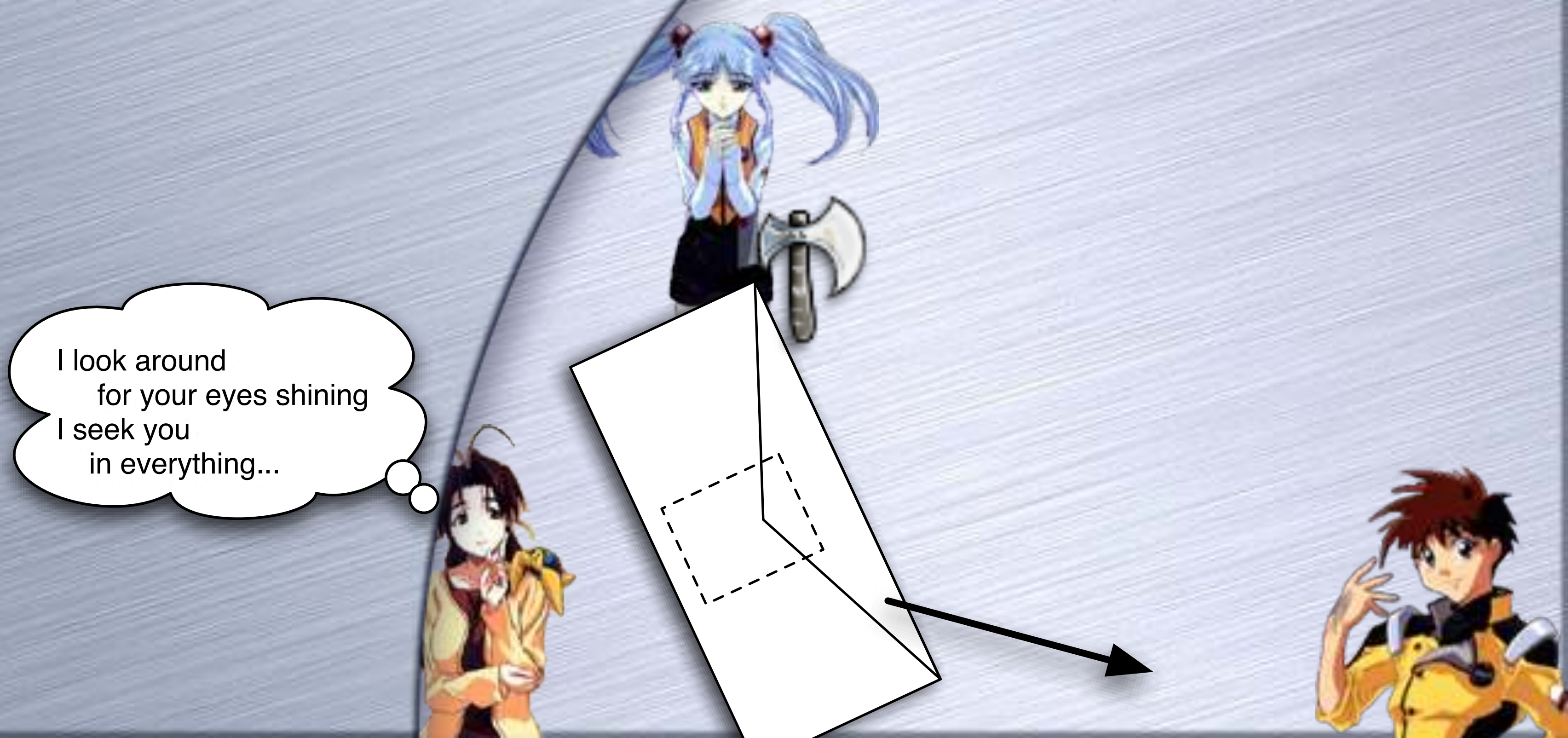

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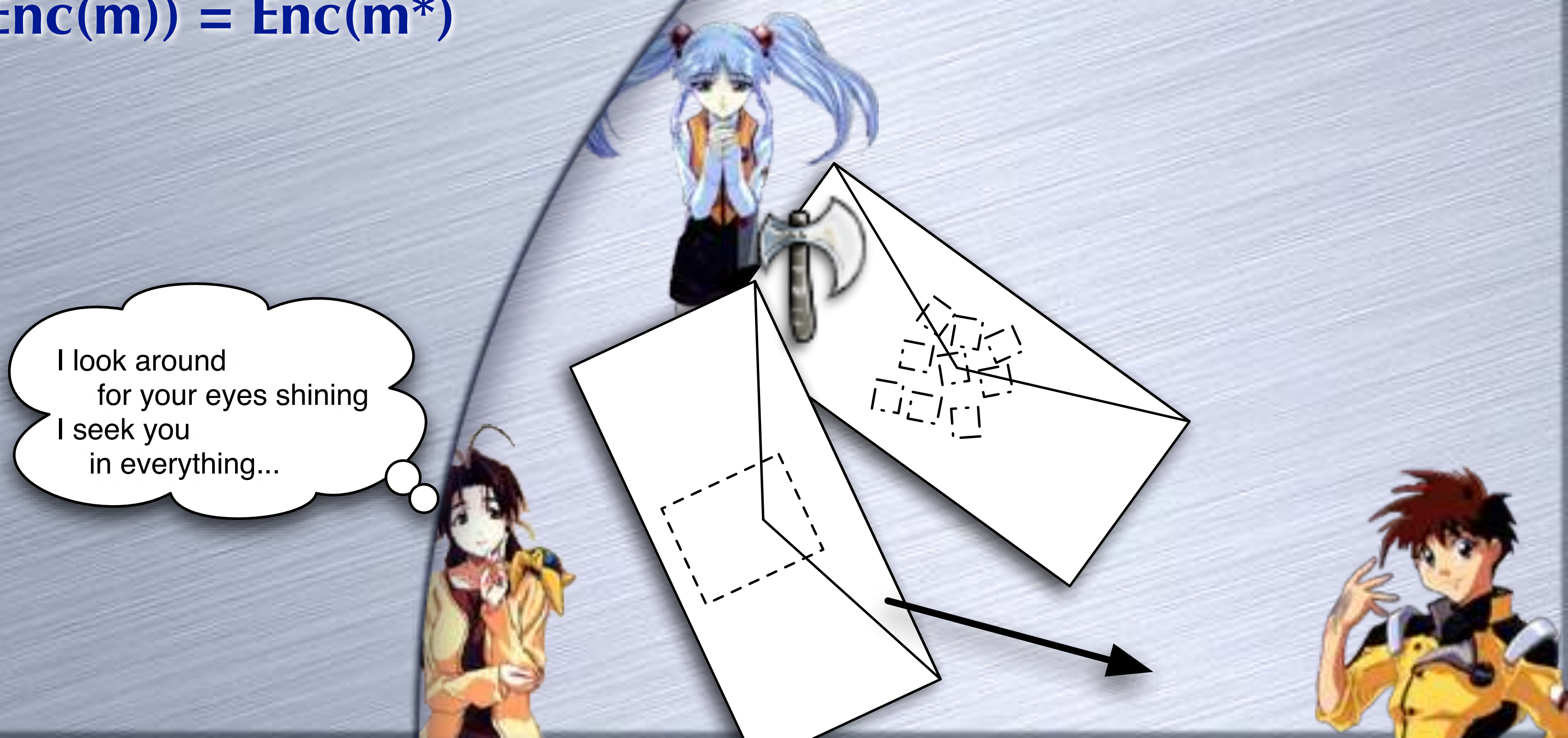

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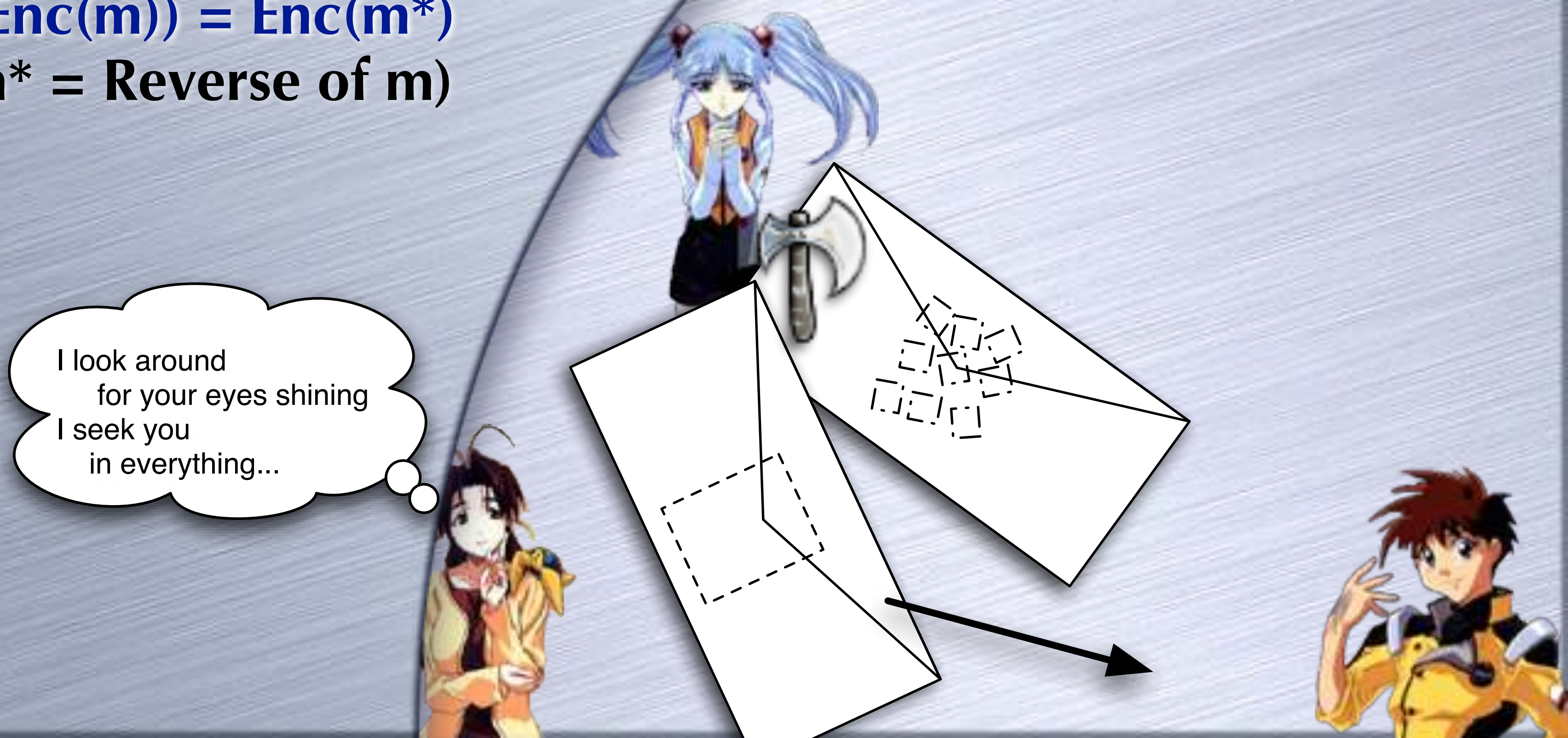

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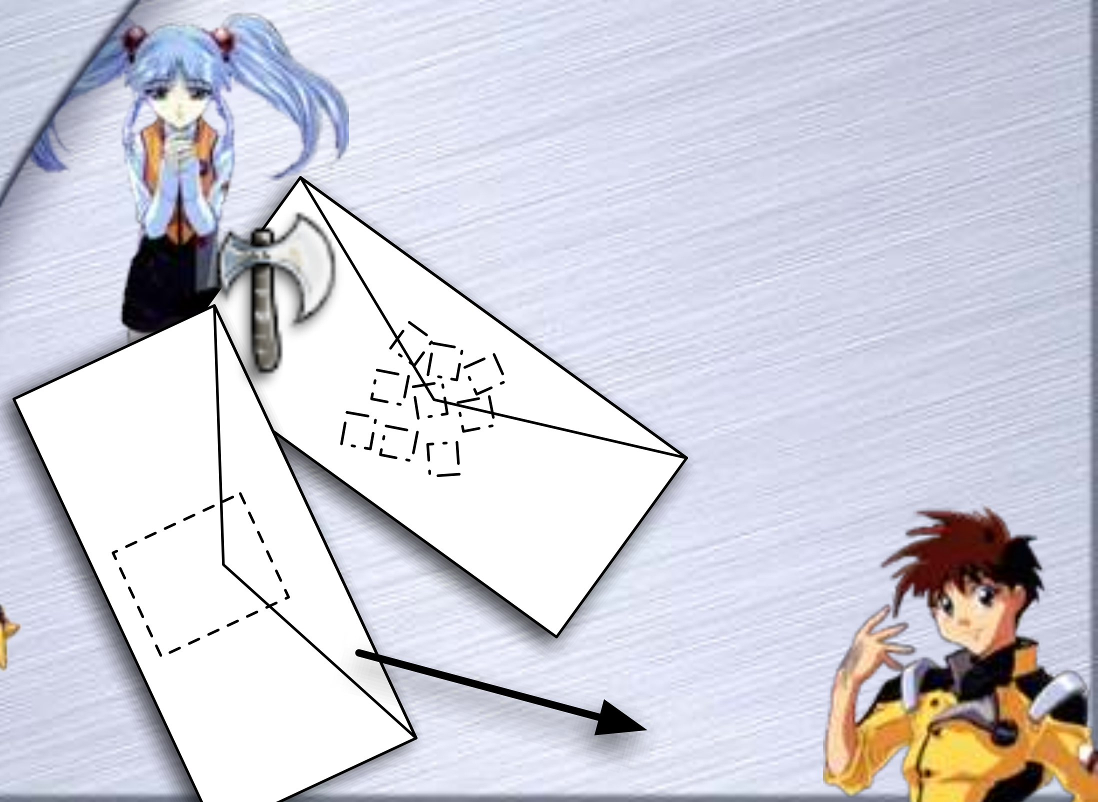

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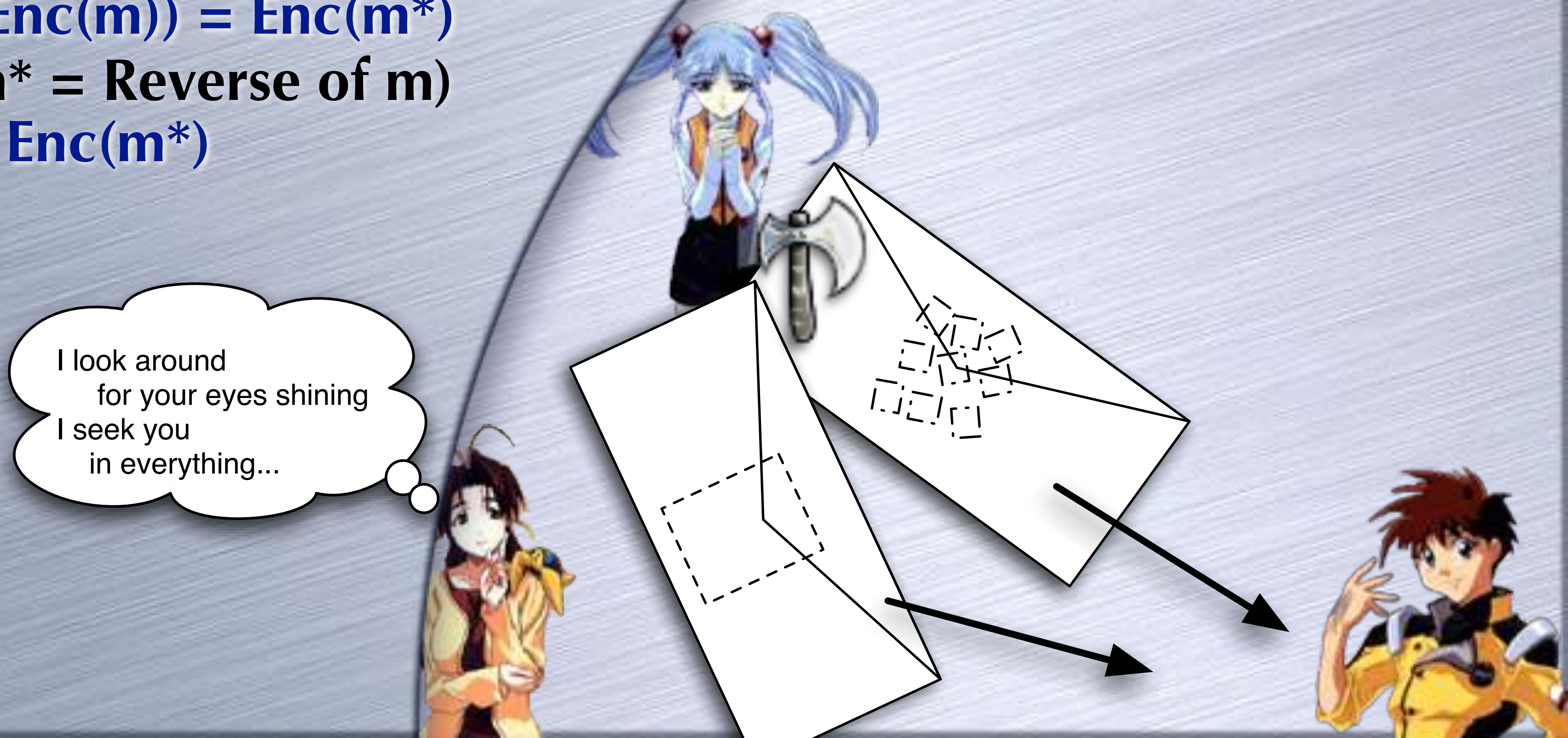

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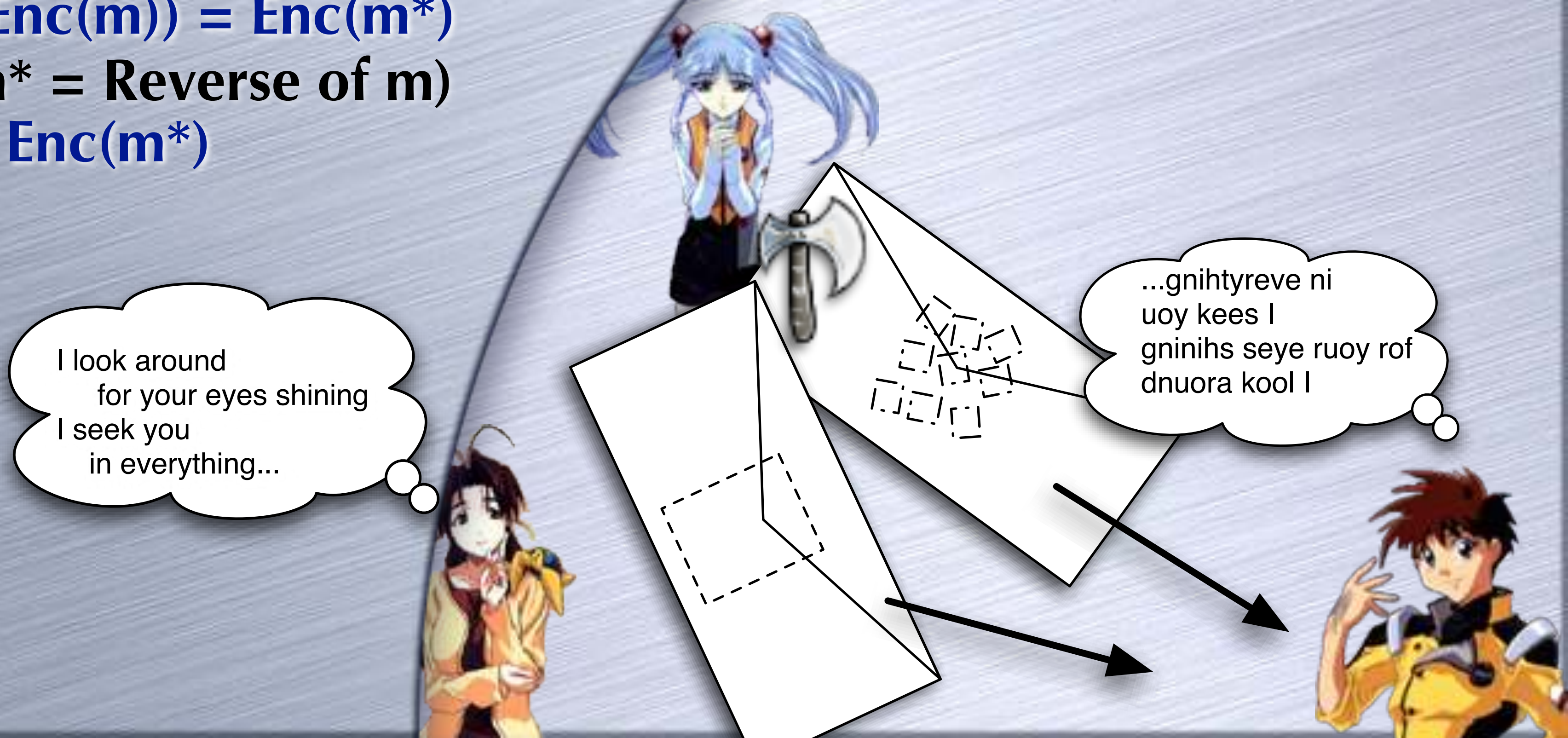

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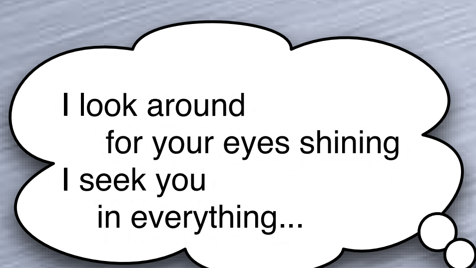

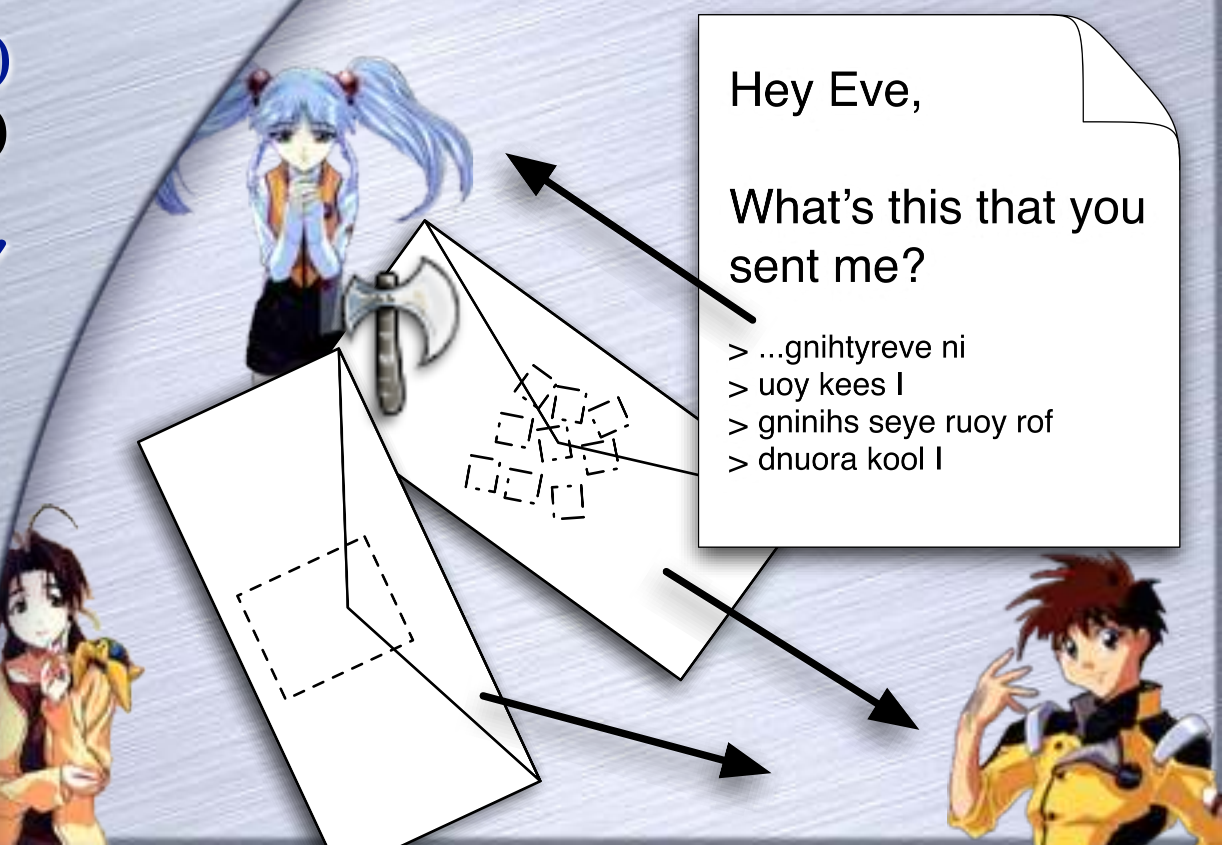

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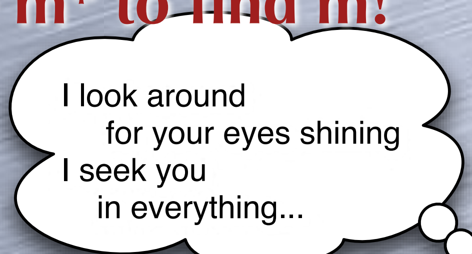

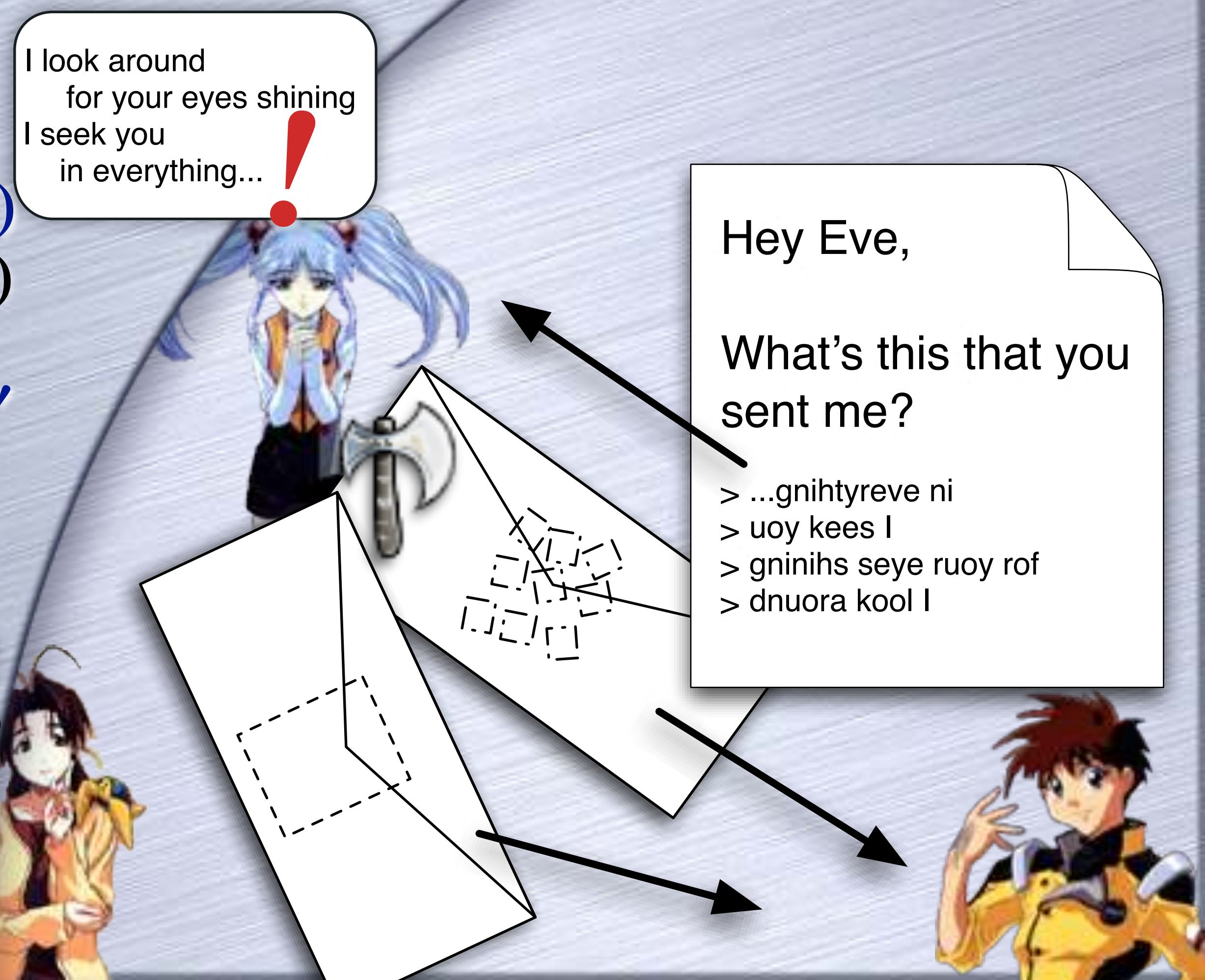

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More subtly, the 1 bit - valid or invalid - may leak information on message or SK

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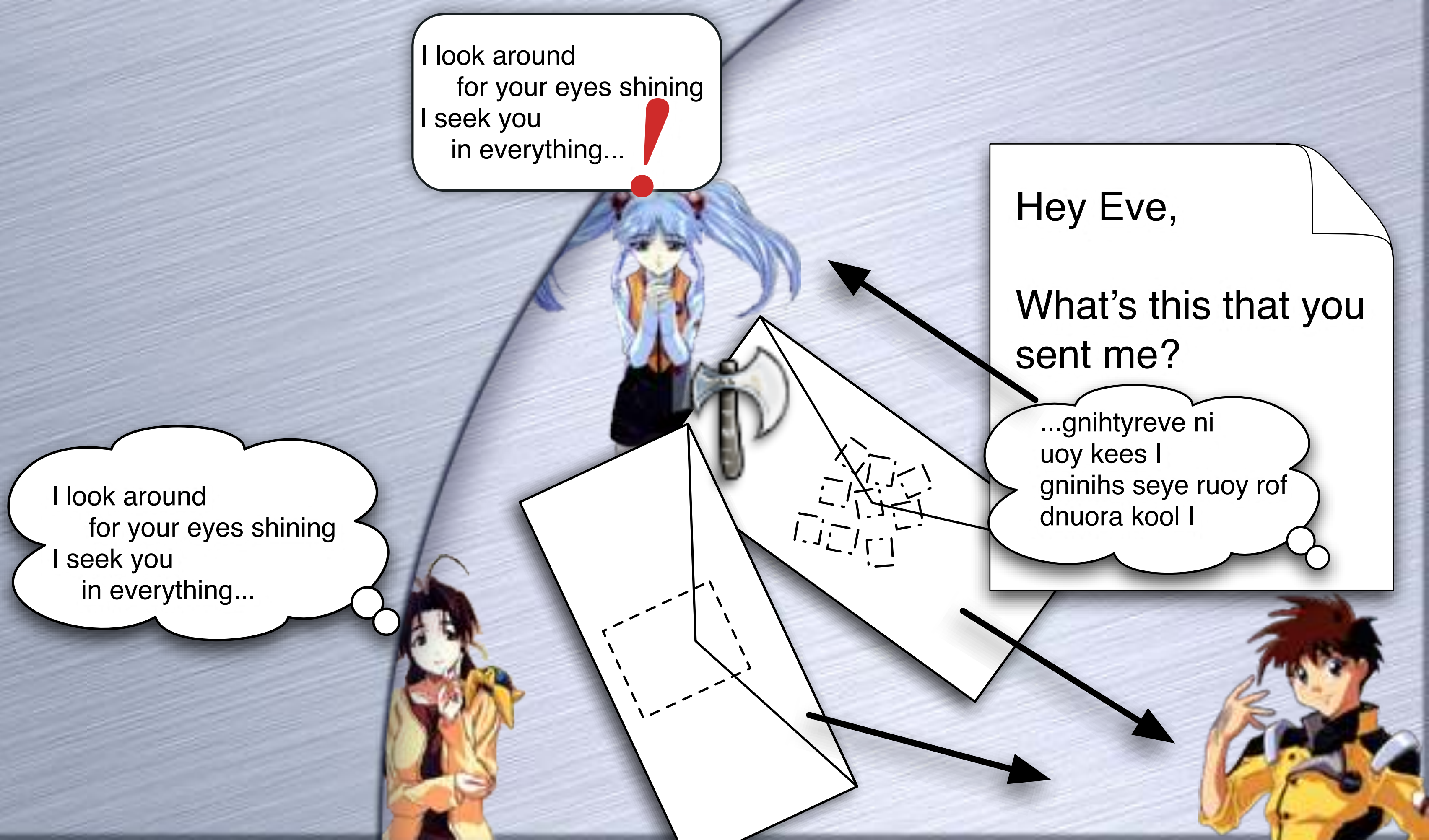

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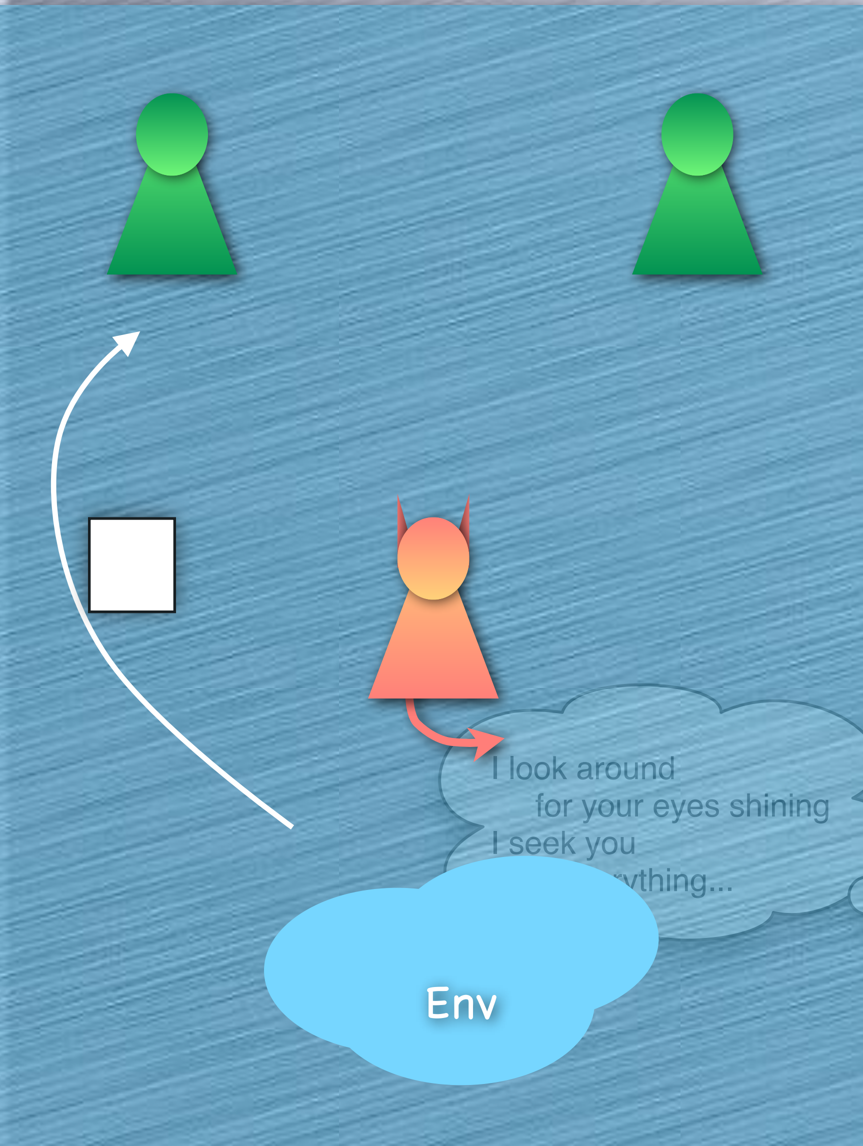

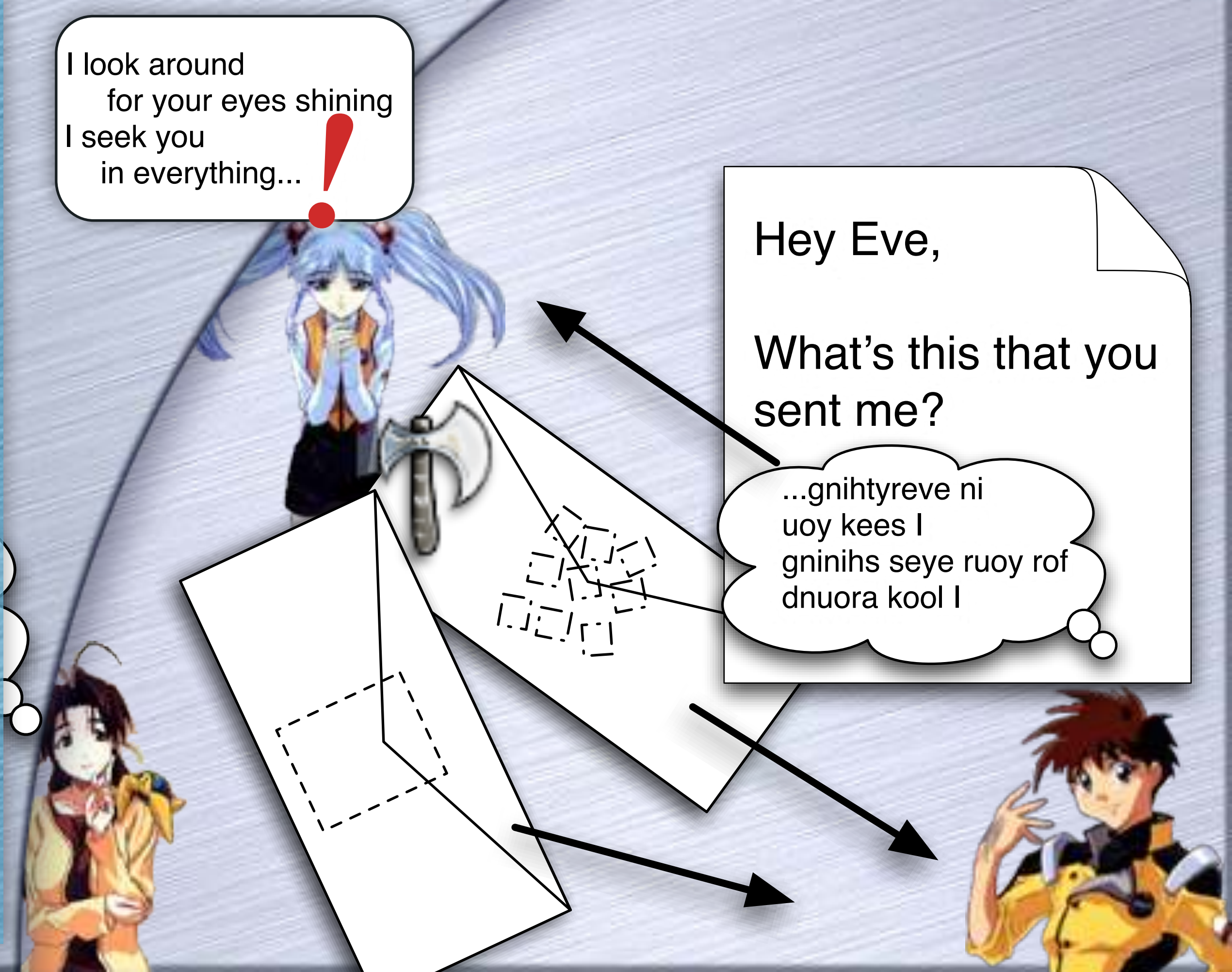

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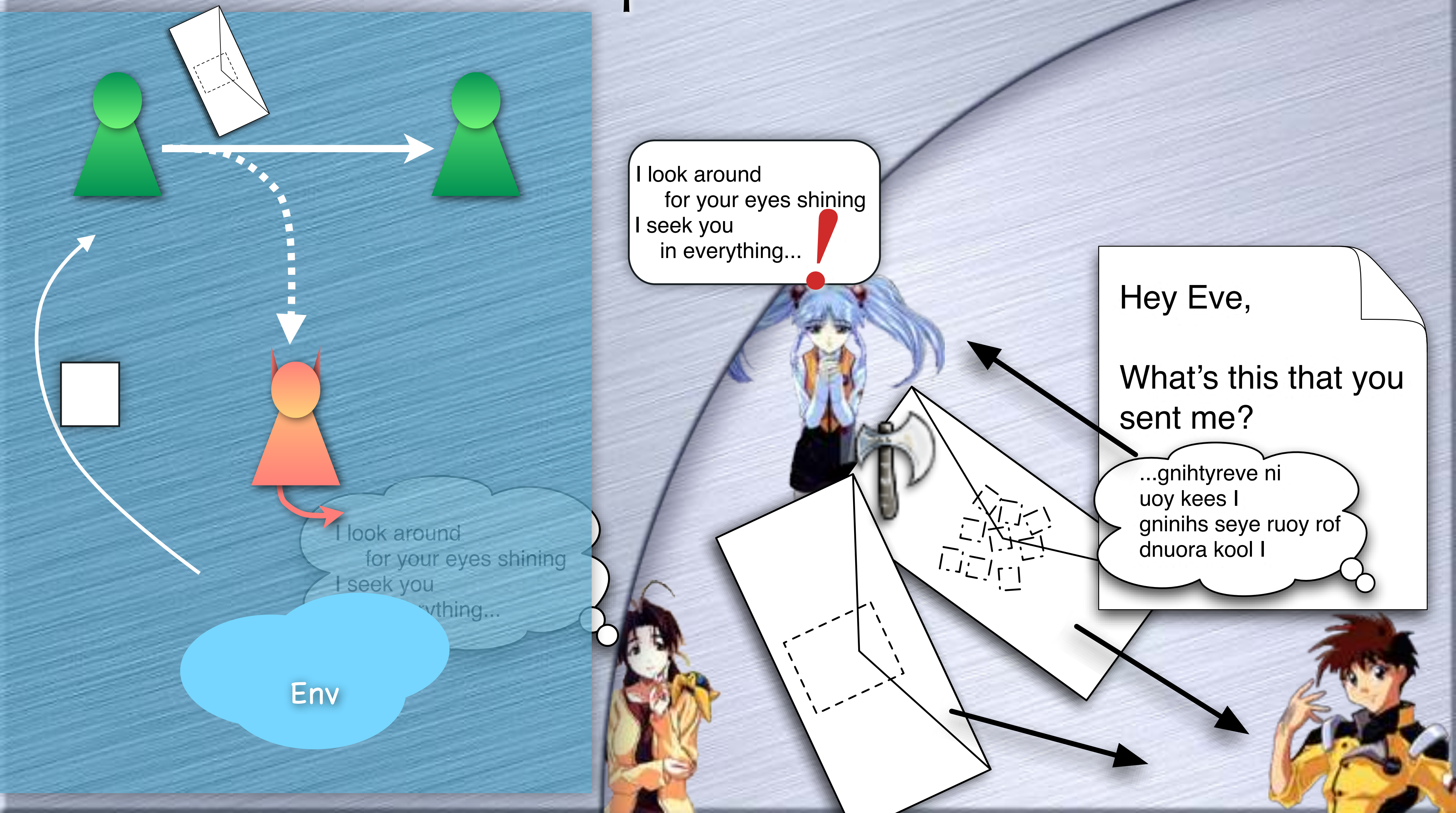

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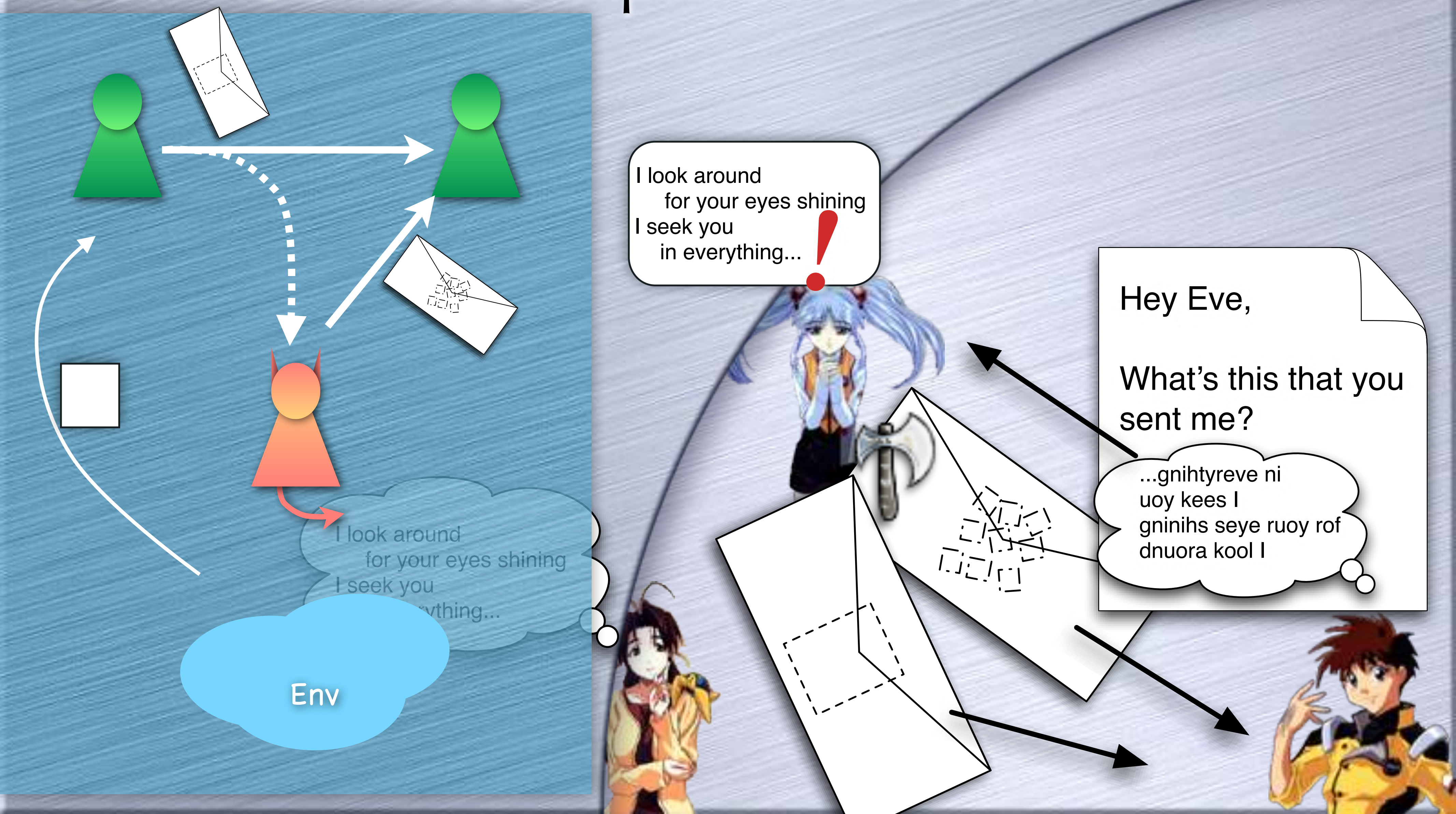

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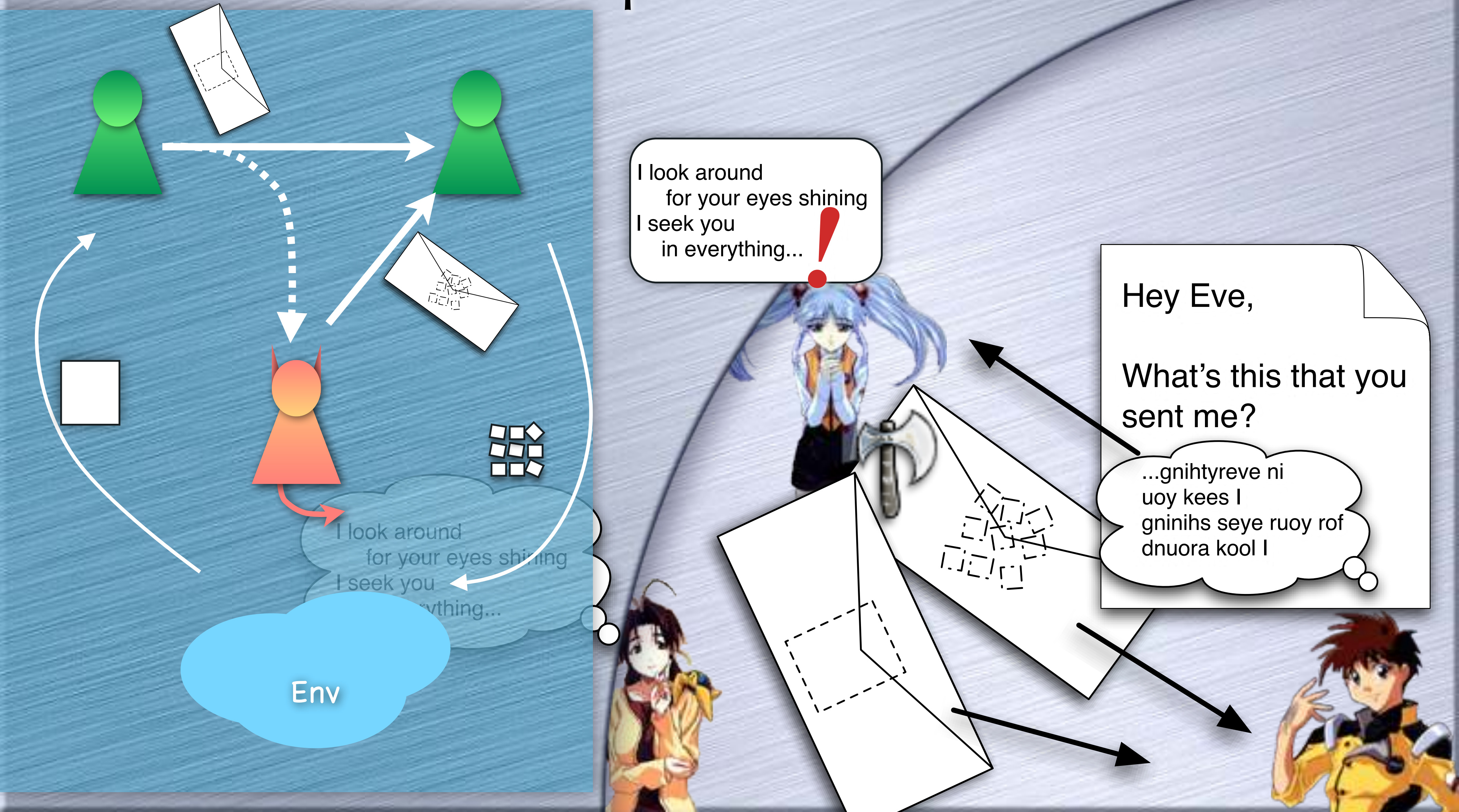

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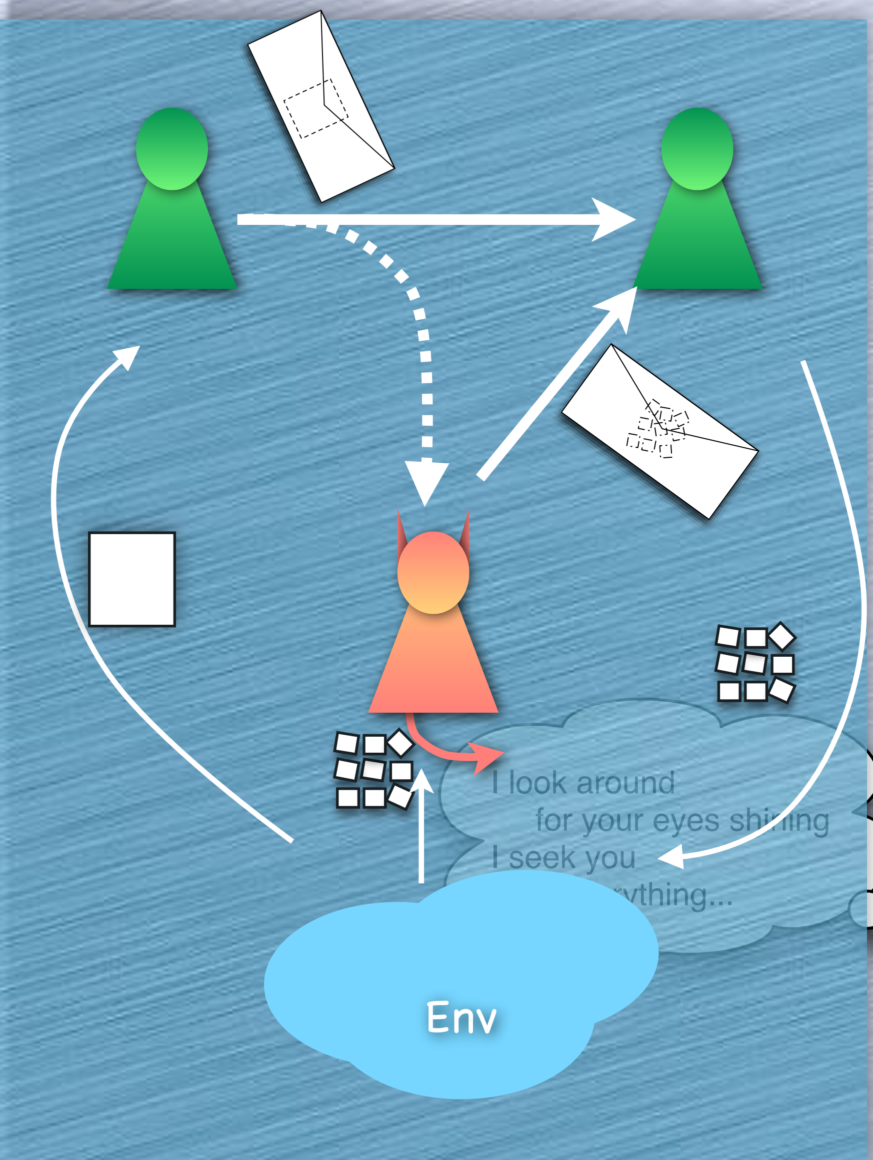

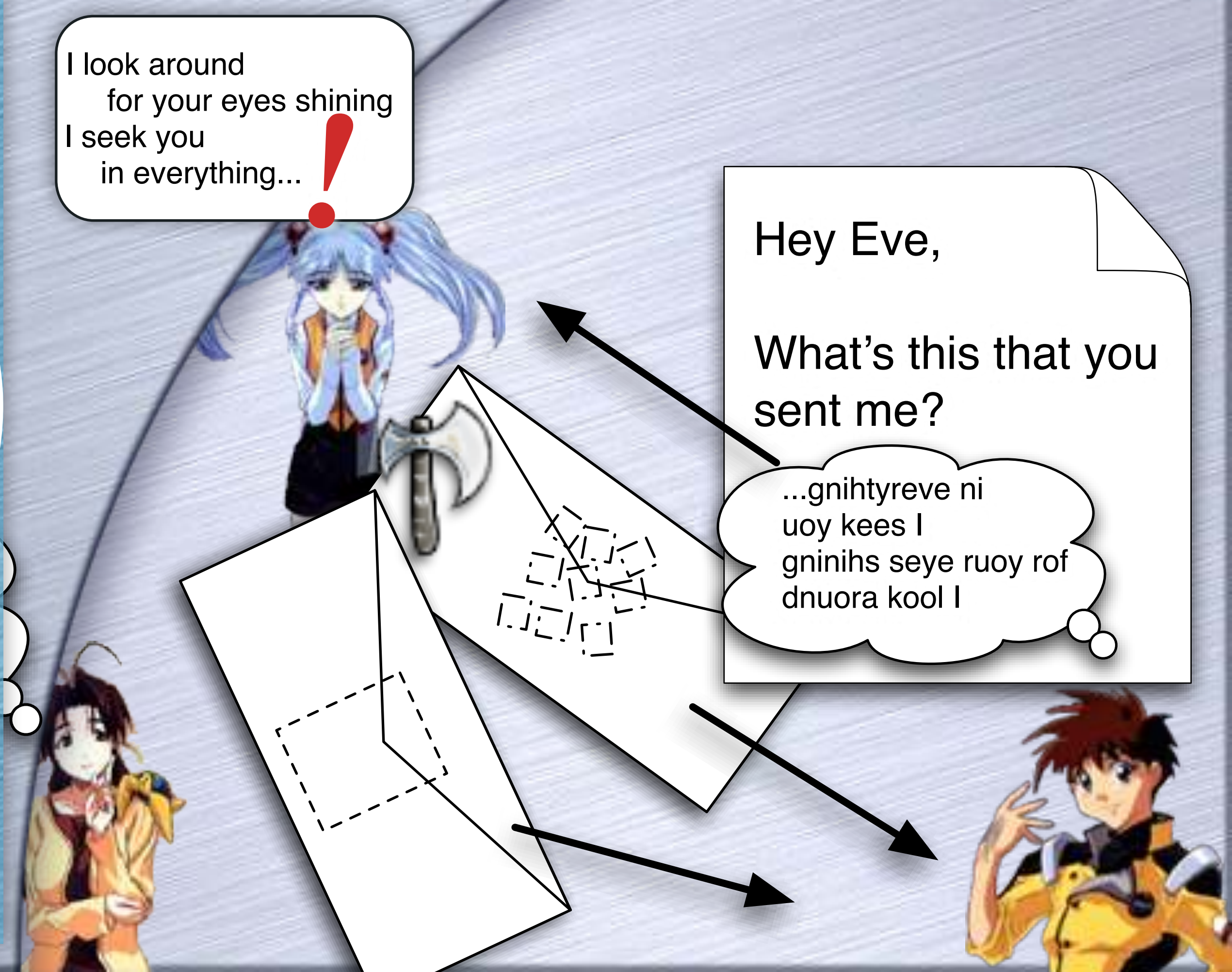

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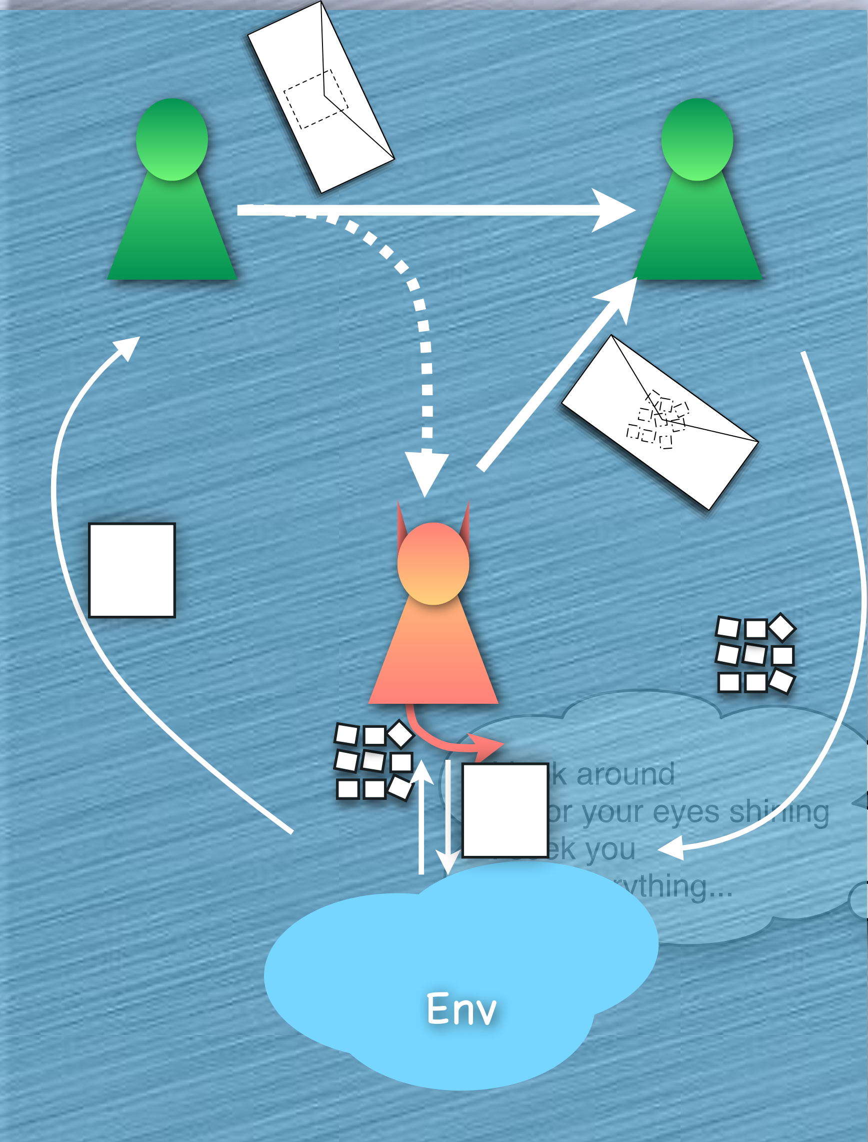

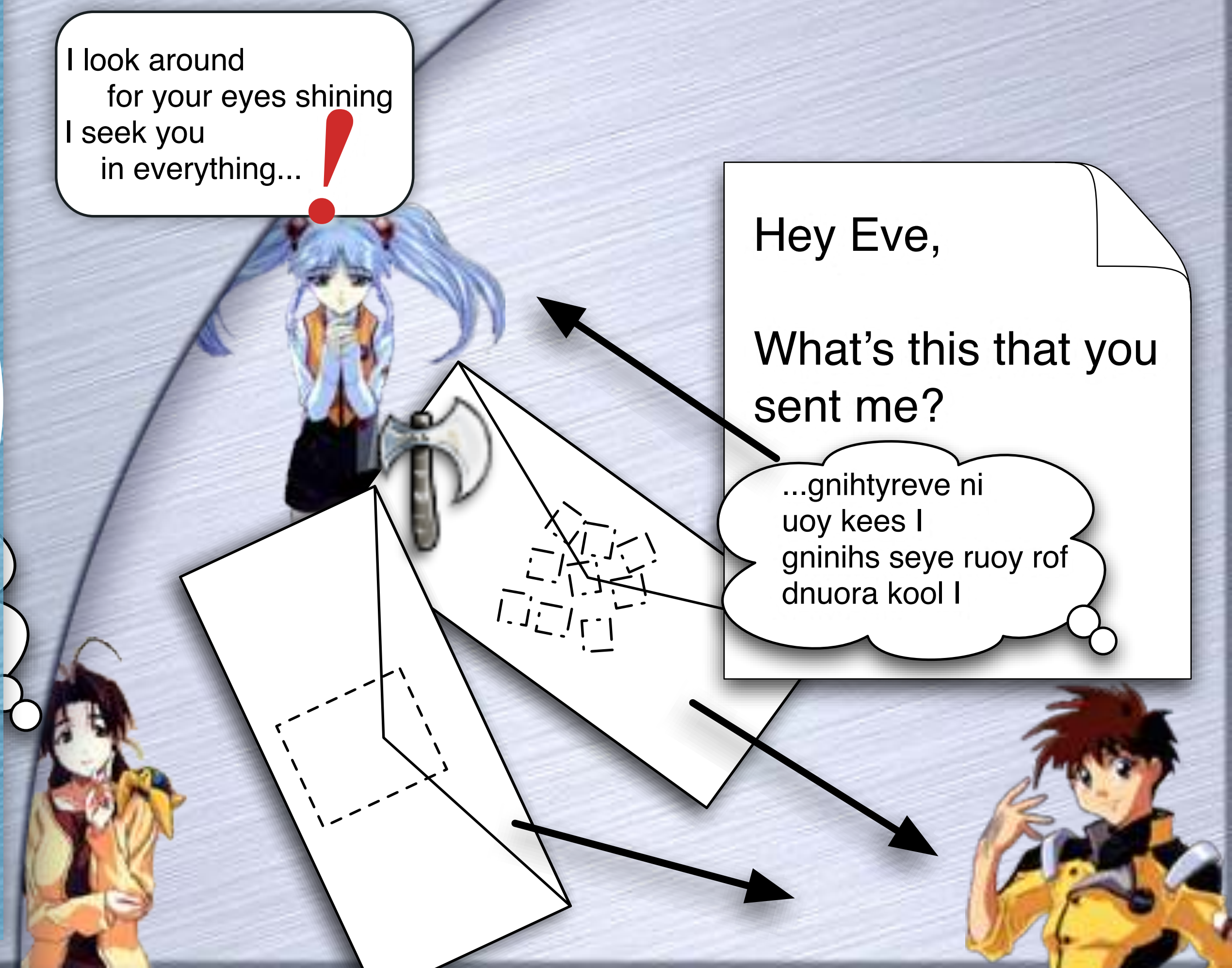

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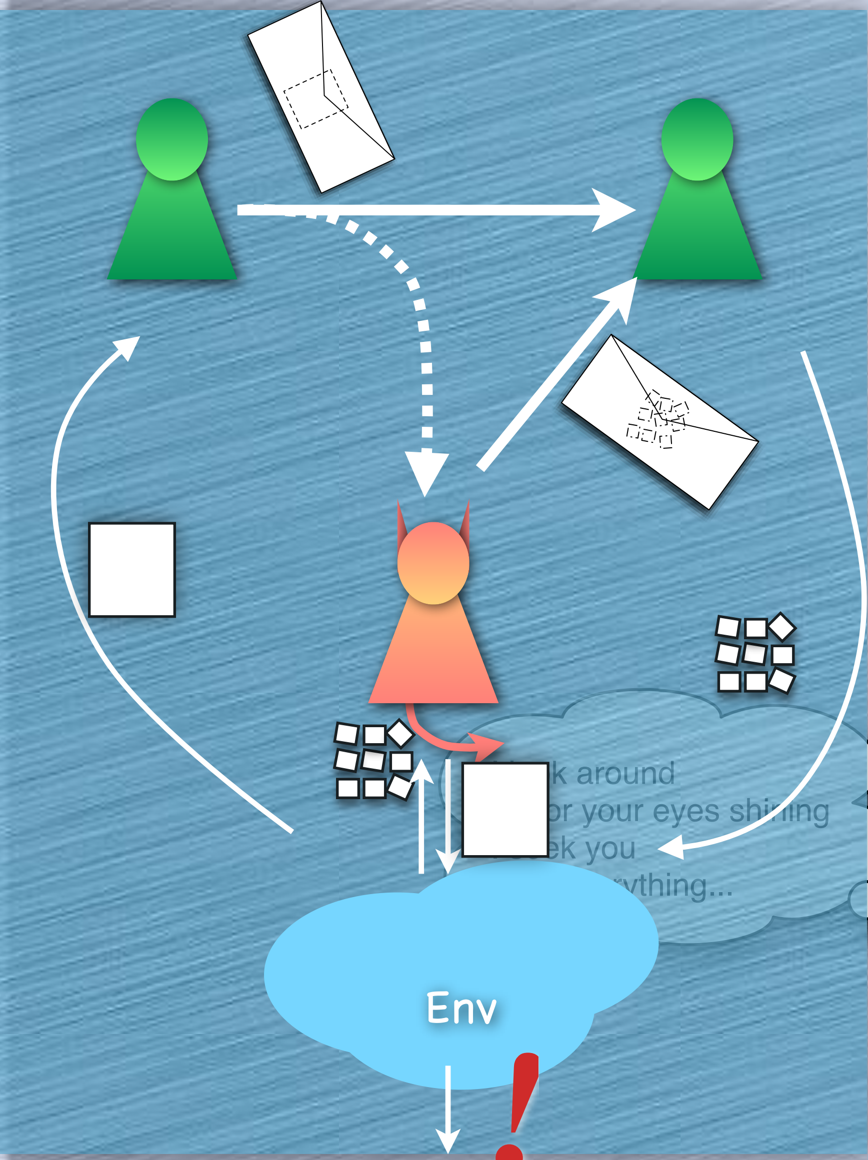
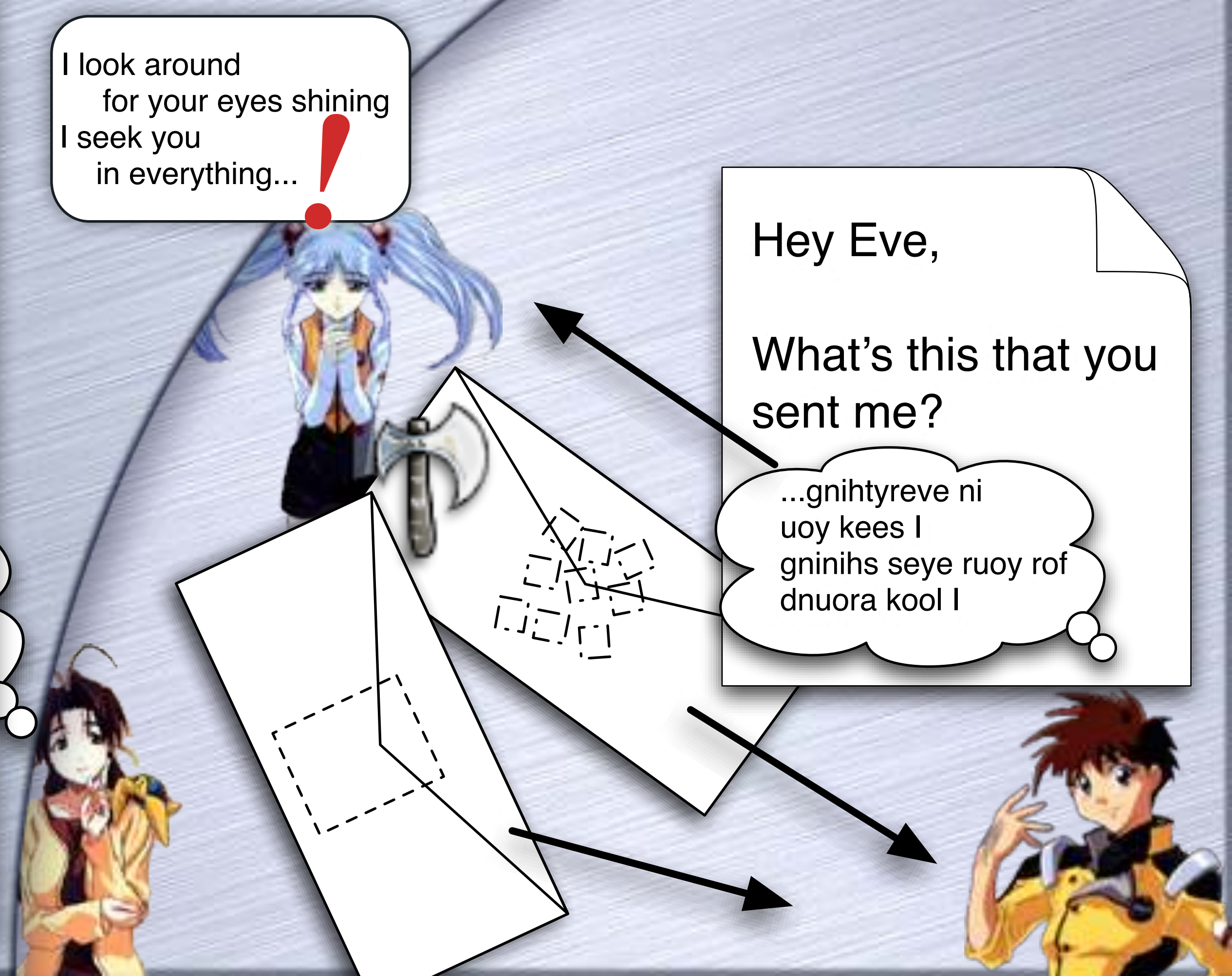
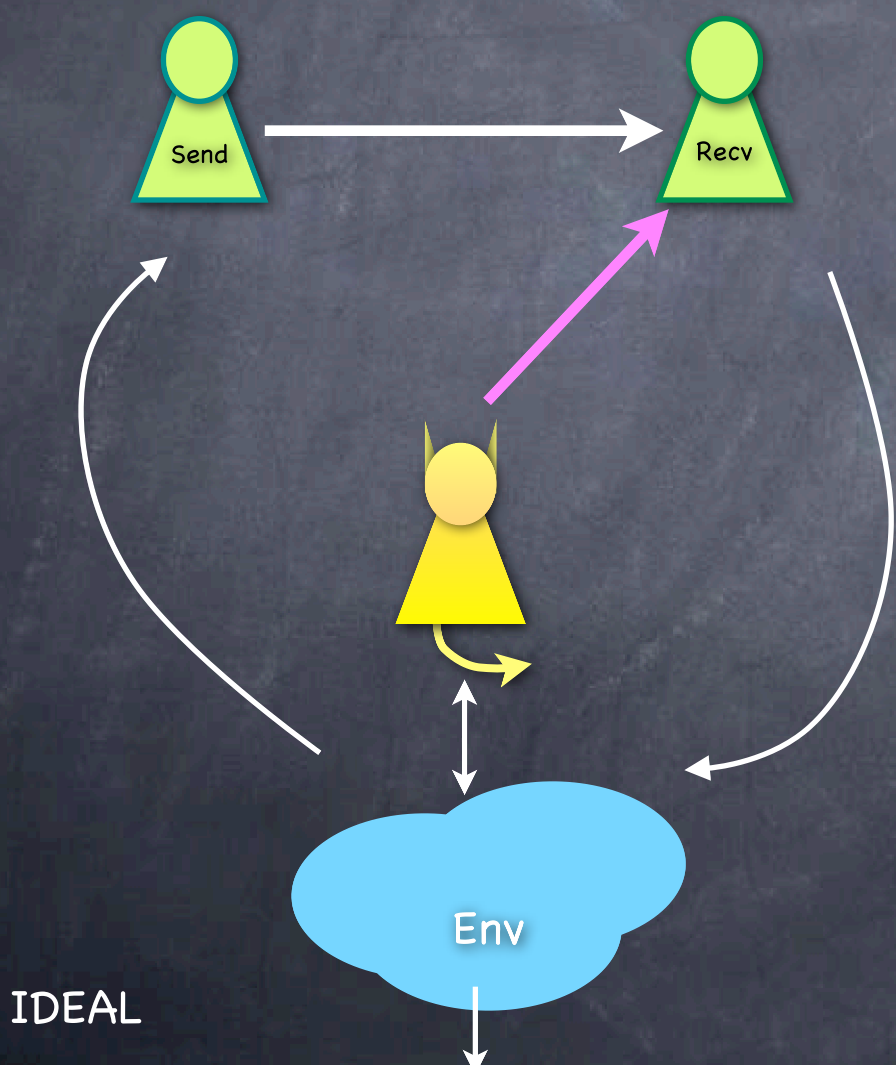




SIM-CCA Security (PKE)



Secure (and correct) if:

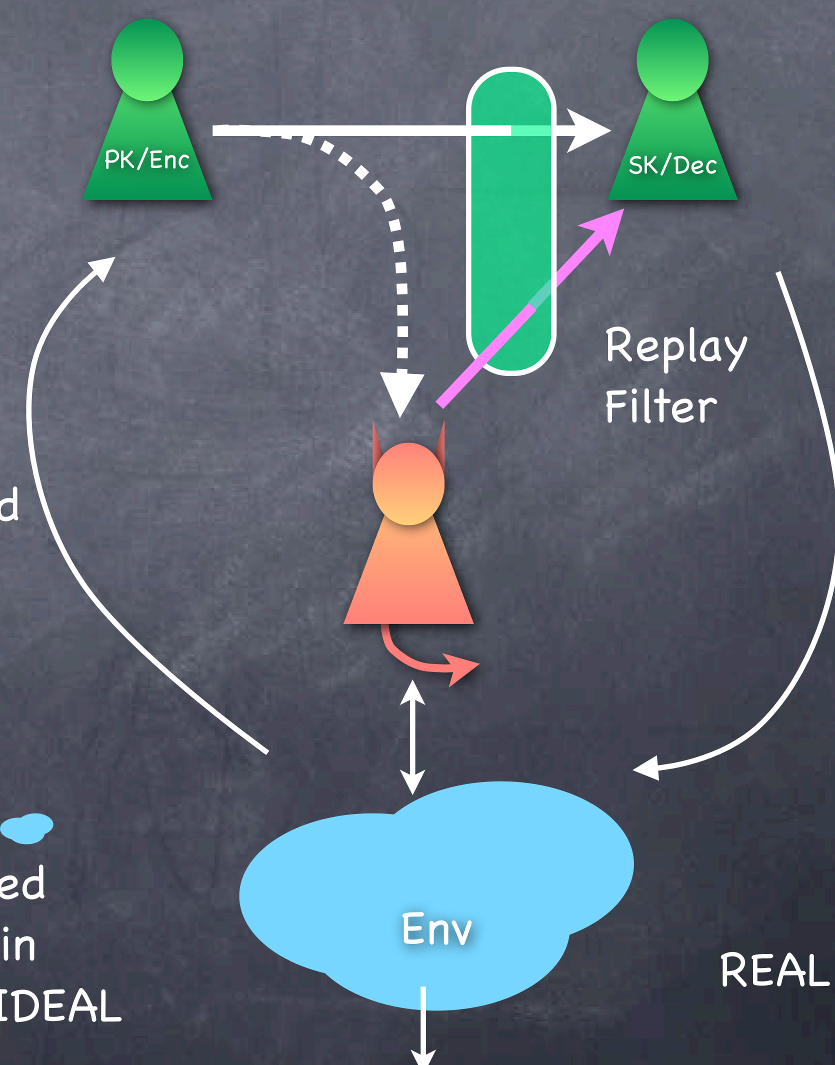
∀

∃ s.t.

∀

output of

is distributed identically in REAL and IDEAL



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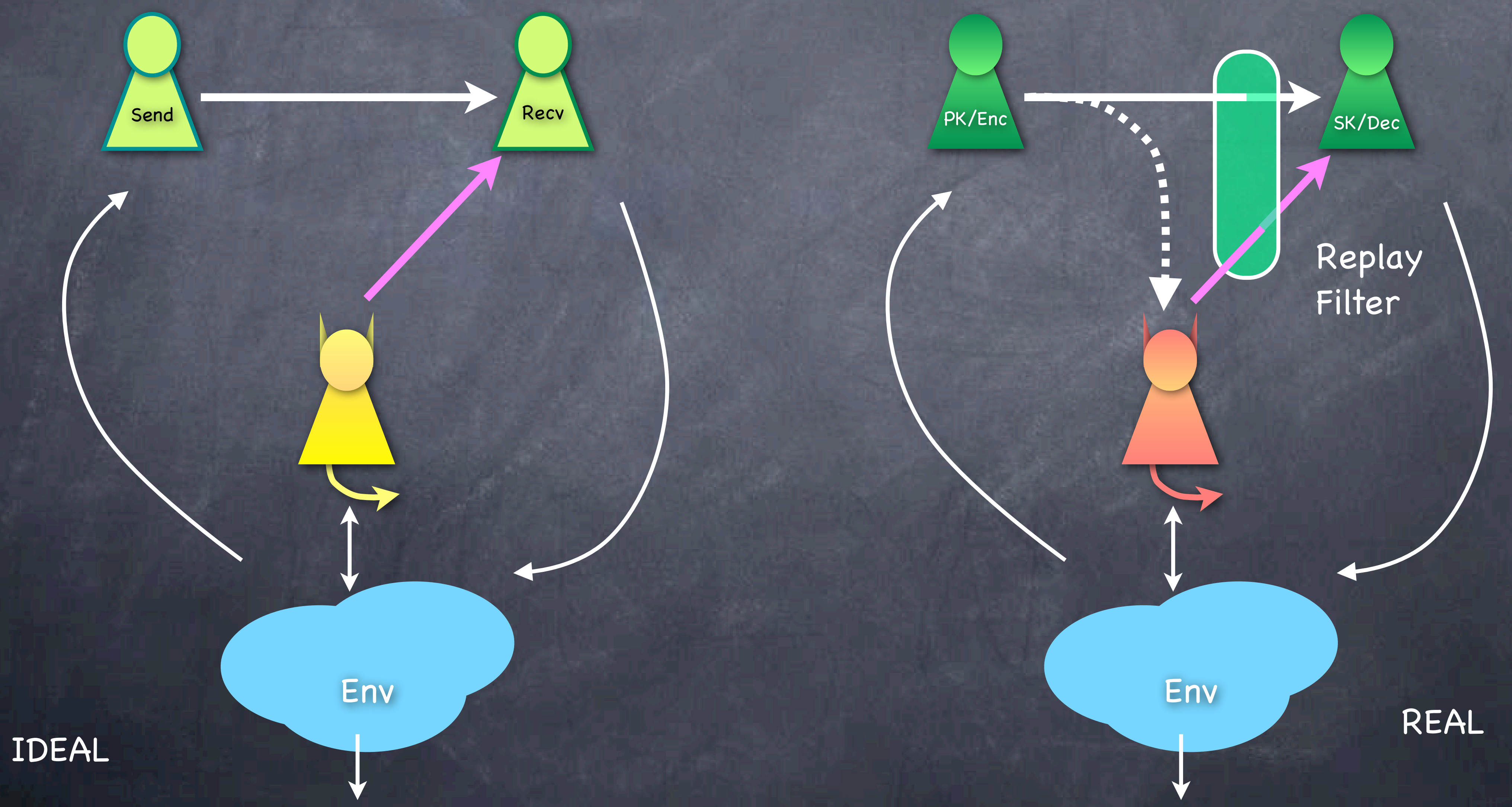

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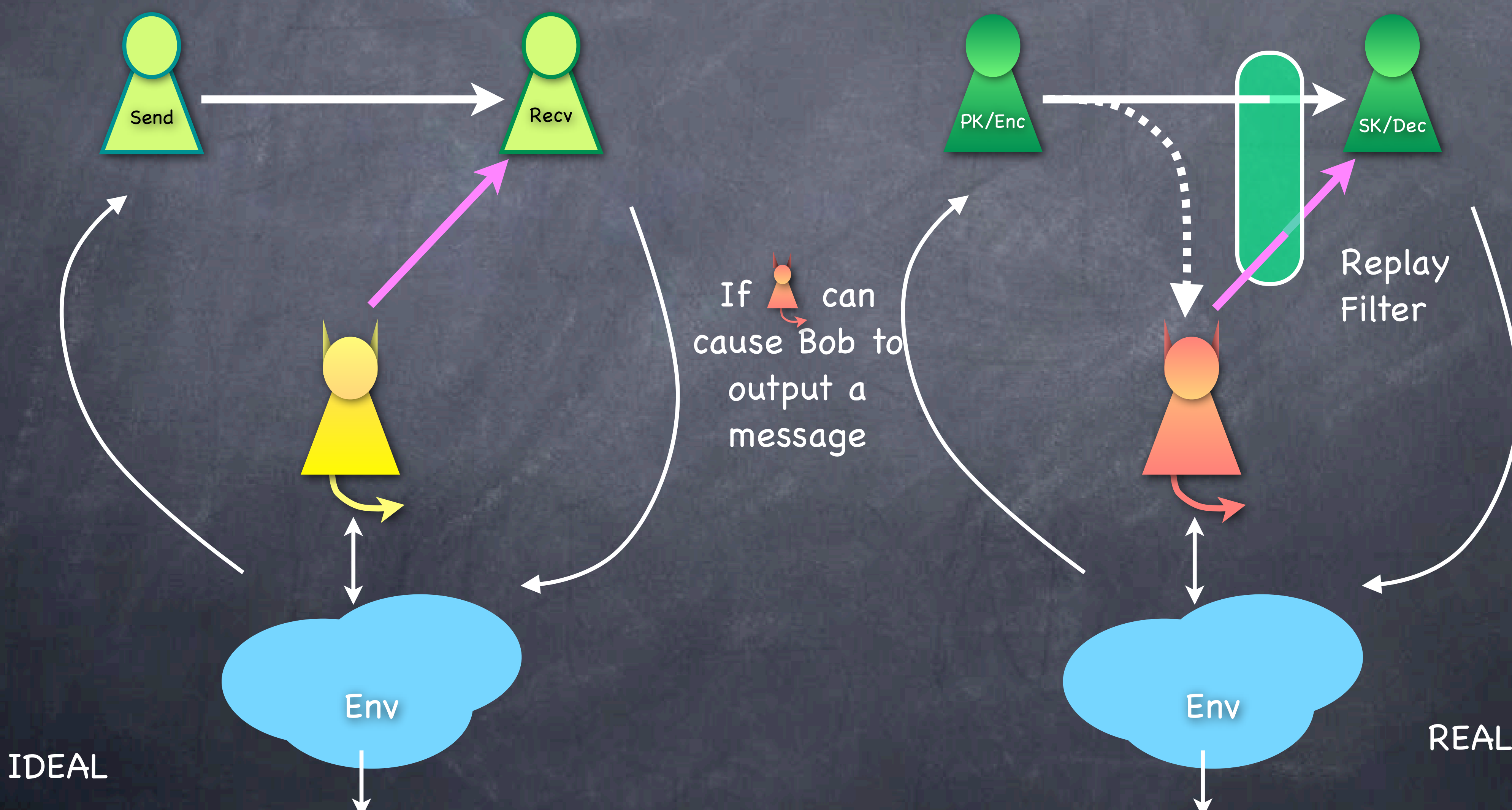

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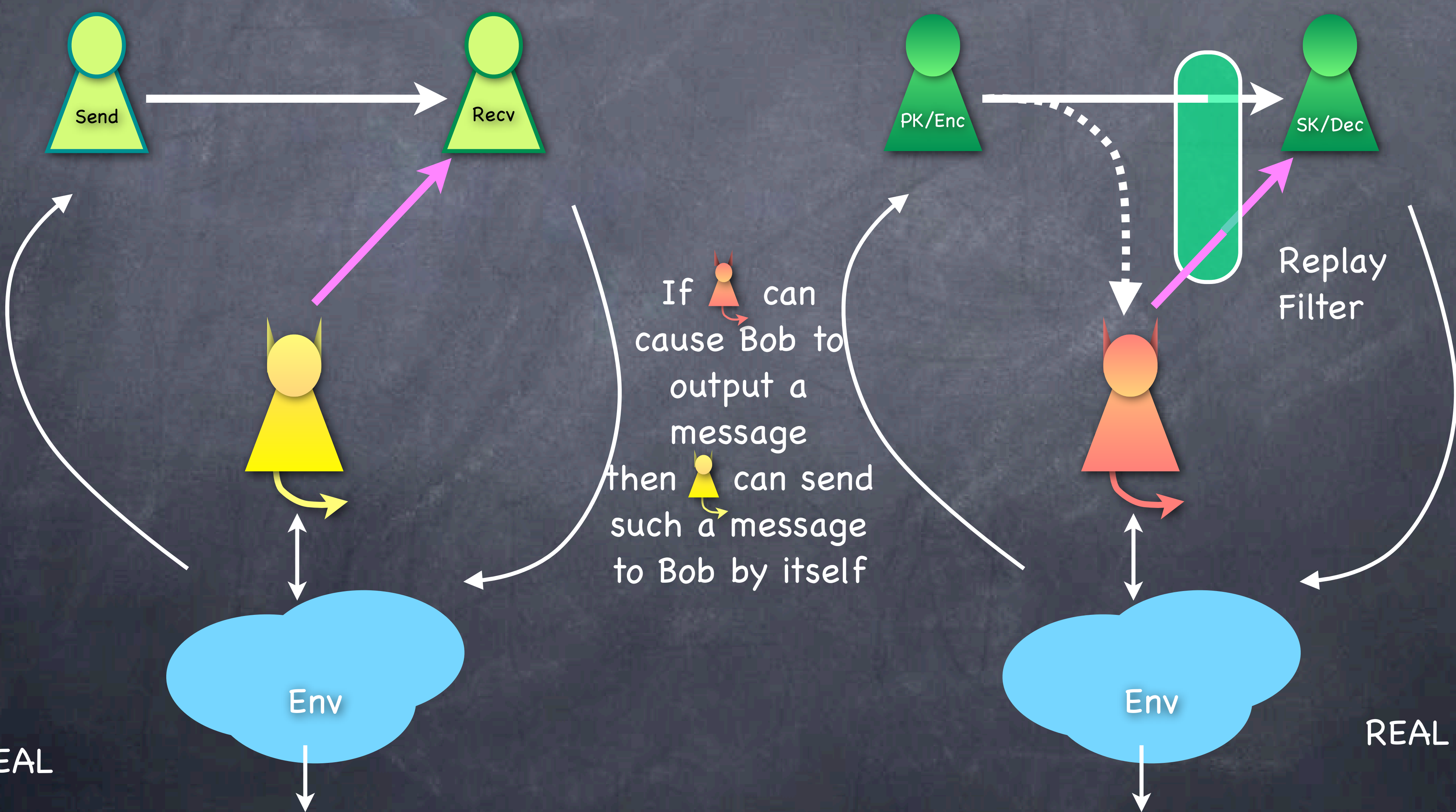

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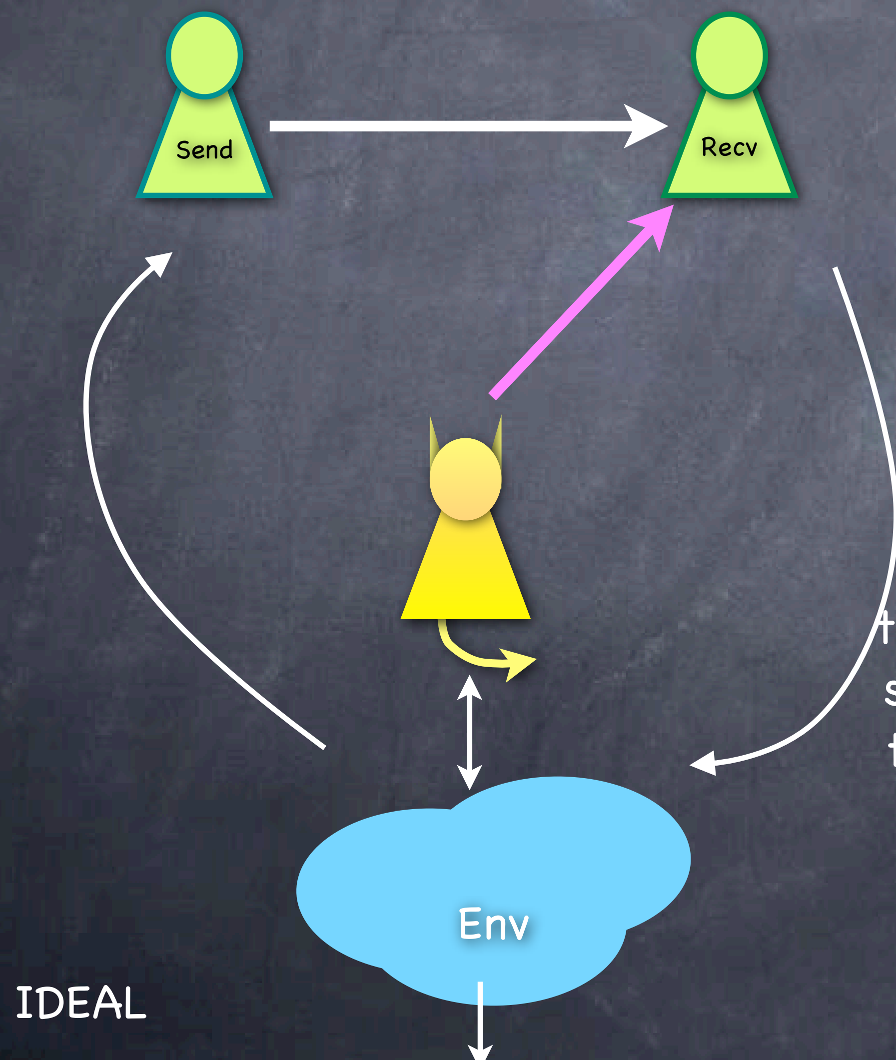
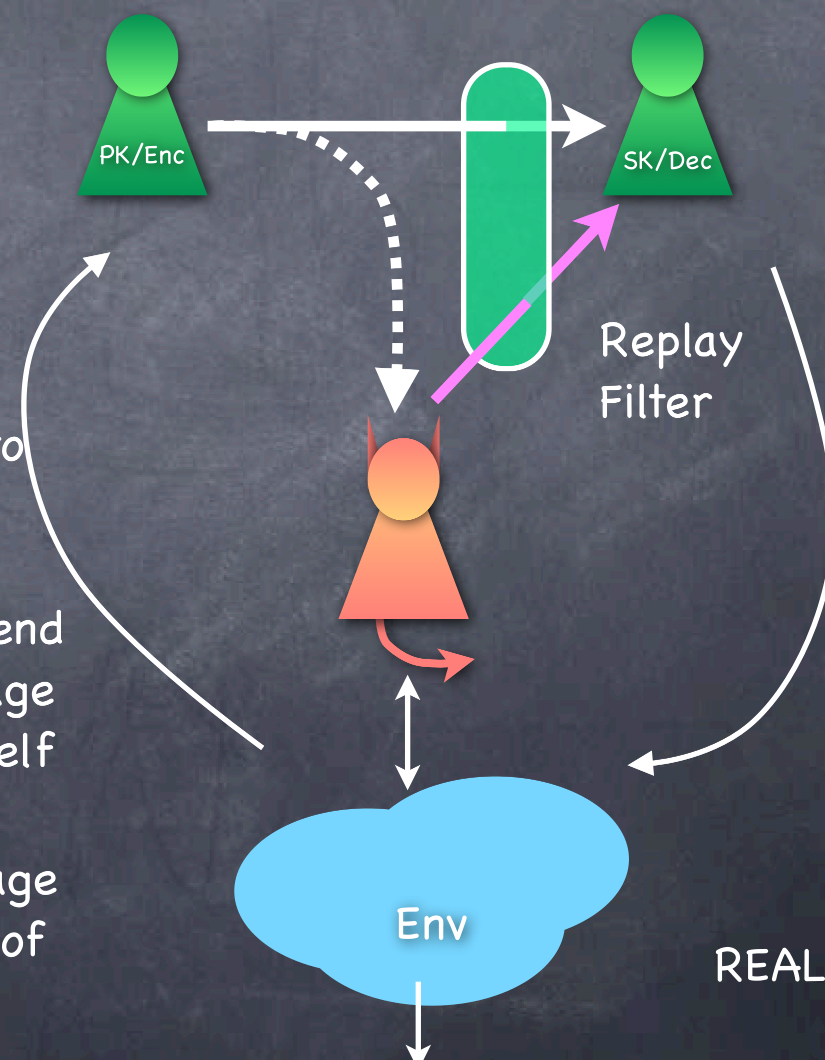


If can cause Bob to output a message then can send such a message to Bob by itself

Hence message not a result of malleating



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    - Relatively low overhead on top of the (fast) SKE encryption

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  - Less security sufficient: KEM used to transfer a random key; DEM uses a new key every time.

Today

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