Defining Encryption (ctd.)

Lecture 3

CPA/CCA security

Computational Indistinguishability

Pseudo-randomness, One-Way Functions

Security of Encryption

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- Coming up: Formalizing notions of "computational" security (as opposed to perfect/statistical security)
  - Then, security definitions used for encryption of multiple messages

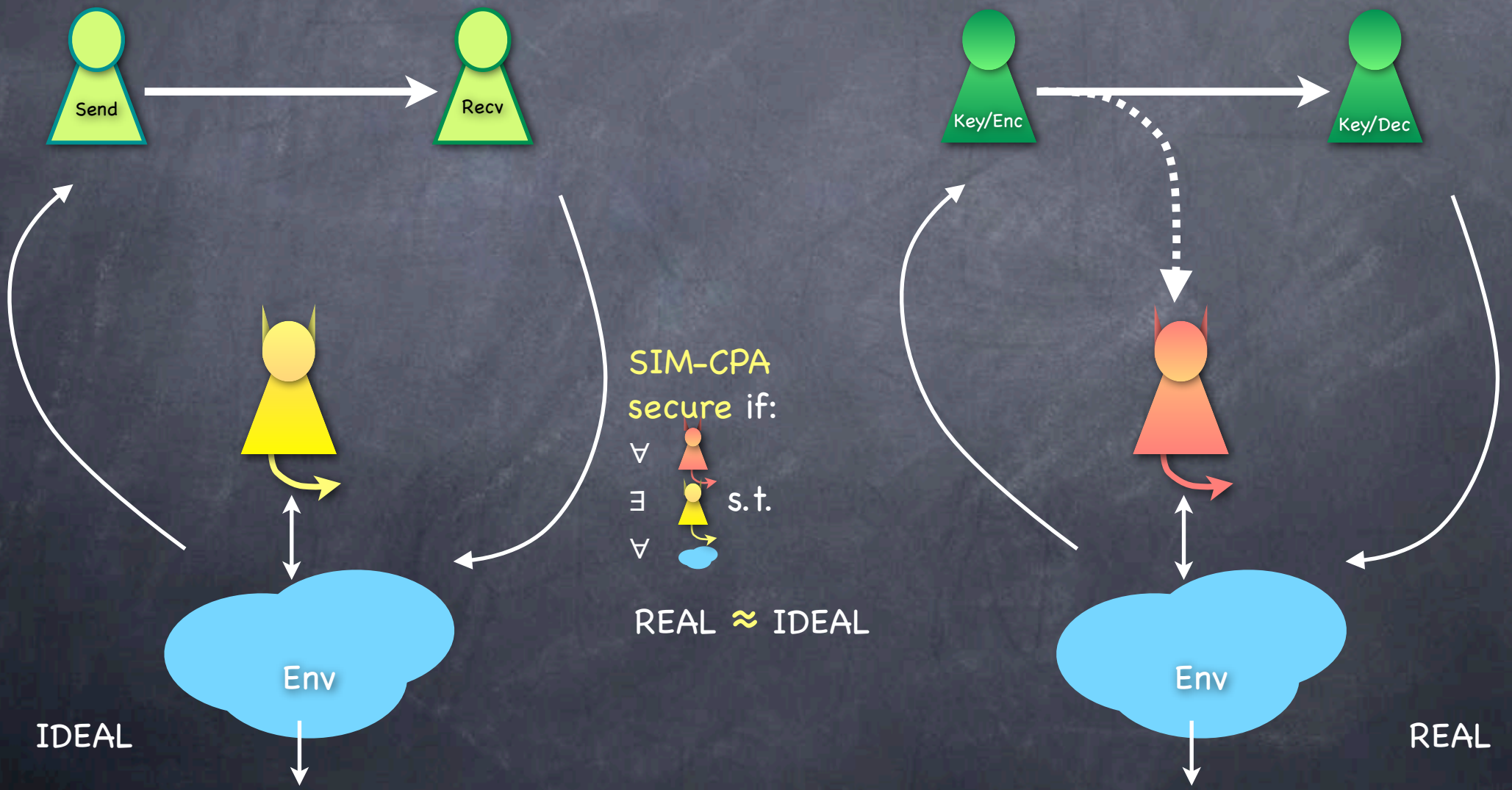
Symmetric-Key Encryption

The Syntax

- Shared-key (Private-key) Encryption
  - Key Generation: Randomized
    - $K \leftarrow \mathcal{K}$, uniformly randomly drawn from the key-space (or according to a key-distribution)
  - Encryption: Randomized
    - Enc: $\mathcal{M} \times \mathcal{K} \times \mathcal{R} \rightarrow \mathcal{C}$. During encryption a fresh random string will be chosen uniformly at random from $\mathcal{R}$
  - Decryption: Deterministic
    - Dec: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$

Symmetric-Key Encryption

SIM-CPA Security



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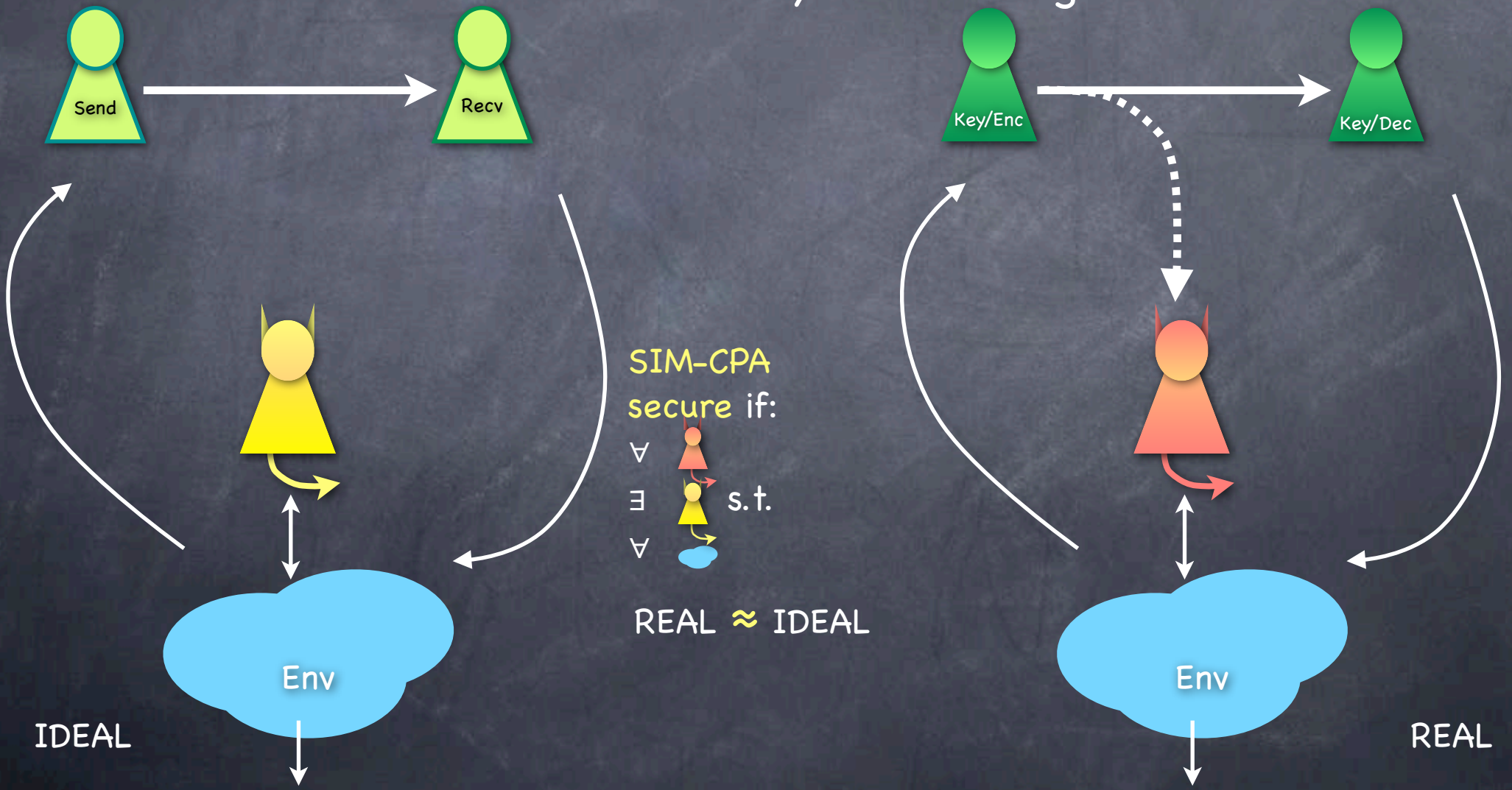

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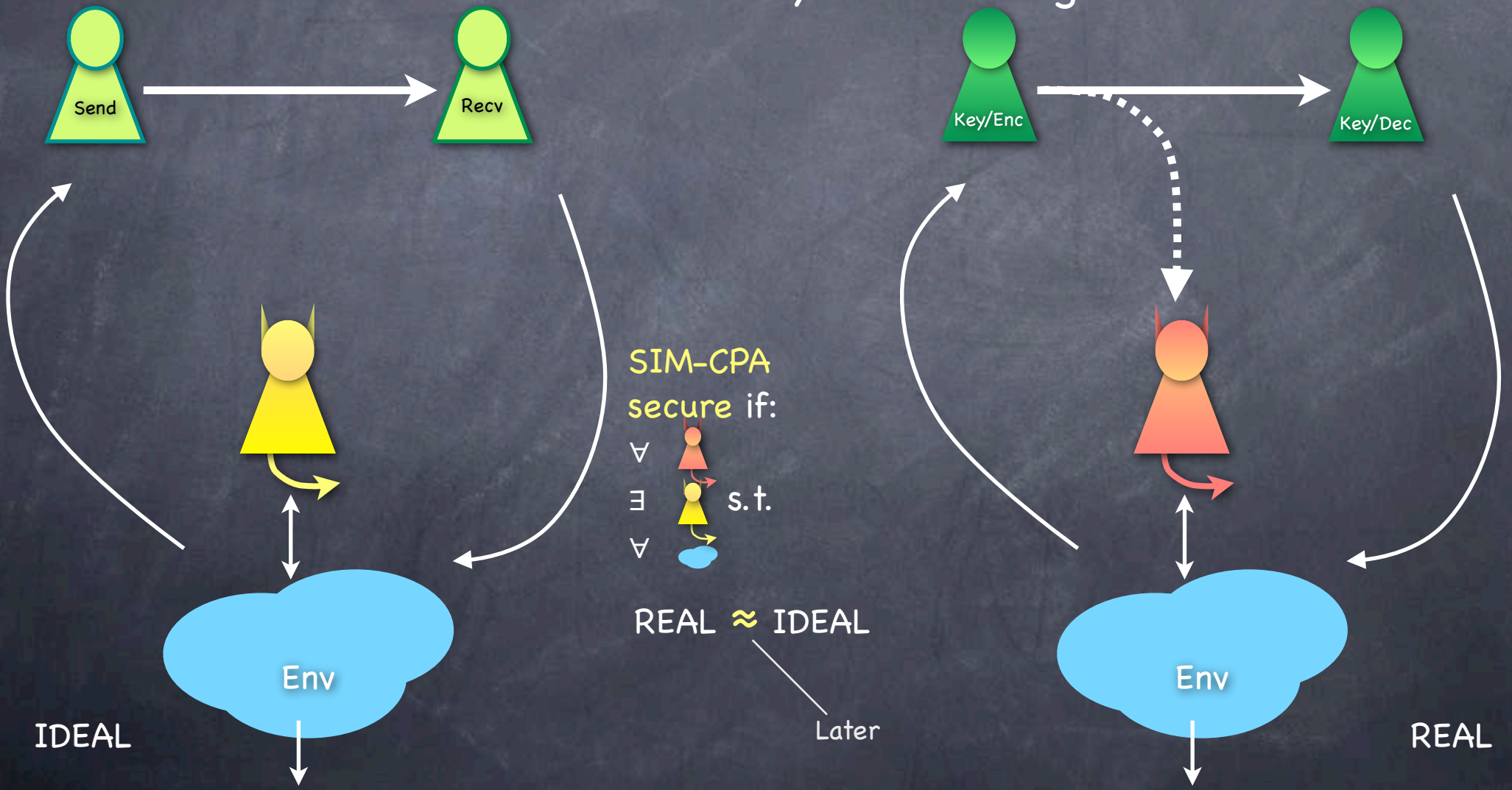

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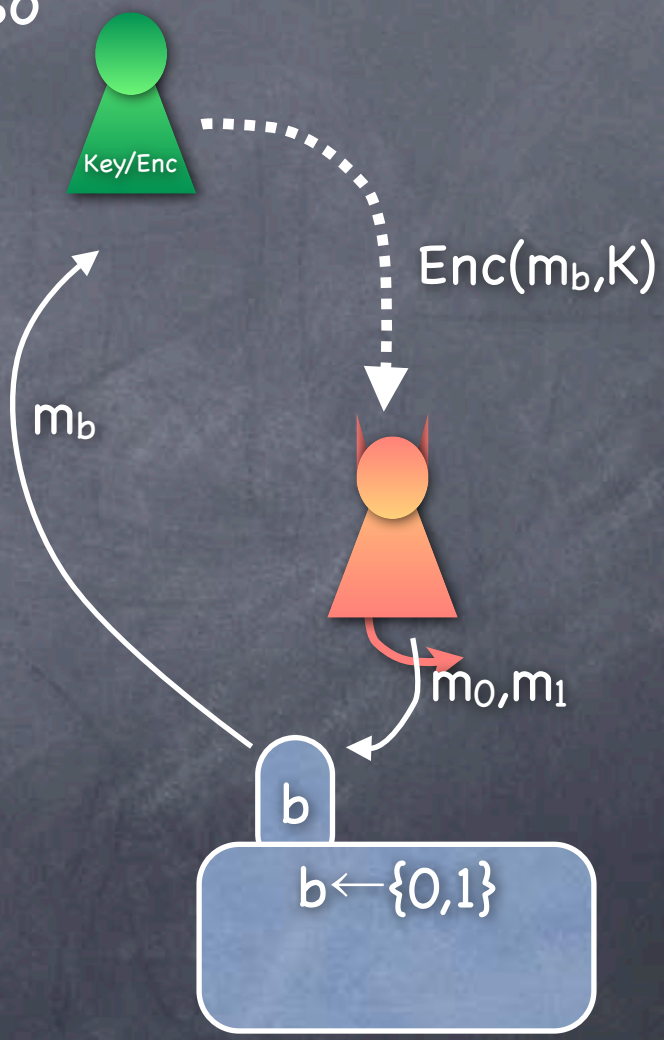

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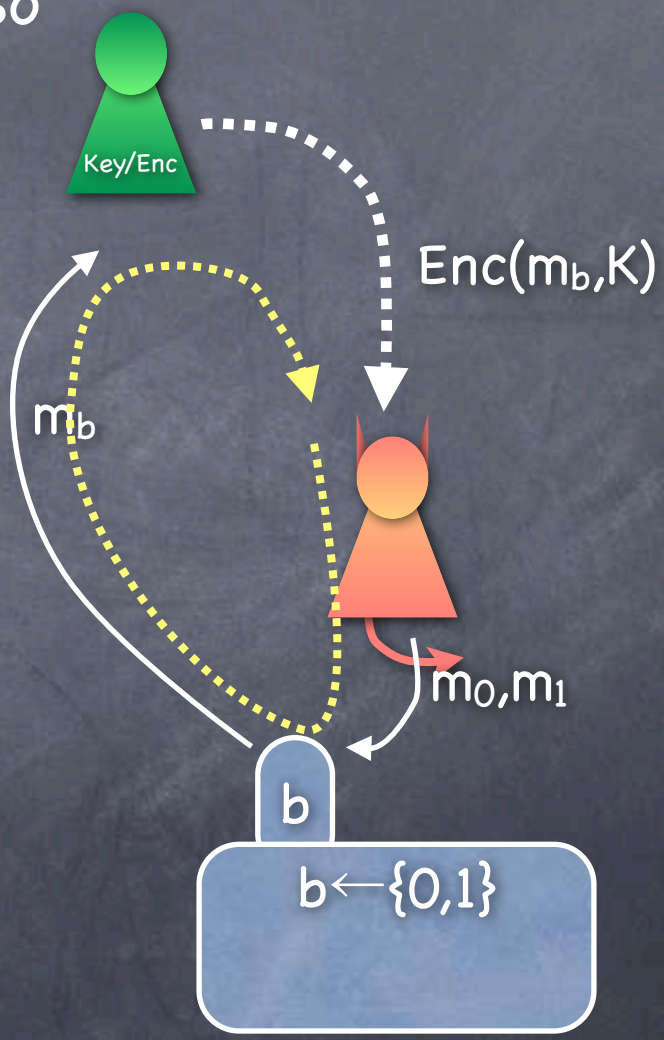

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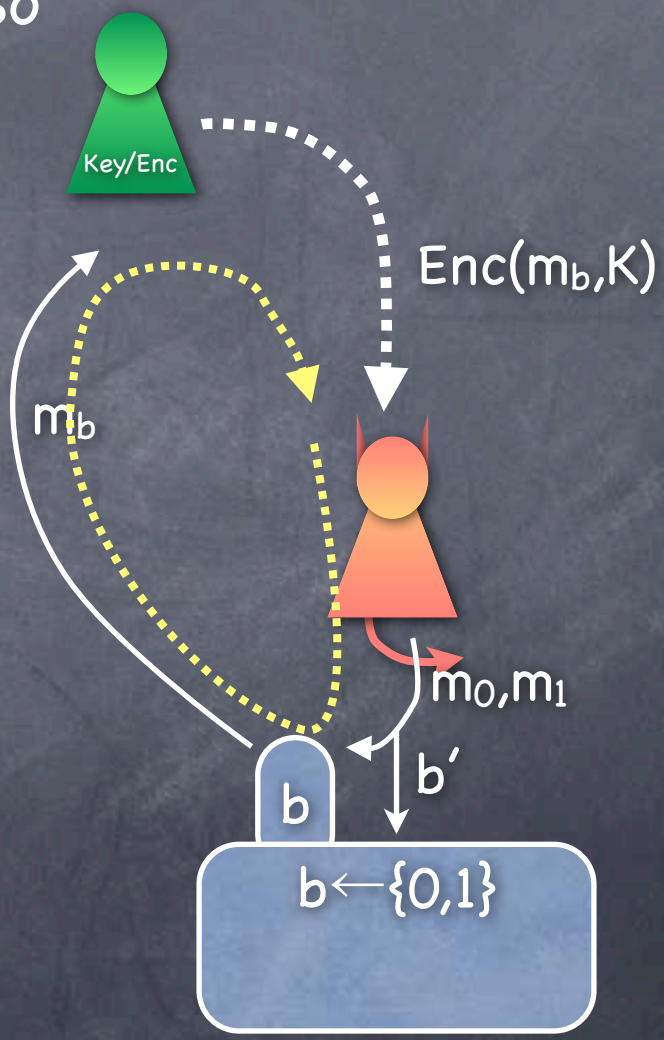

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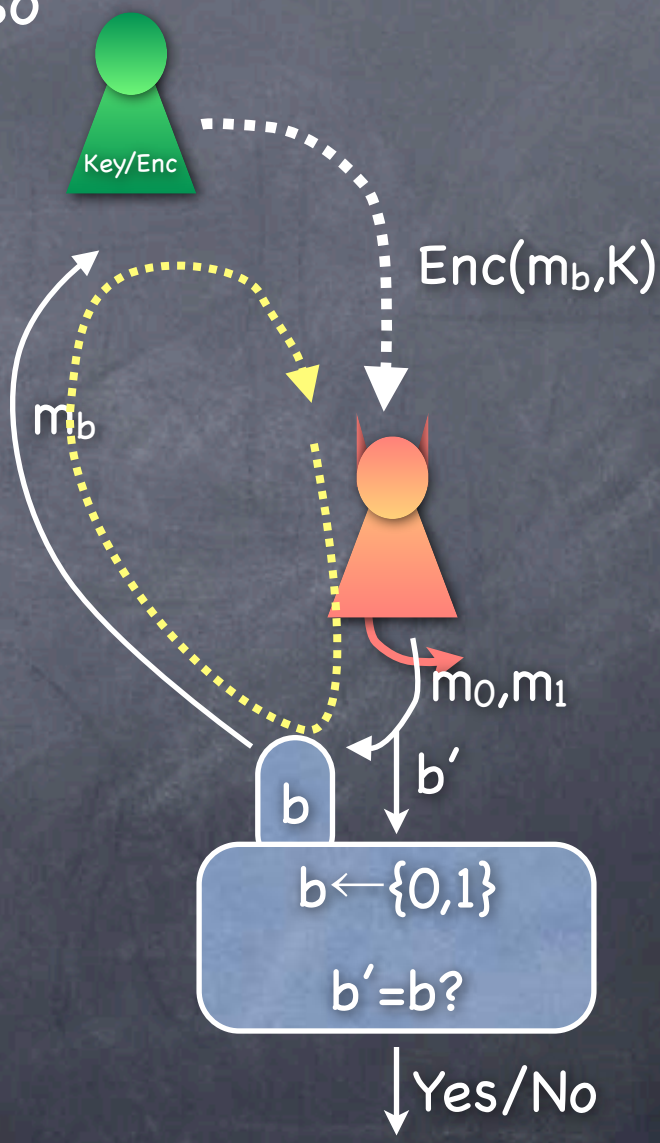

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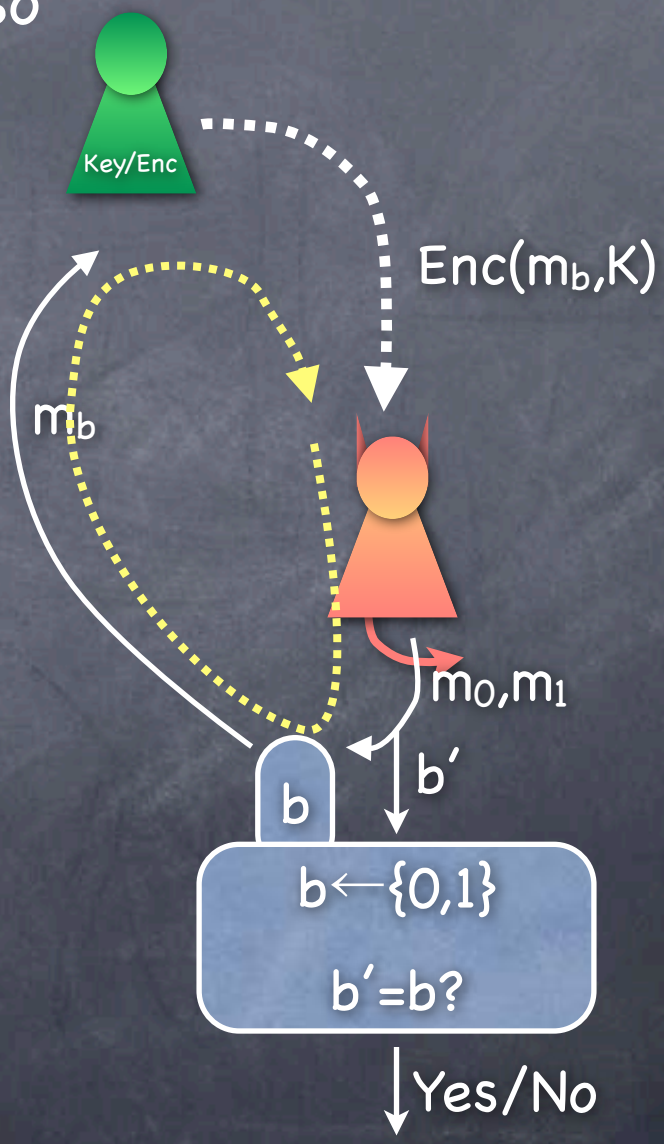

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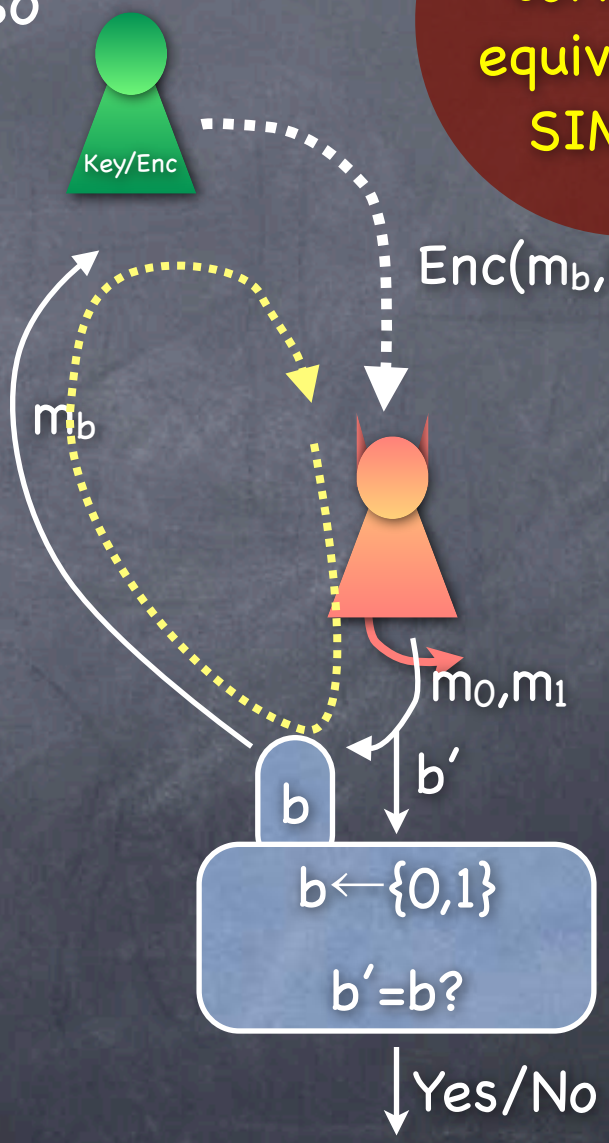

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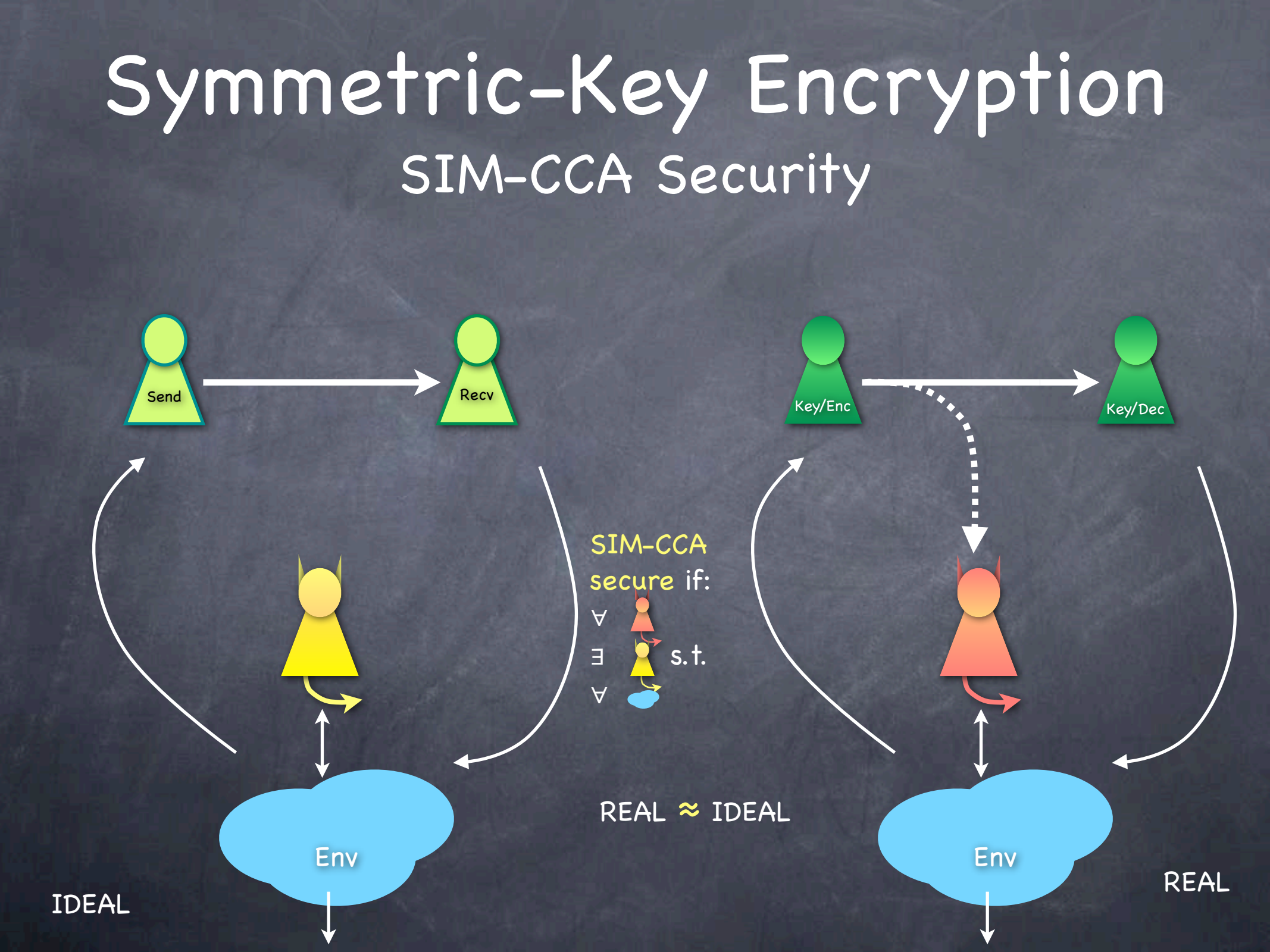

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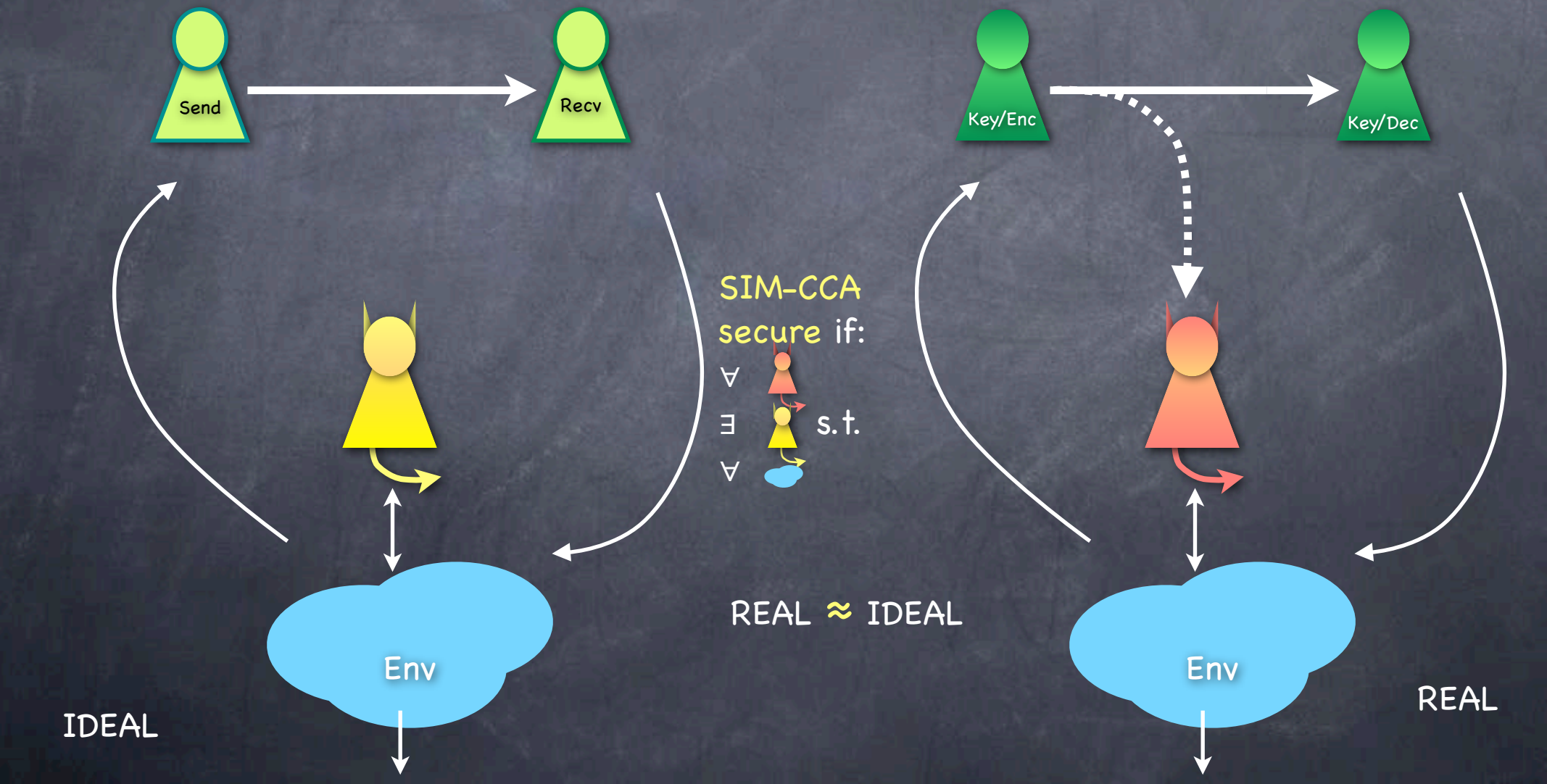

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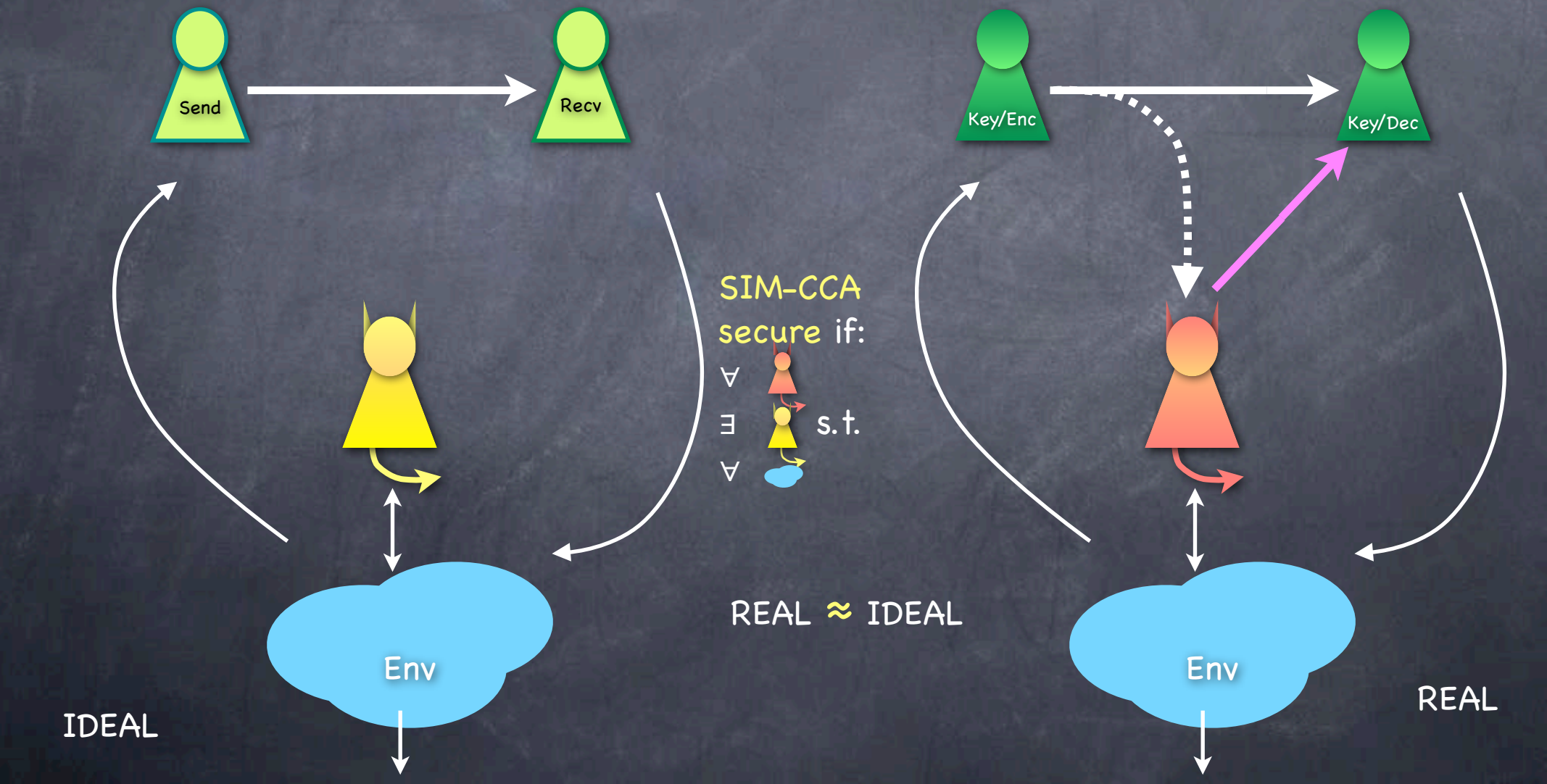

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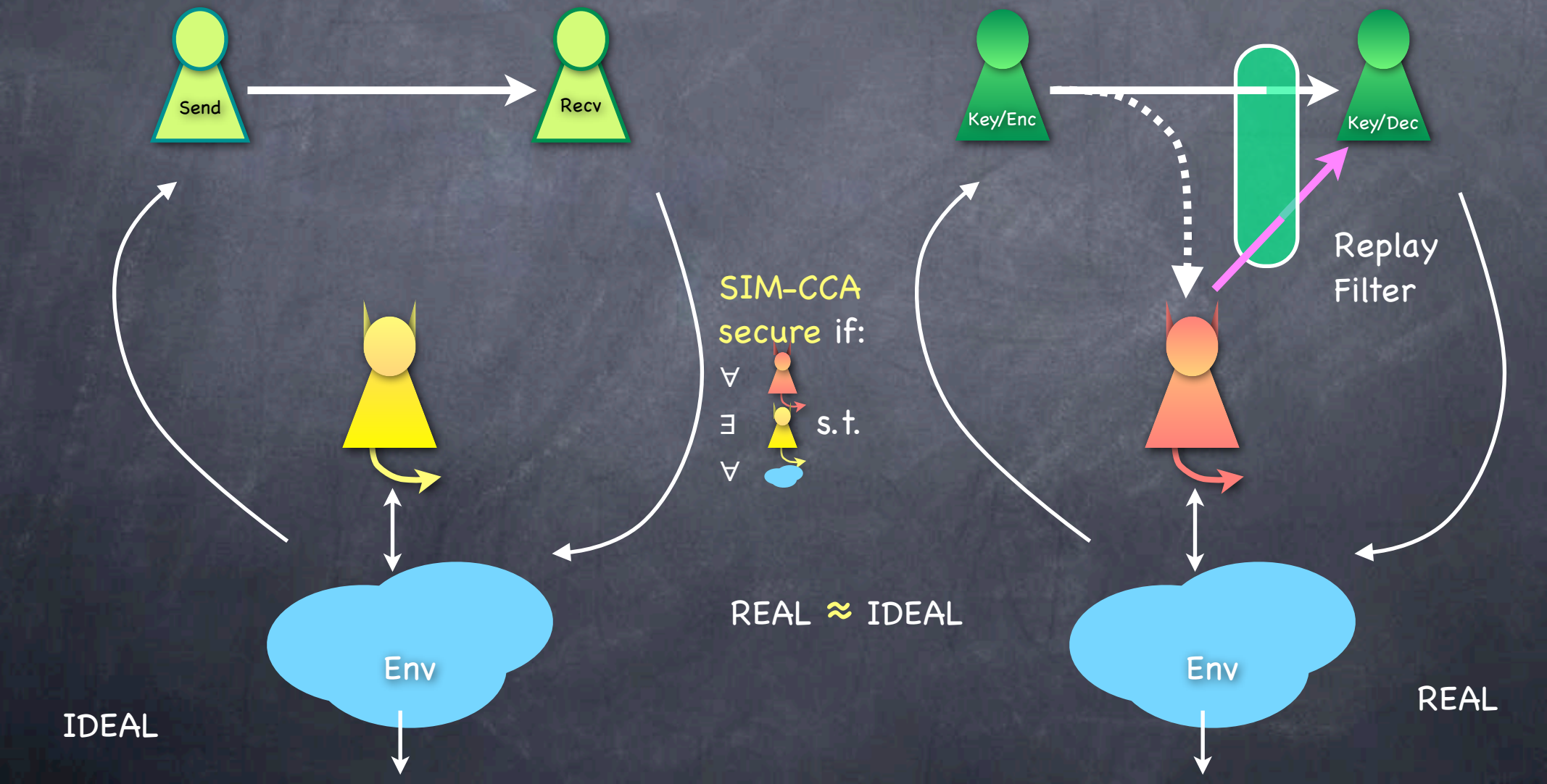

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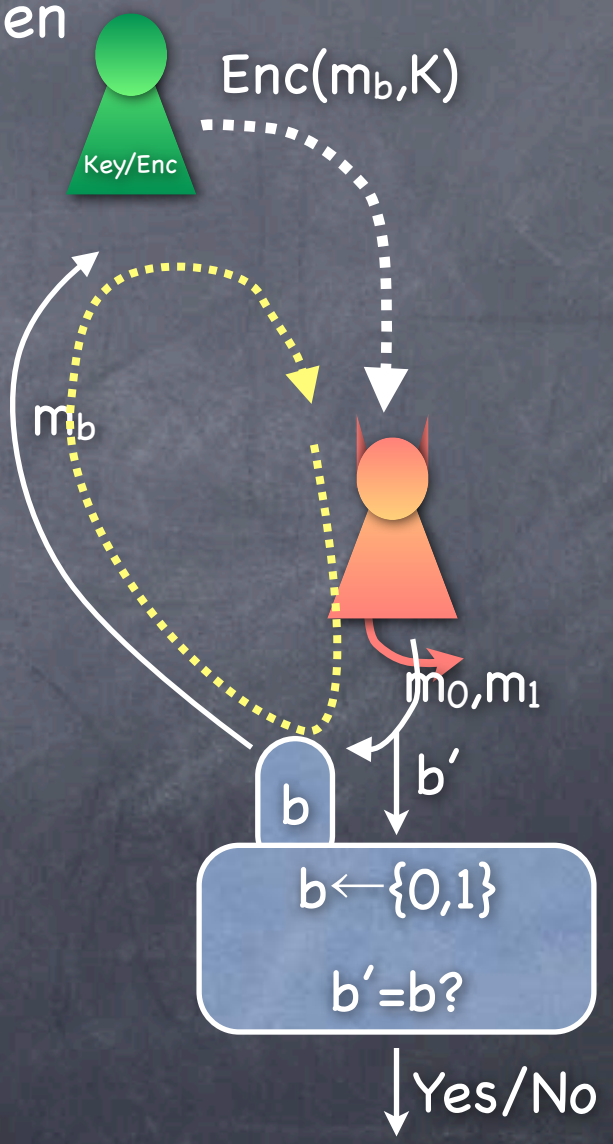

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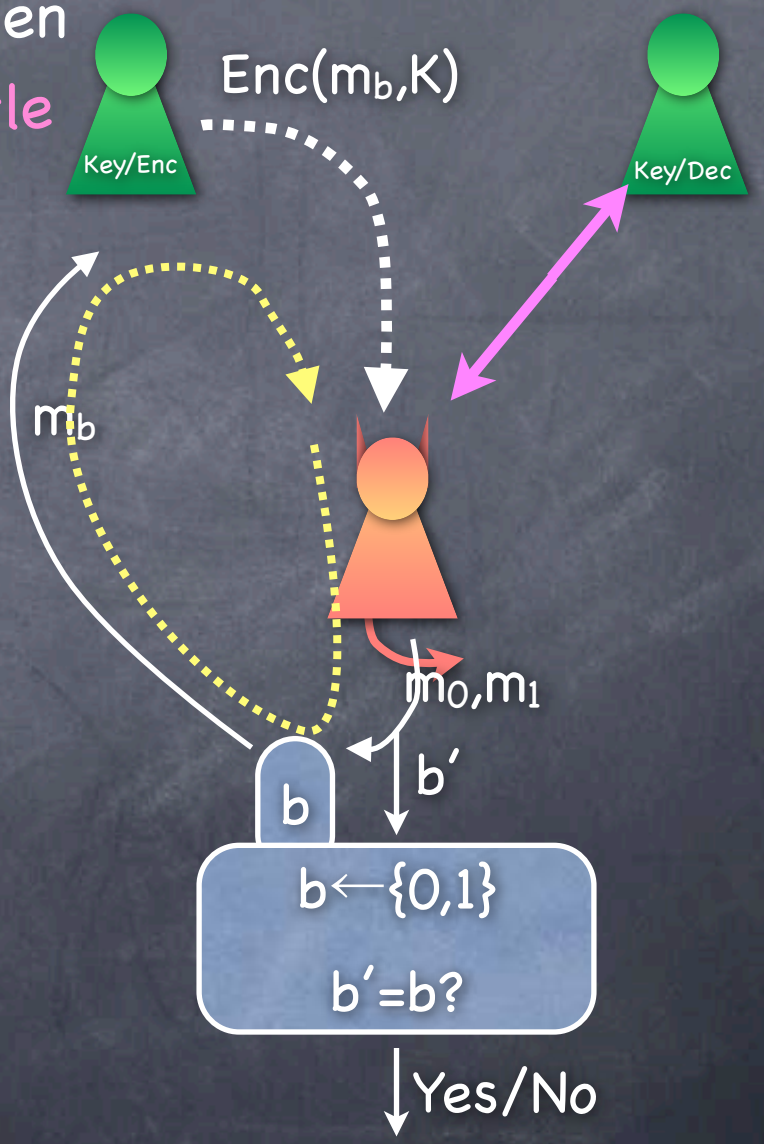

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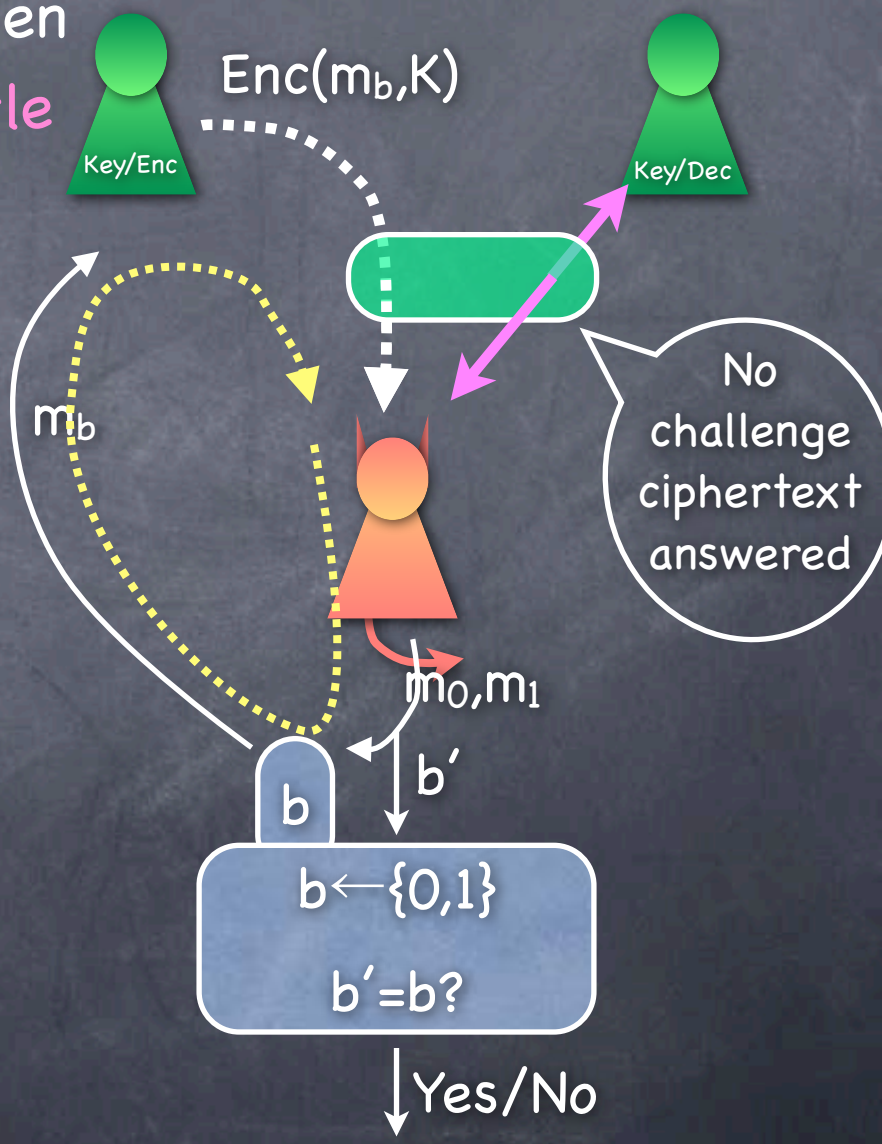

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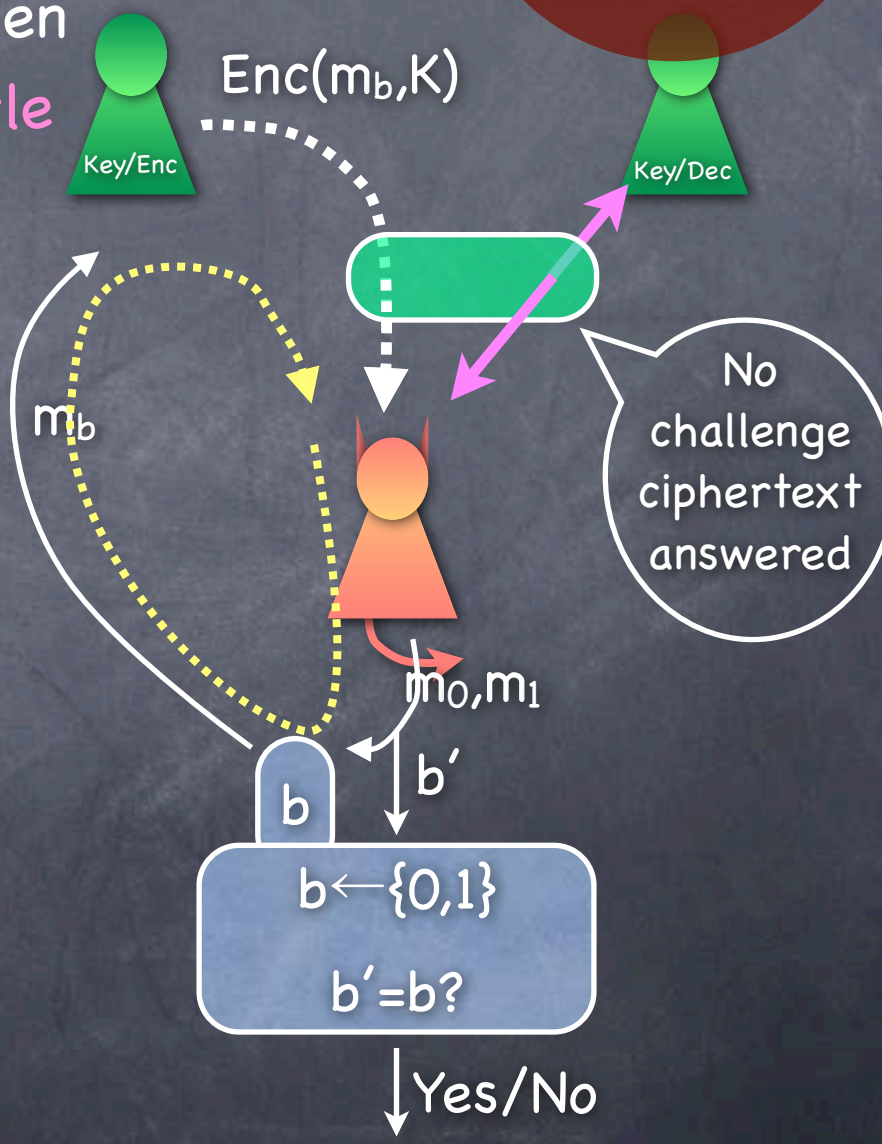

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  - But what is $\approx$?

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- Security guarantees are given asymptotically as a function of the security parameter

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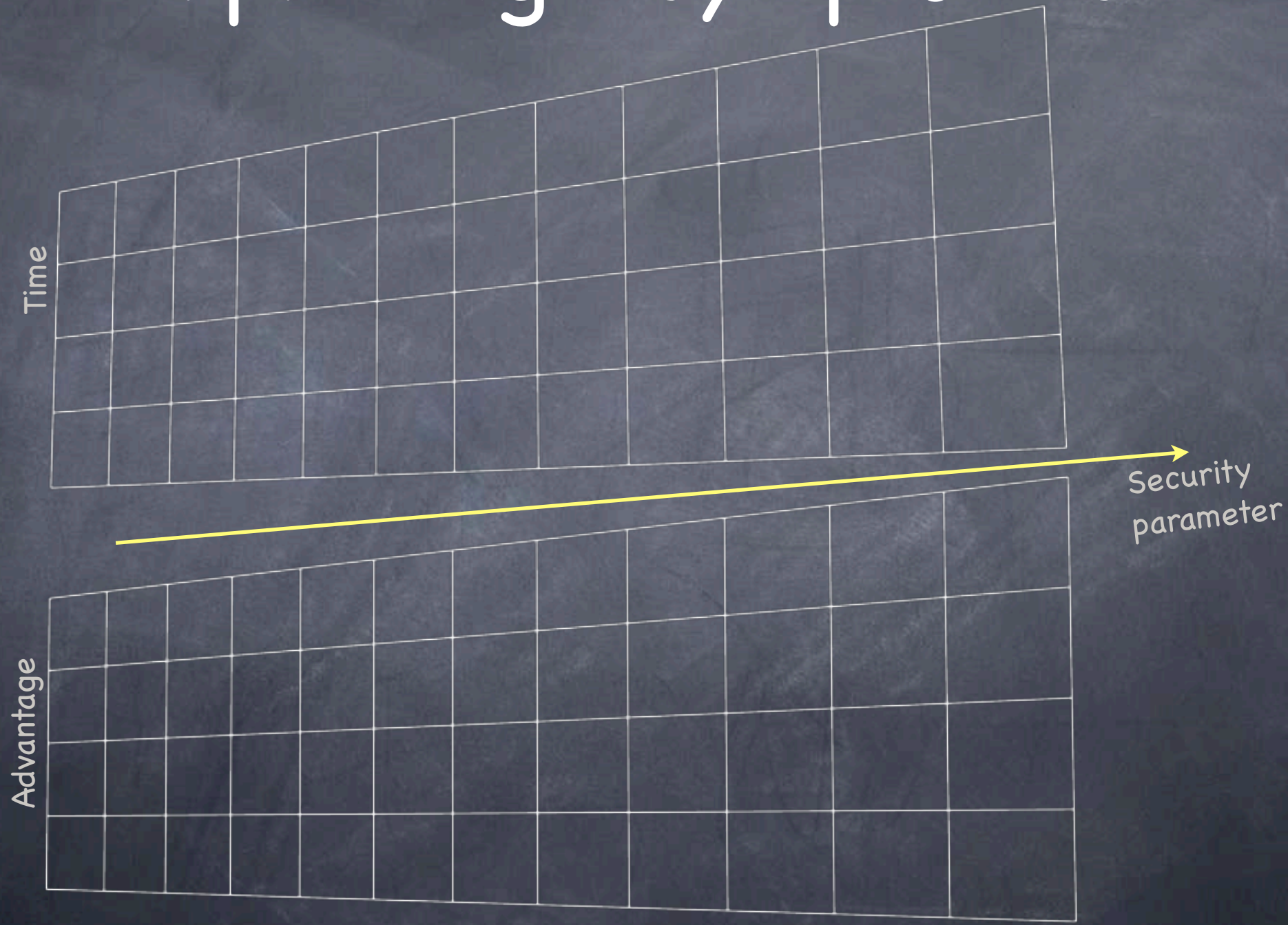

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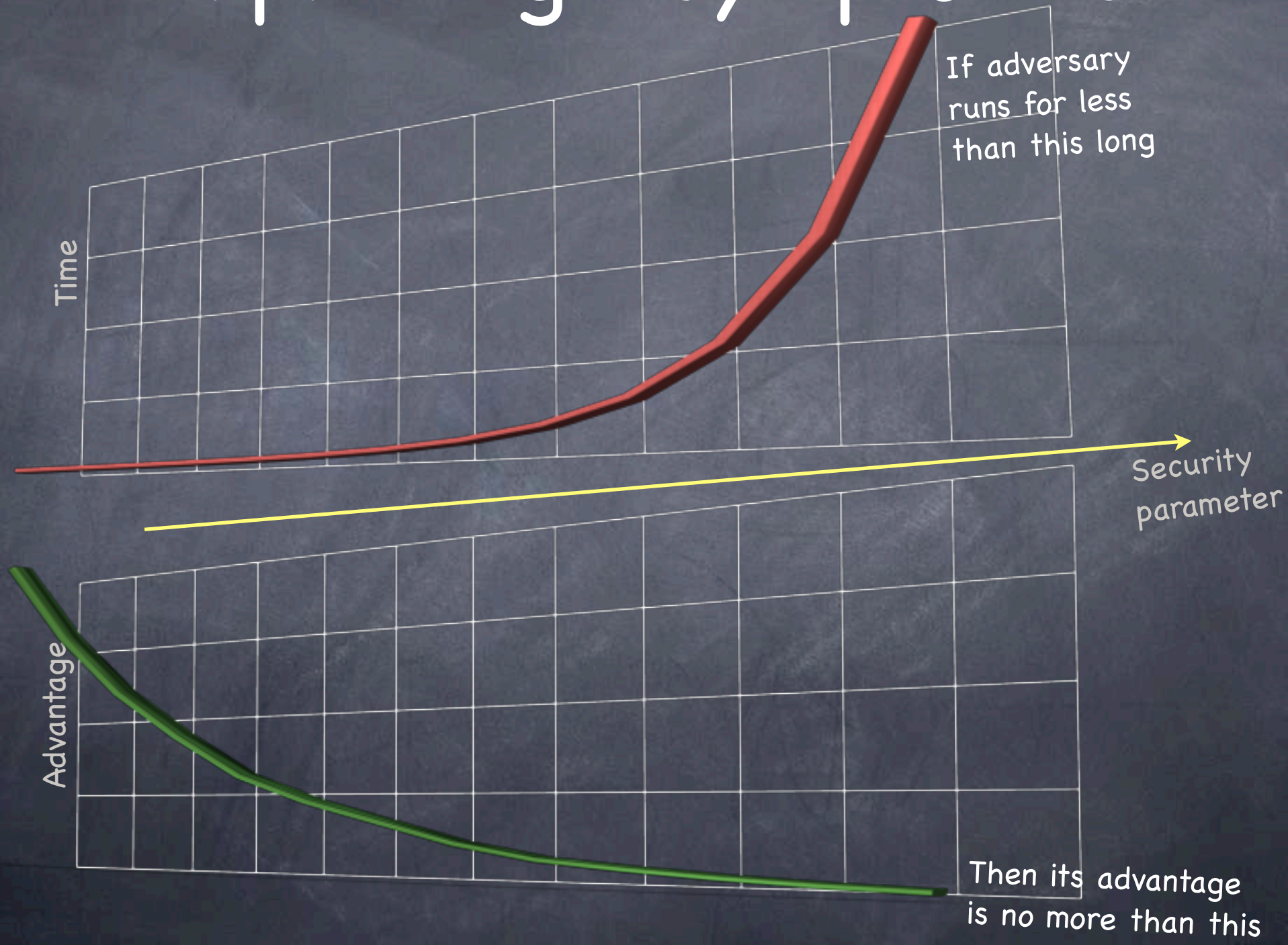

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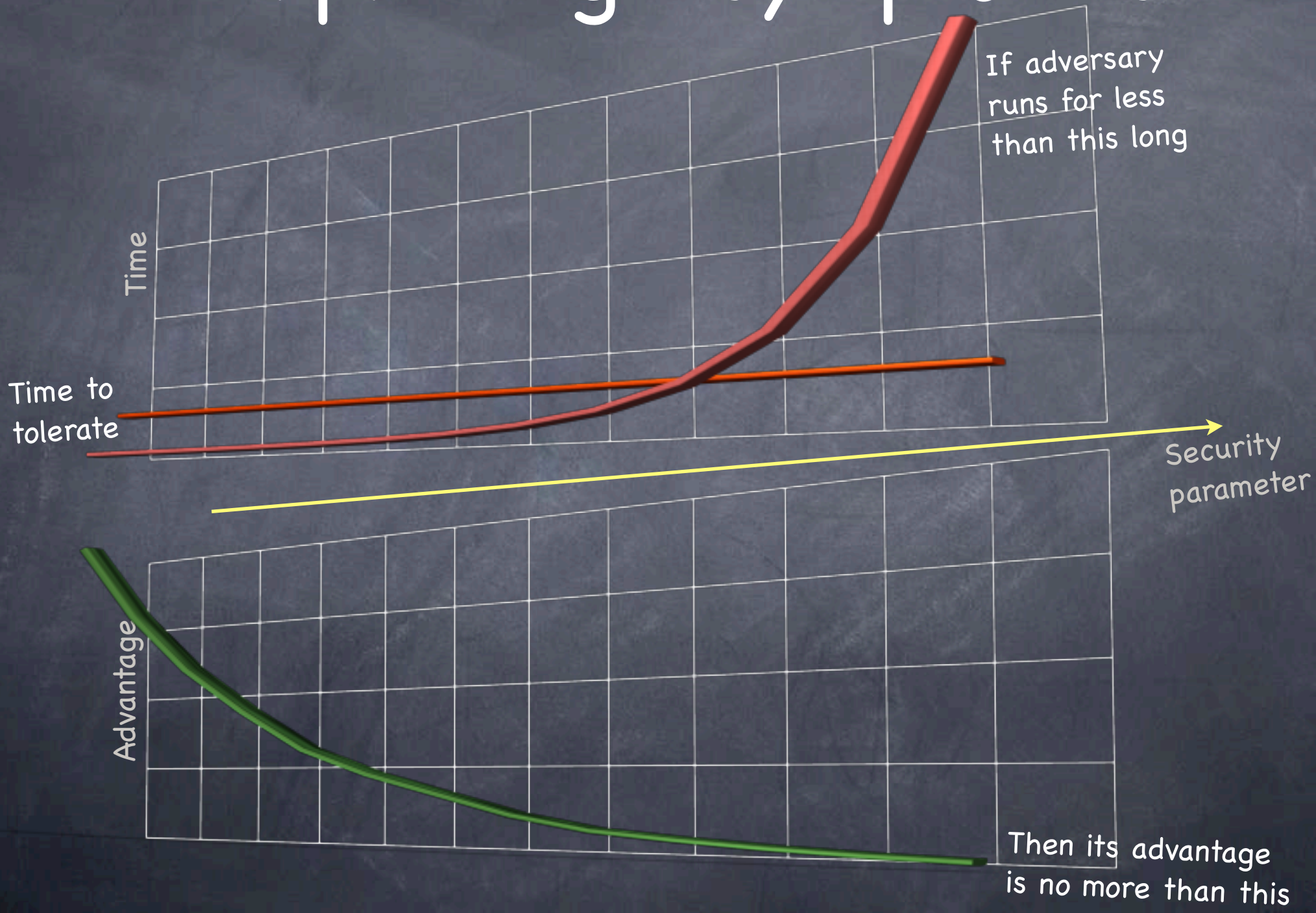

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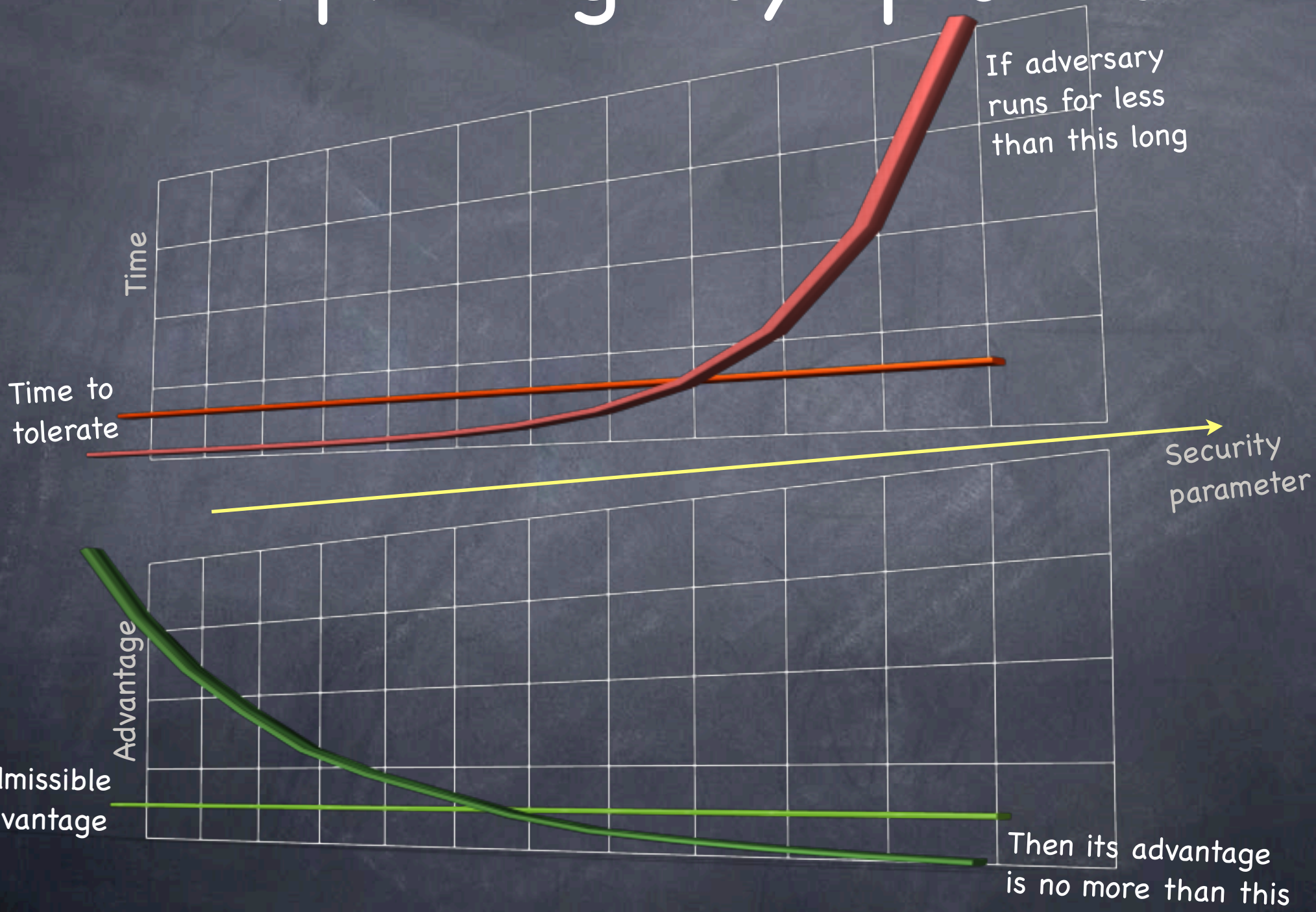

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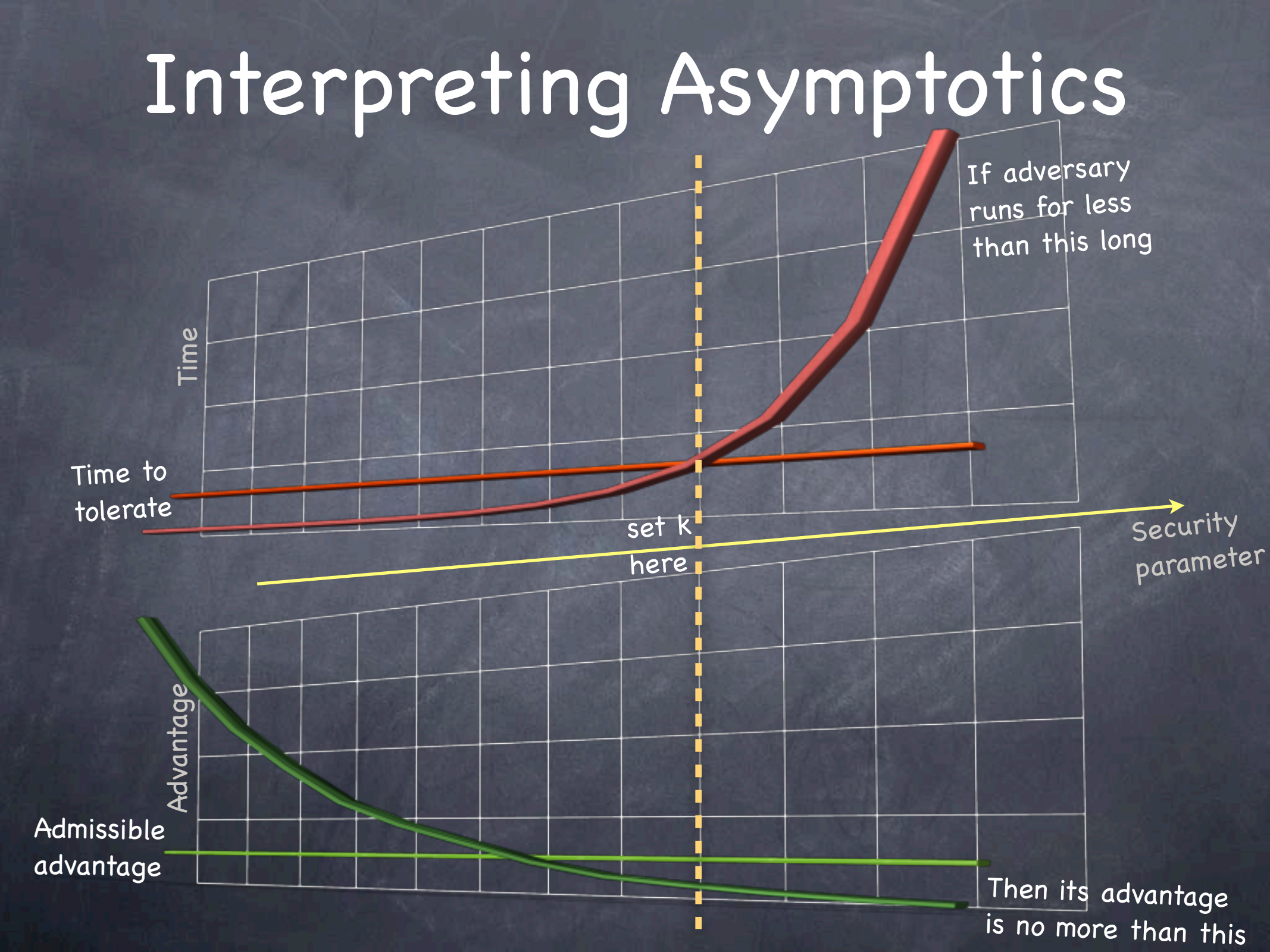

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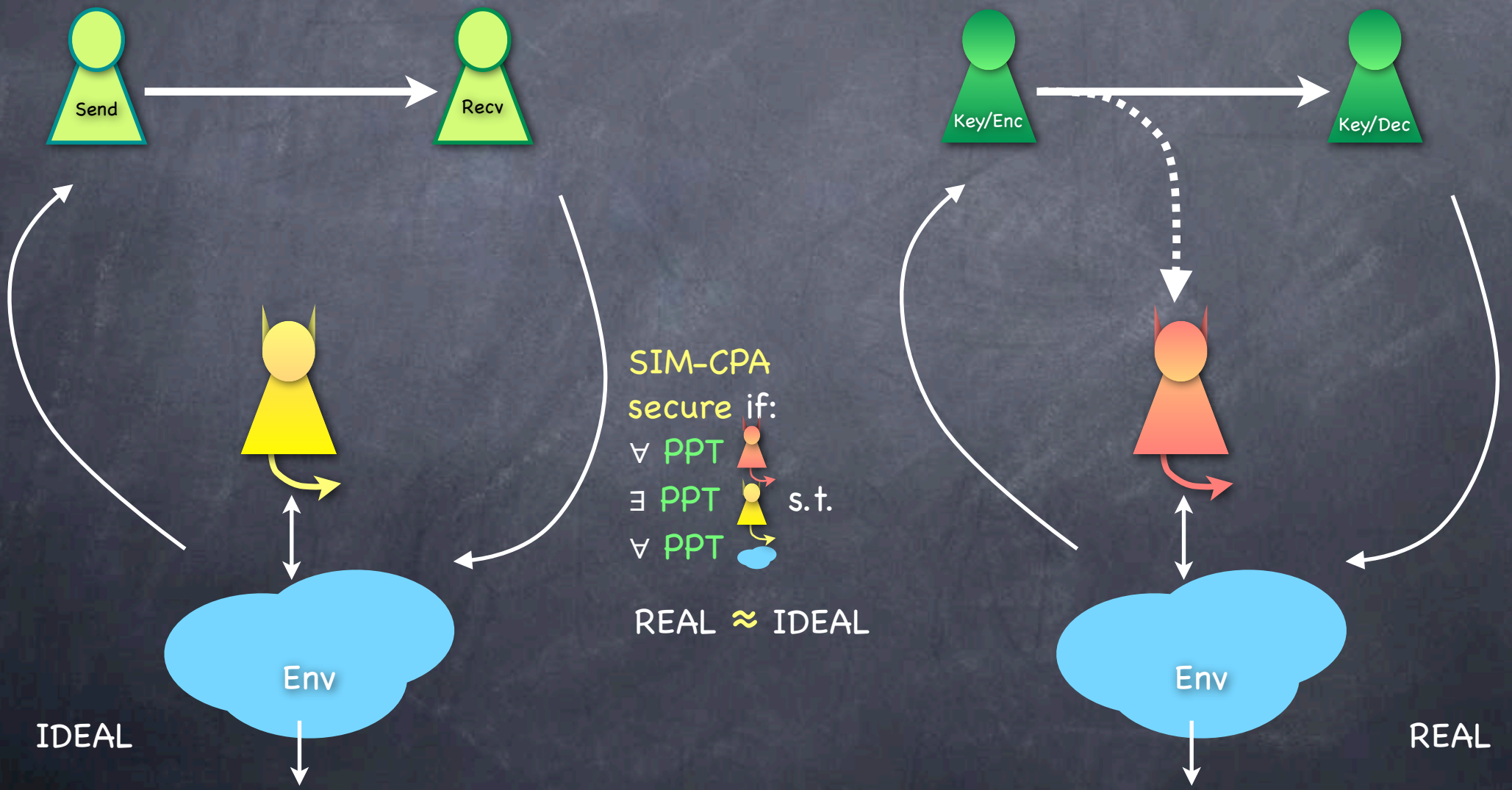
  - So that negl(k) $\times$ poly(k) = negl'(k)
    - Needed, because Eve can often increase advantage polynomially by spending that much more time/by seeing that many more messages

Symmetric-Key Encryption

SIM-CPA Security



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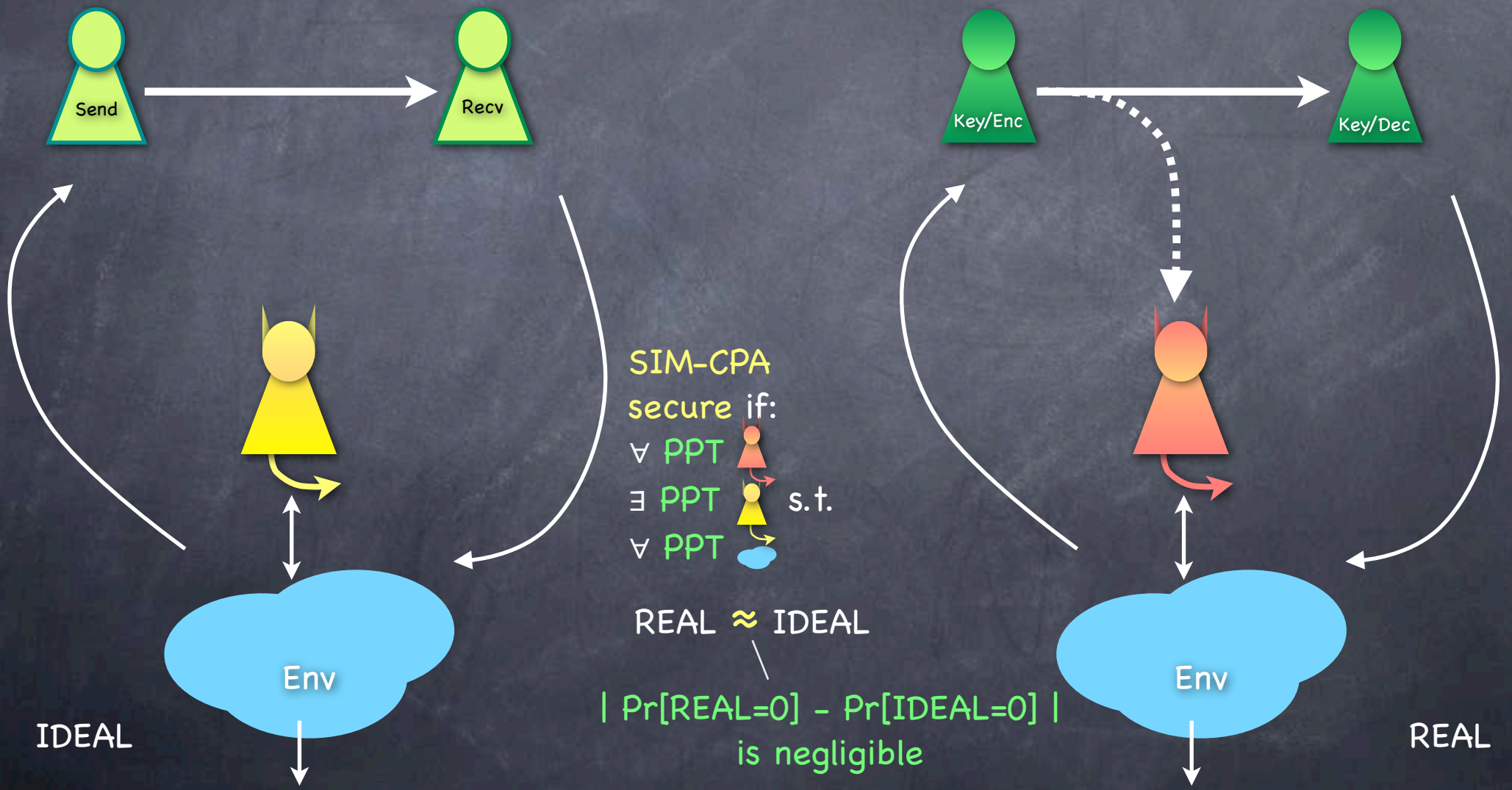

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  - Coming up: One-Way Functions, Hardcore predicates, PRG, ...

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  - Turns out they are equivalent!

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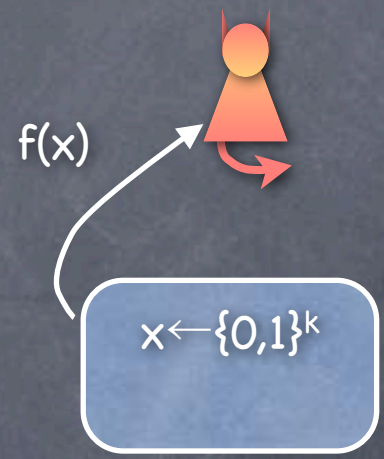

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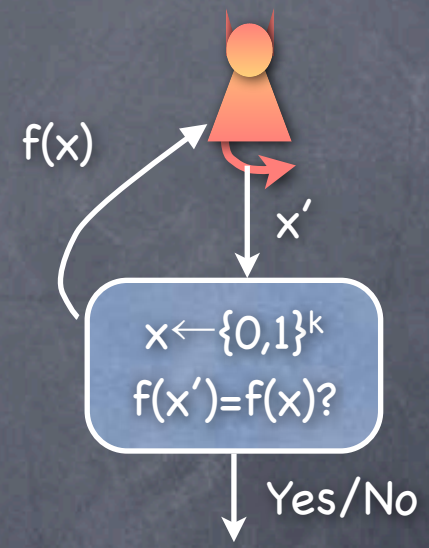

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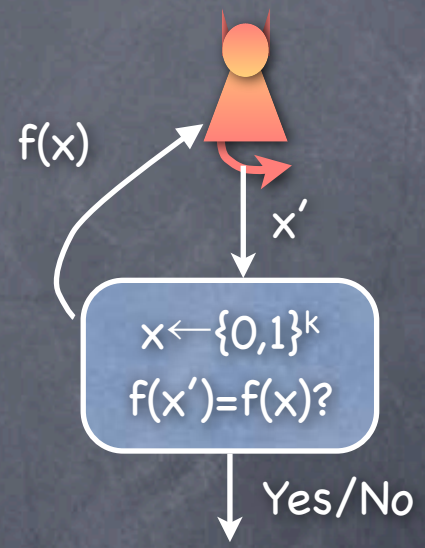

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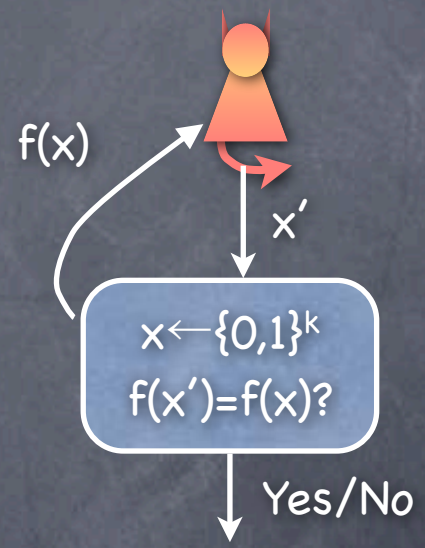

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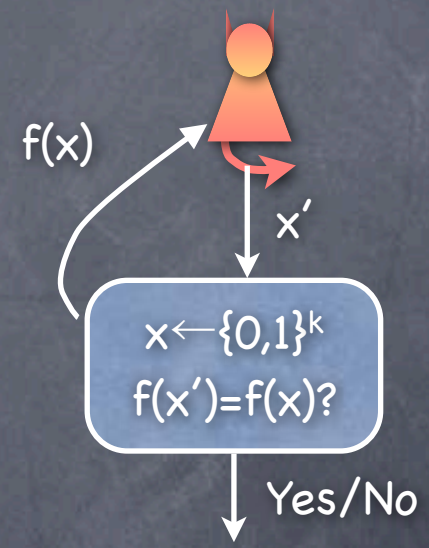

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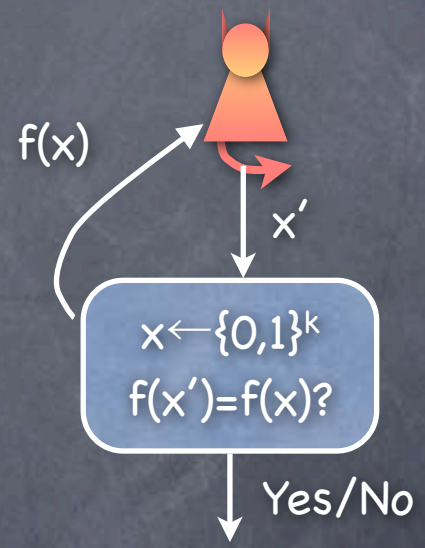

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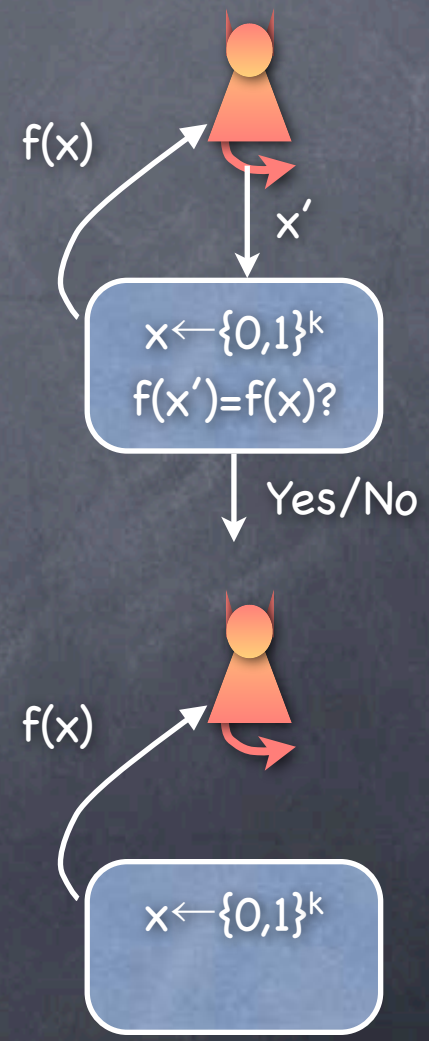

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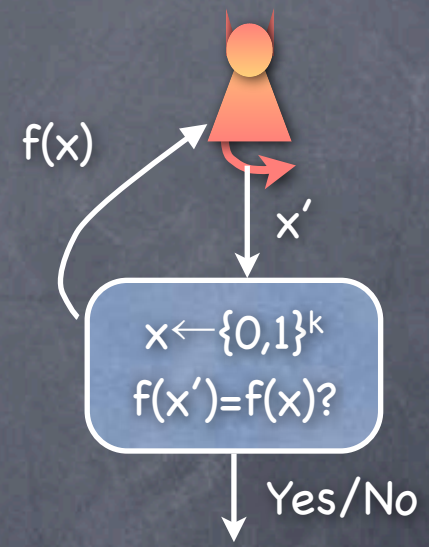

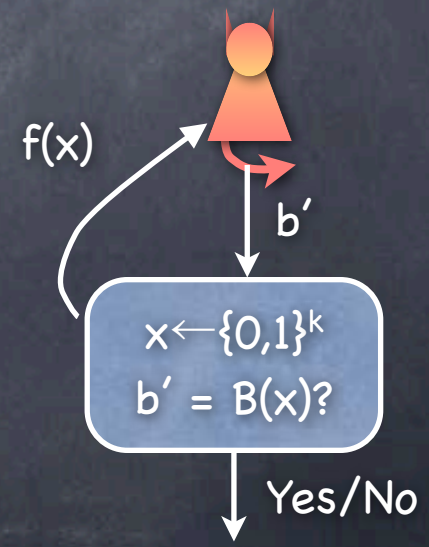

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- B is a hardcore predicate of a OWF f if
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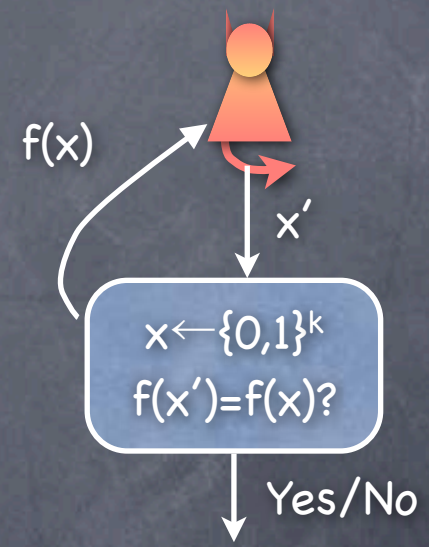
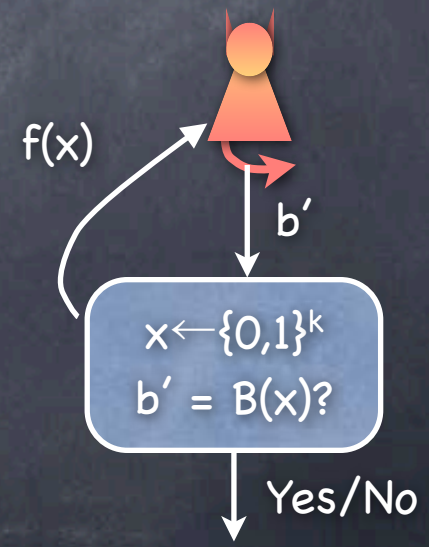
  - For all (non-uniform) PPT adversary, advantage in the Hardcore-predicate experiment is negligible
  - B(x) remains "completely" hidden, given f(x)





Next

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- Candidate OWFs

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- Using OWF/Hardcore-predicates to build PRG and (CPA-secure) SKE