Cryptography

Lecture 1

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Our first encounter with secrecy:
Secret-Sharing

Secrecy

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  - Access to learning and/or influencing information

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  - Access to learning and/or influencing information
- One of the aspects of access control is secrecy

A Game

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- Other ideas?

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- Note: any one share can be chosen before knowing the message [why?]

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Can't get Pr[msg_1|view] = Pr[msg_2|view], unless prior is uniform

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    - Leakage resilience ...

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- our previous example: (2,2) secret-sharing

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    - Pick $s_1,\ldots,s_{n-1}$ uniformly at random from G
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  - Claim: This is an (n,n) secret-sharing scheme [Why?]

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PROOF

Additive Secret-Sharing: Proof

- Share(M):
  - Pick $s_1,\ldots,s_{n-1}$ uniformly at random from G
  - Let $s_n = M - (s_1 + \ldots + s_{n-1})$
- Reconstruct($s_1,\ldots,s_n$): $M = s_1 + \ldots + s_n$
- Claim: Upto n-1 shares give no information about M
- Proof: Let $T \subseteq \{1,\ldots,n\}$, $|T| = n-1$. We shall show that $\{ s_i \}_{i\in T}$ is distributed the same way (in fact, uniformly) irrespective of what M is.
  - For concreteness consider $T = \{2,\ldots,n\}$. Fix any (n-1)-tuple of elements in G, $(g_1,\ldots,g_{n-1}) \in G^{n-1}$. To prove $Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})]$ is same for all M.
  - Fix any M.
  - $(s_2,\ldots,s_n) = (g_1,\ldots,g_{n-1}) \Leftrightarrow (s_2,\ldots,s_{n-1}) = (g_1,\ldots,g_{n-2})$ and $s_1 = M-(g_1+\ldots+g_{n-1})$.
  - So $Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})] = Pr[(s_1,\ldots,s_{n-1})=(a,g_1,\ldots,g_{n-2})]$, $a:=(M-(g_1+\ldots+g_{n-1})$
  - But $Pr[(s_1,\ldots,s_{n-1})=(a,g_1,\ldots,g_{n-2})] = 1/|G|^{n-1}$, since $(s_1,\ldots,s_{n-1})$ are picked uniformly at random from G
  - Hence $Pr[(s_2,\ldots,s_n)=(g_1,\ldots,g_{n-1})] = 1/|G|^{n-1}$, irrespective of M. □

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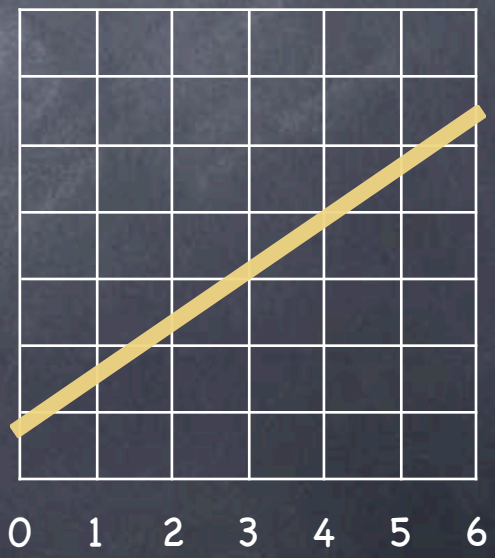

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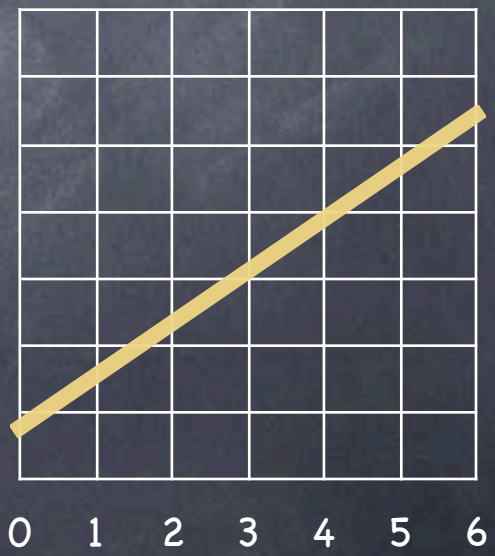

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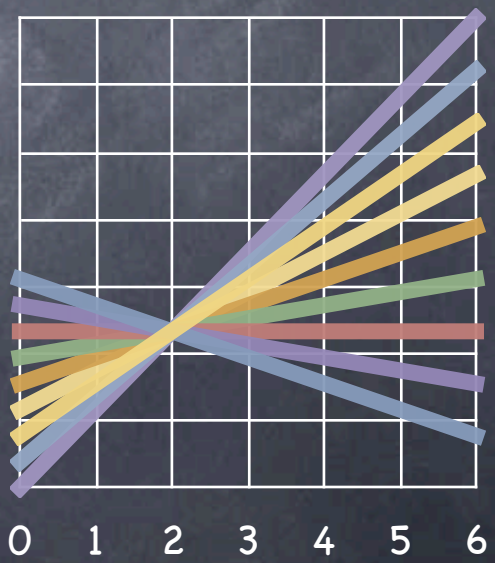

Threshold Secret-Sharing

- Construction: (n,2) secret-sharing
- Message-space = share-space = F, a field (e.g. integers mod a prime, $\mathbb{F}_p$)
  - Share(M): pick random r. Let $s_i = r \cdot a_i + M$ (for $i=1,\ldots,n < |F|$)
  - Reconstruct(s_i, s_j): $r = (s_i - s_j)/(a_i - a_j)$; $M = s_i - r \cdot a_i$
  - Each s_i by itself is uniformly distributed, irrespective of M [Why?]
  - "Geometric" interpretation
    - Sharing picks a random "line" y = f(x), such that f(0)=M. Shares $s_i = f(a_i)$.
    - s_i is independent of M: exactly one line passing through (a_i,s_i) and (0,M') for any secret M'
    - But can reconstruct the line from two points!

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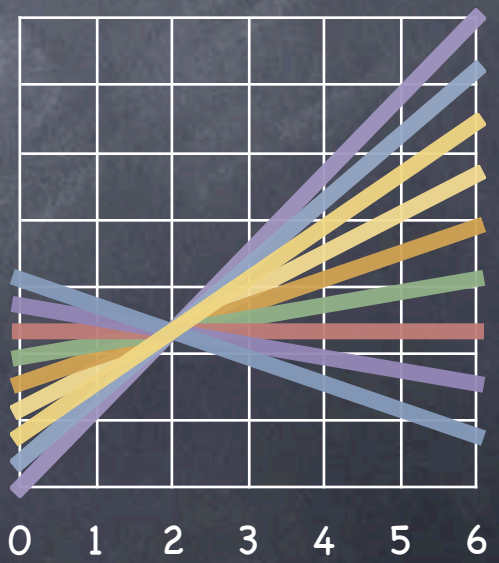

PROOF

(n,2) Secret-Sharing: Proof

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- Reconstruct(s_i, s_j): $r = (s_i - s_j)/(a_i - a_j)$; $M = s_i - r \cdot a_i$

- Claim: Any one share gives no information about M
- Proof: For any $i \in \{1,..,n\}$ we shall show that s_i is distributed the same way (in fact, uniformly) irrespective of what M is.
- Consider any $g \in F$. We shall show that Pr[$s_i=g$] is independent of M.
- Fix any M.
- For any $g \in F$, $s_i = g \Leftrightarrow r \cdot i + M = g \Leftrightarrow r = (g-M) \cdot a_i^{-1}$ (since $a_i \neq 0$)
- So, Pr[$s_i=g$] = Pr[$r=(g-M) \cdot a_i^{-1}$] = $1/|F|$, since r is chosen uniformly at random

□

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  - Reconstruct($s_1,\ldots,s_t$): Lagrange interpolation to find M=c_0
    - Need t points to reconstruct the polynomial. Given t-1 points, out of $|F|^{t-1}$ polynomials passing through (0,M') (for any M') there is exactly one that passes through the t-1 points

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- Shamir's secret-sharing solves threshold secret-sharing. How about the others?

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  - More efficient schemes known for large classes of access structures

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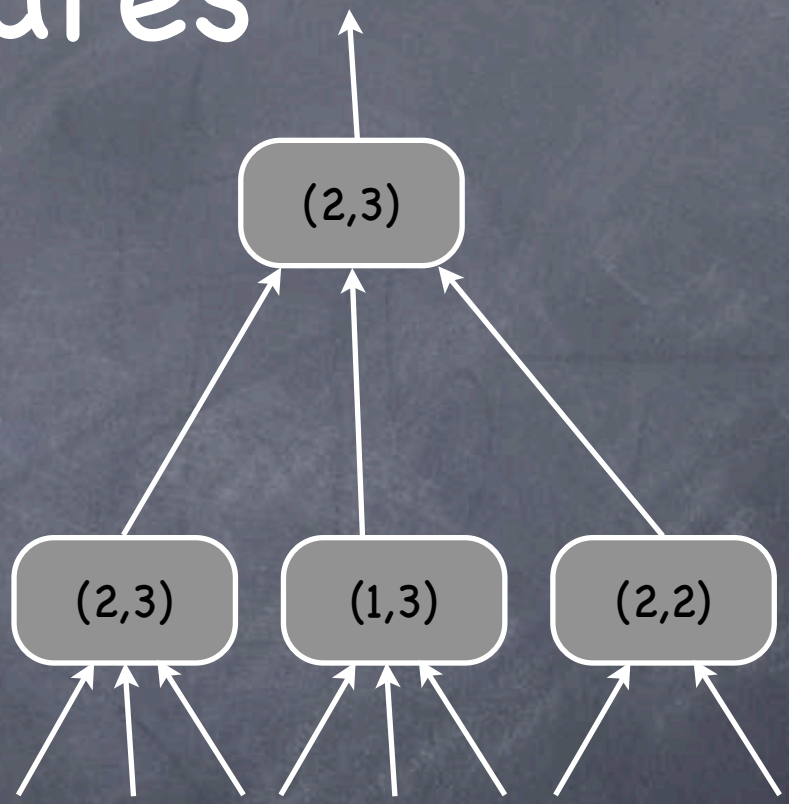

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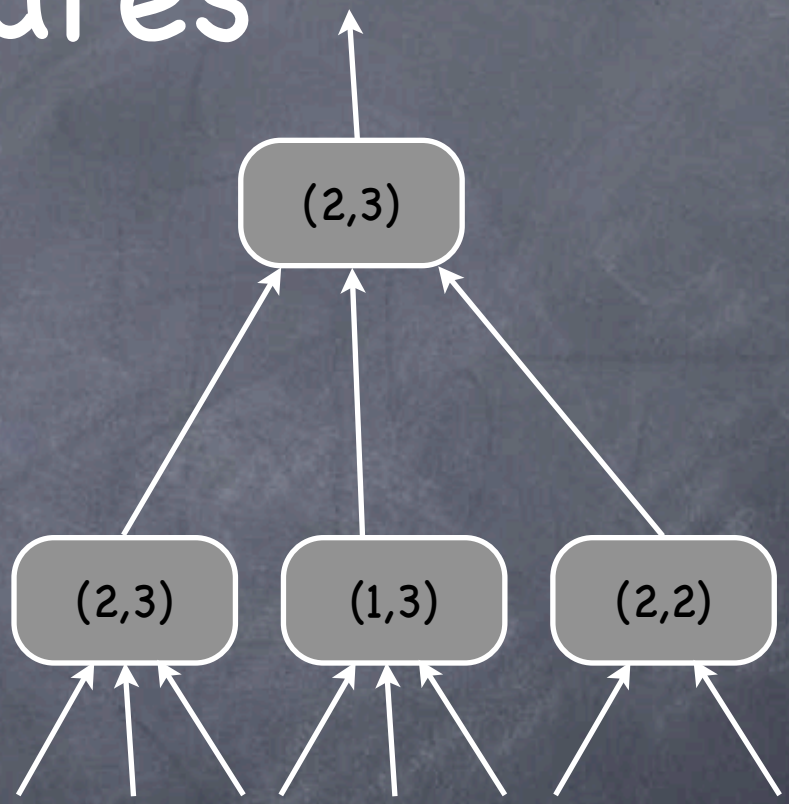

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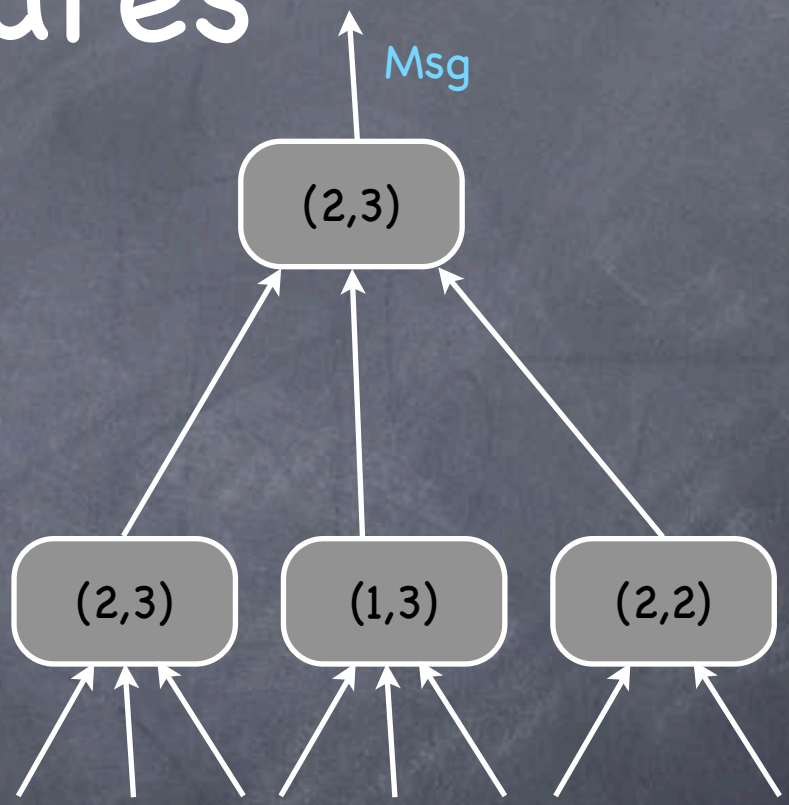

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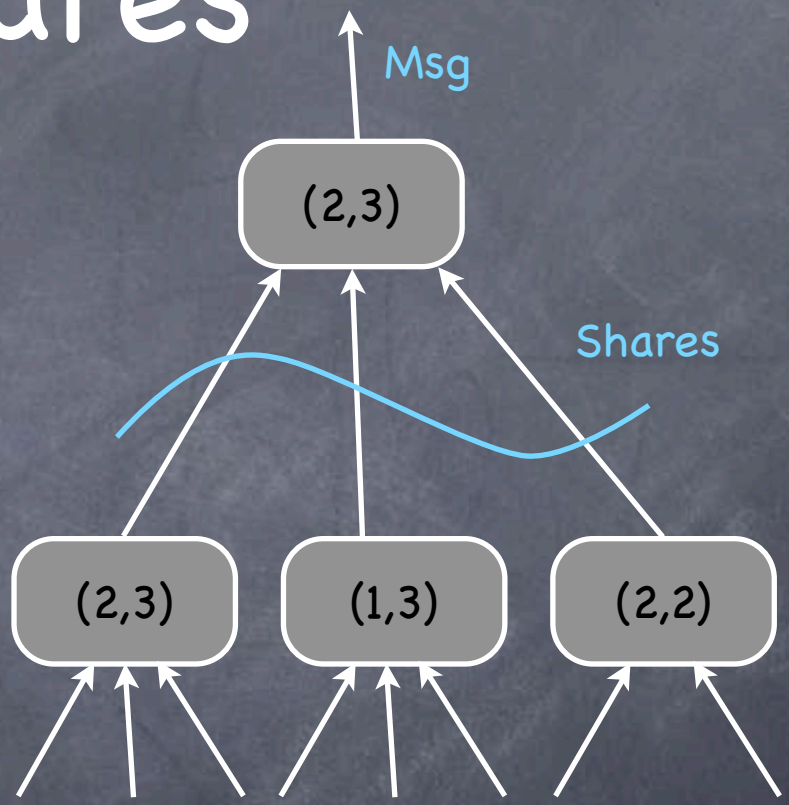

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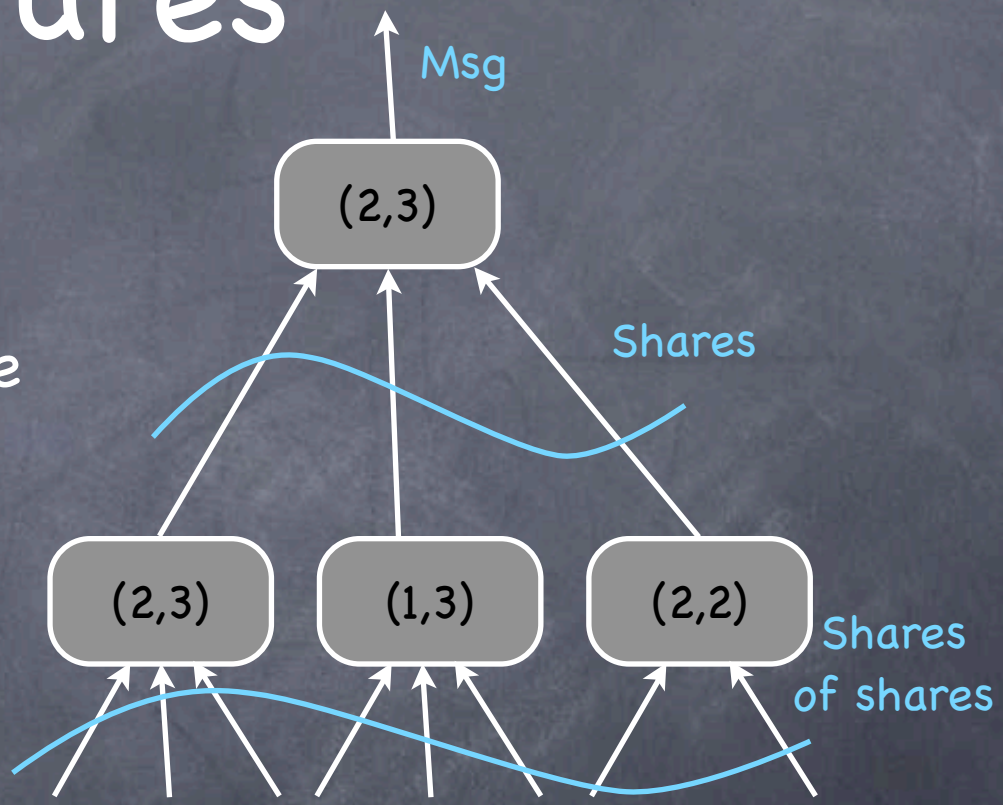

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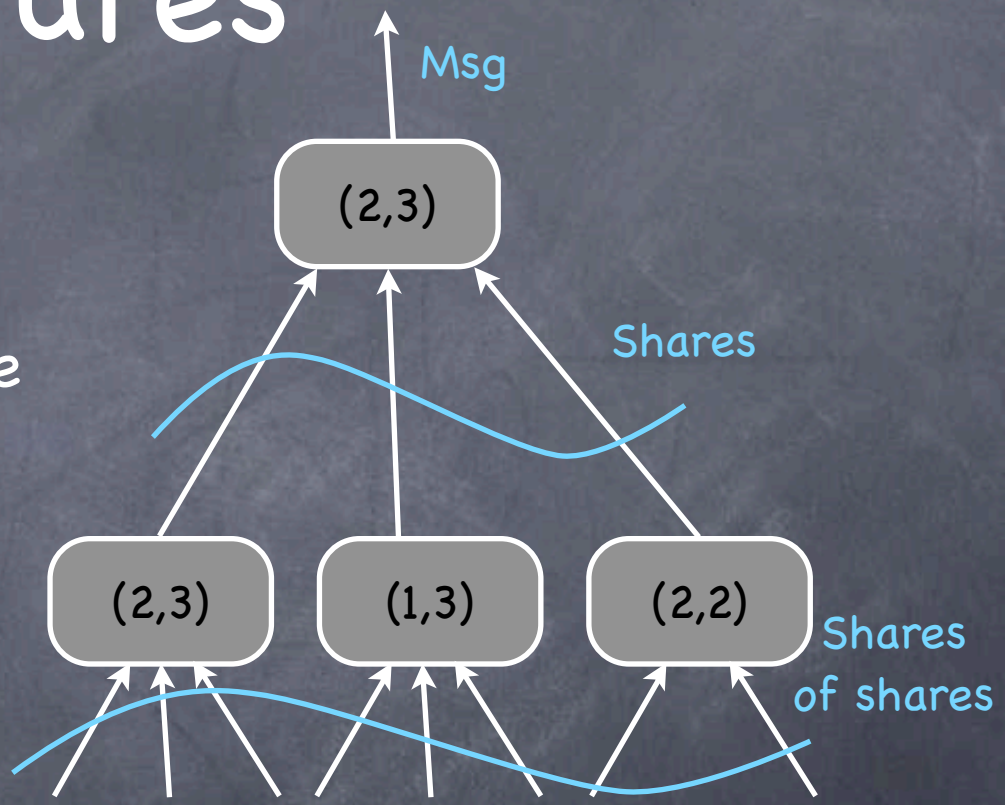

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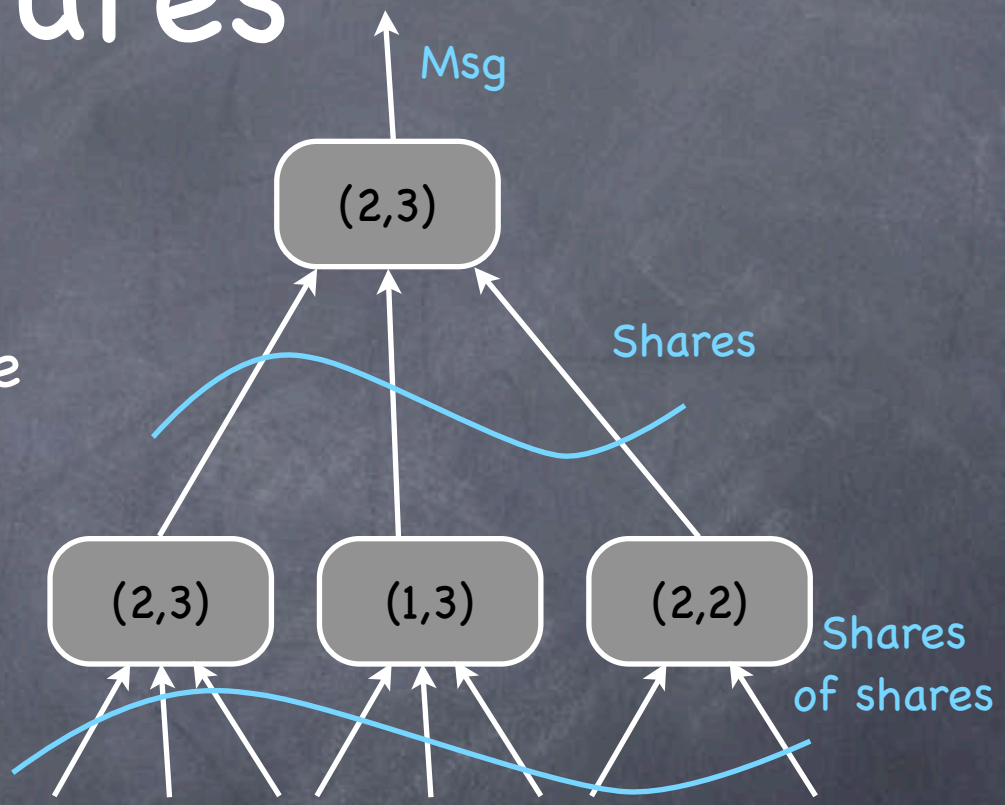
  - Admits linear secret-sharing



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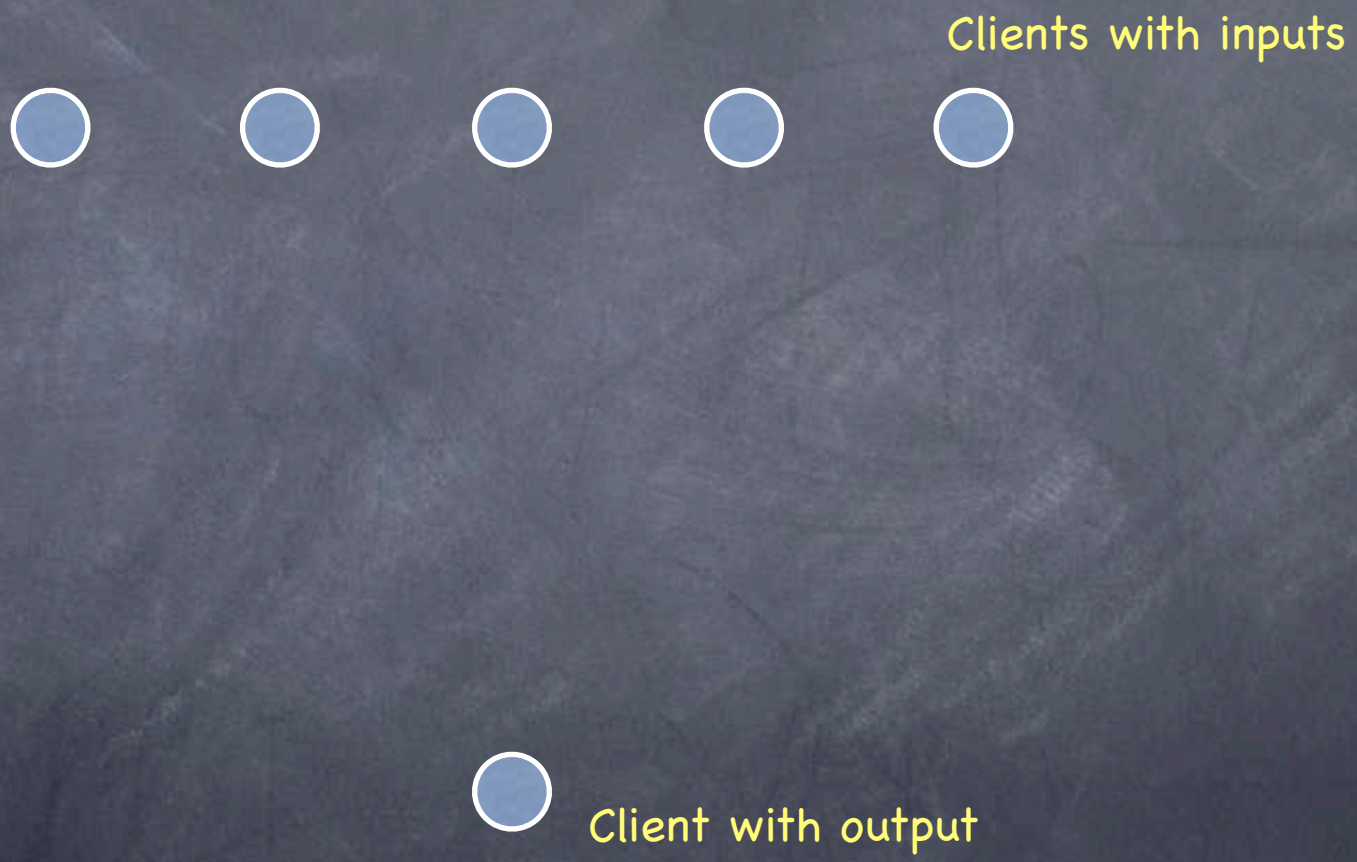

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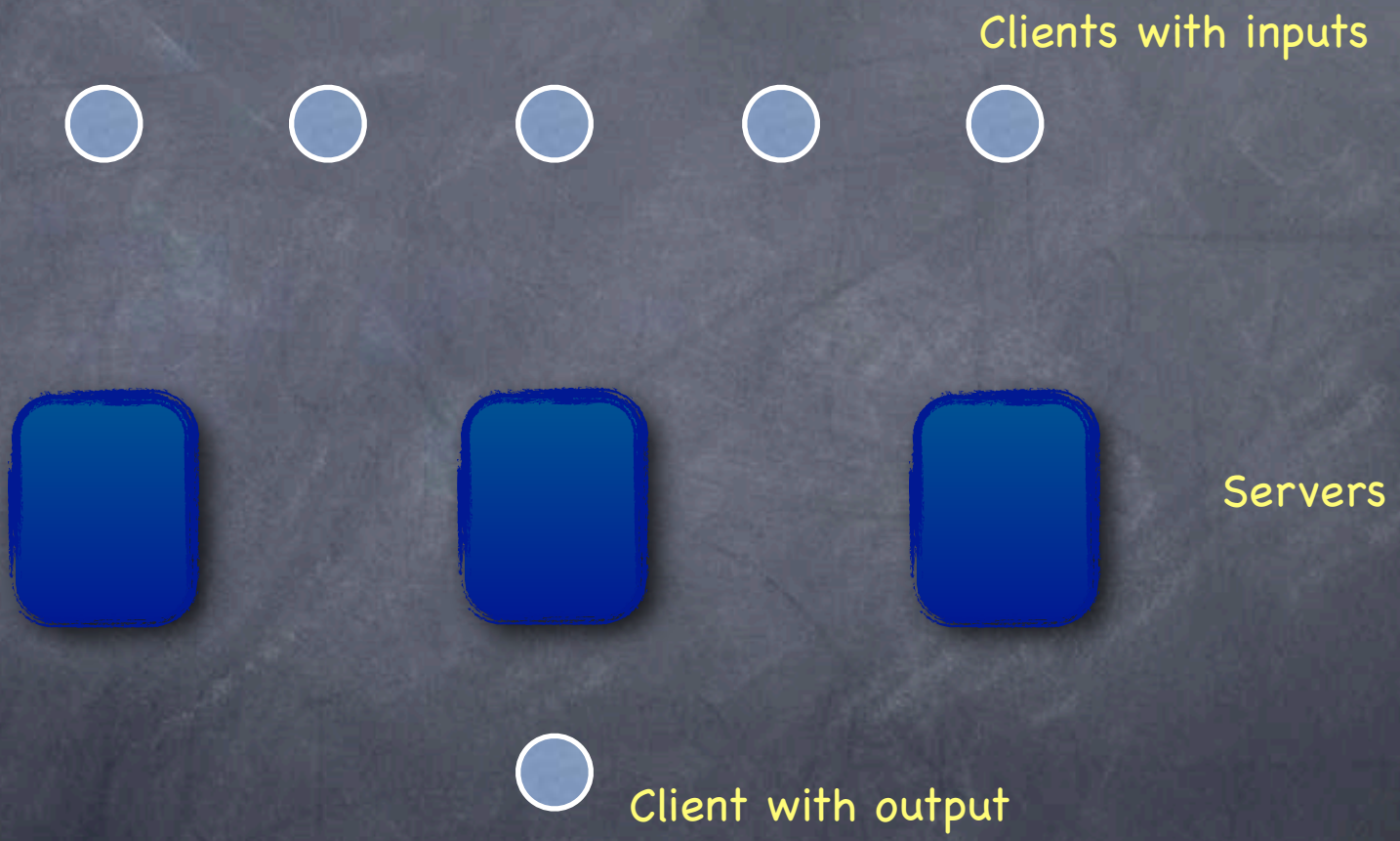

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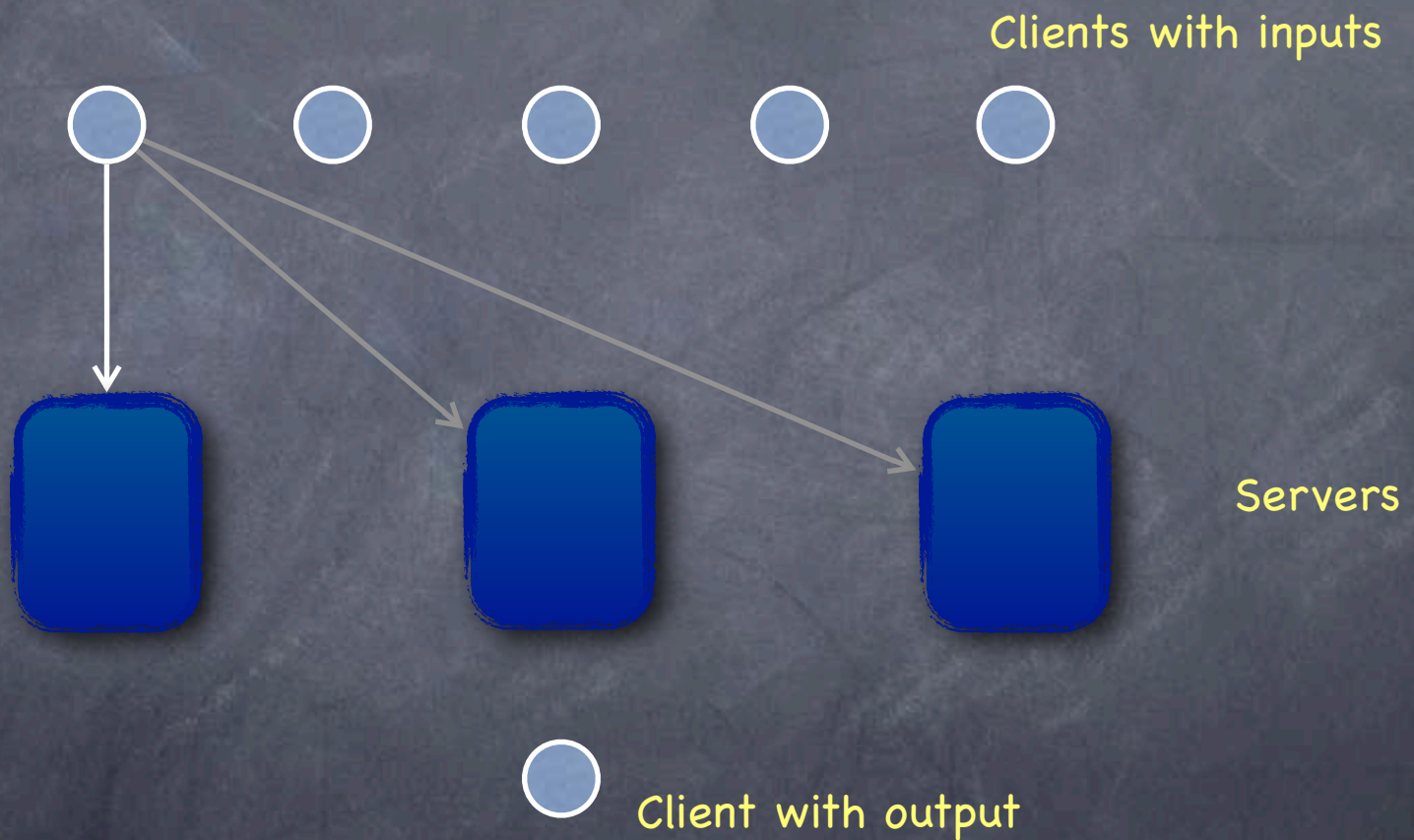

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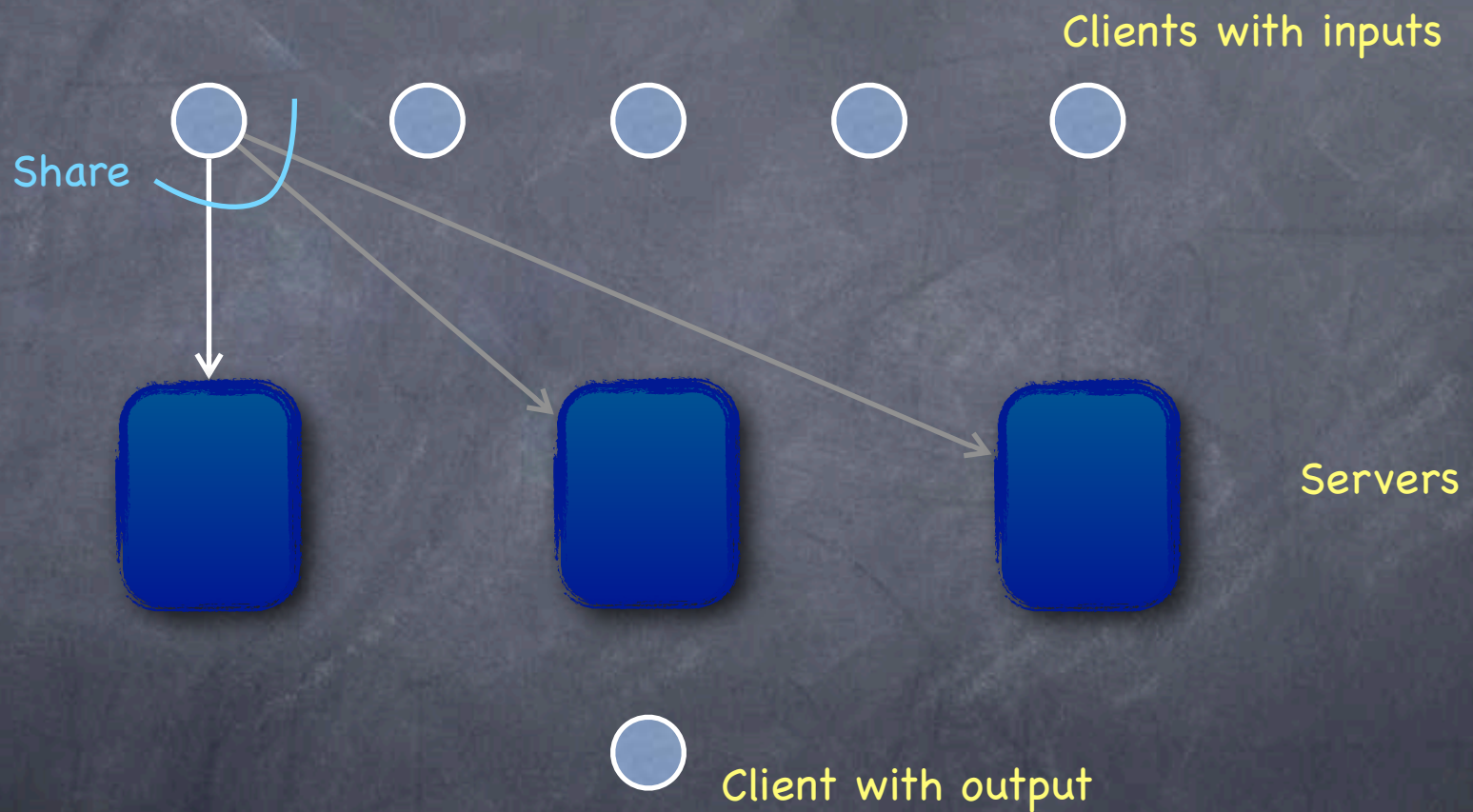

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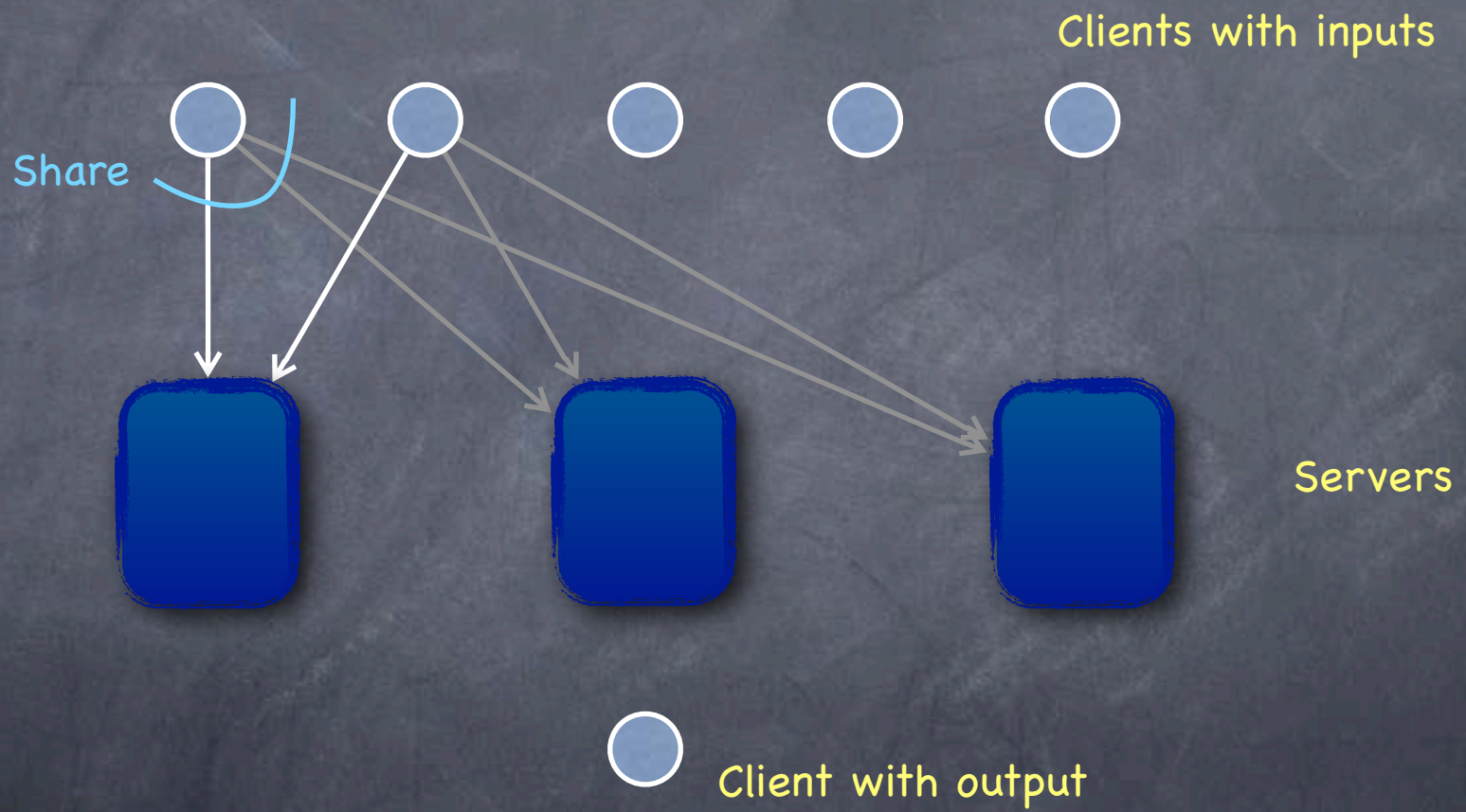

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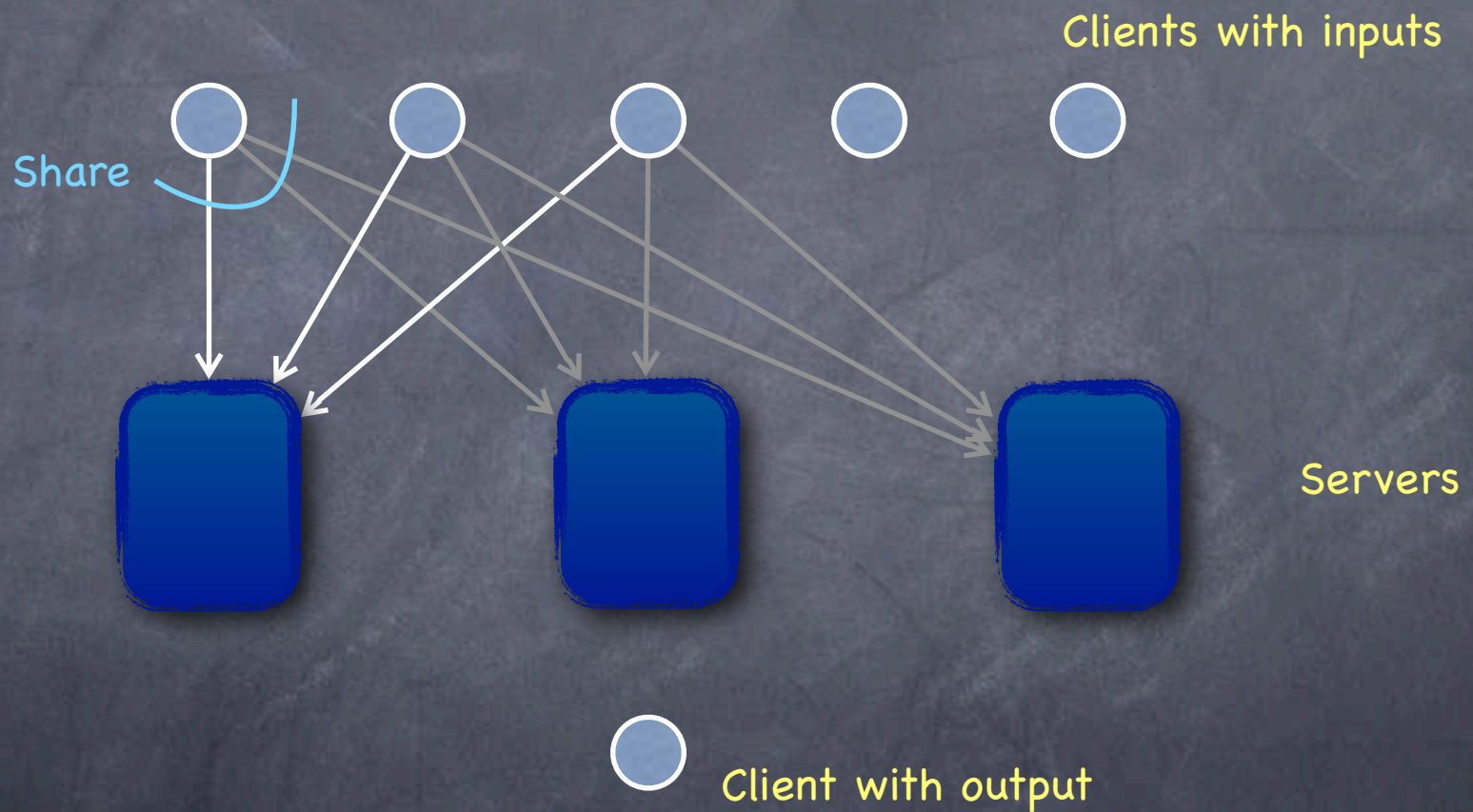

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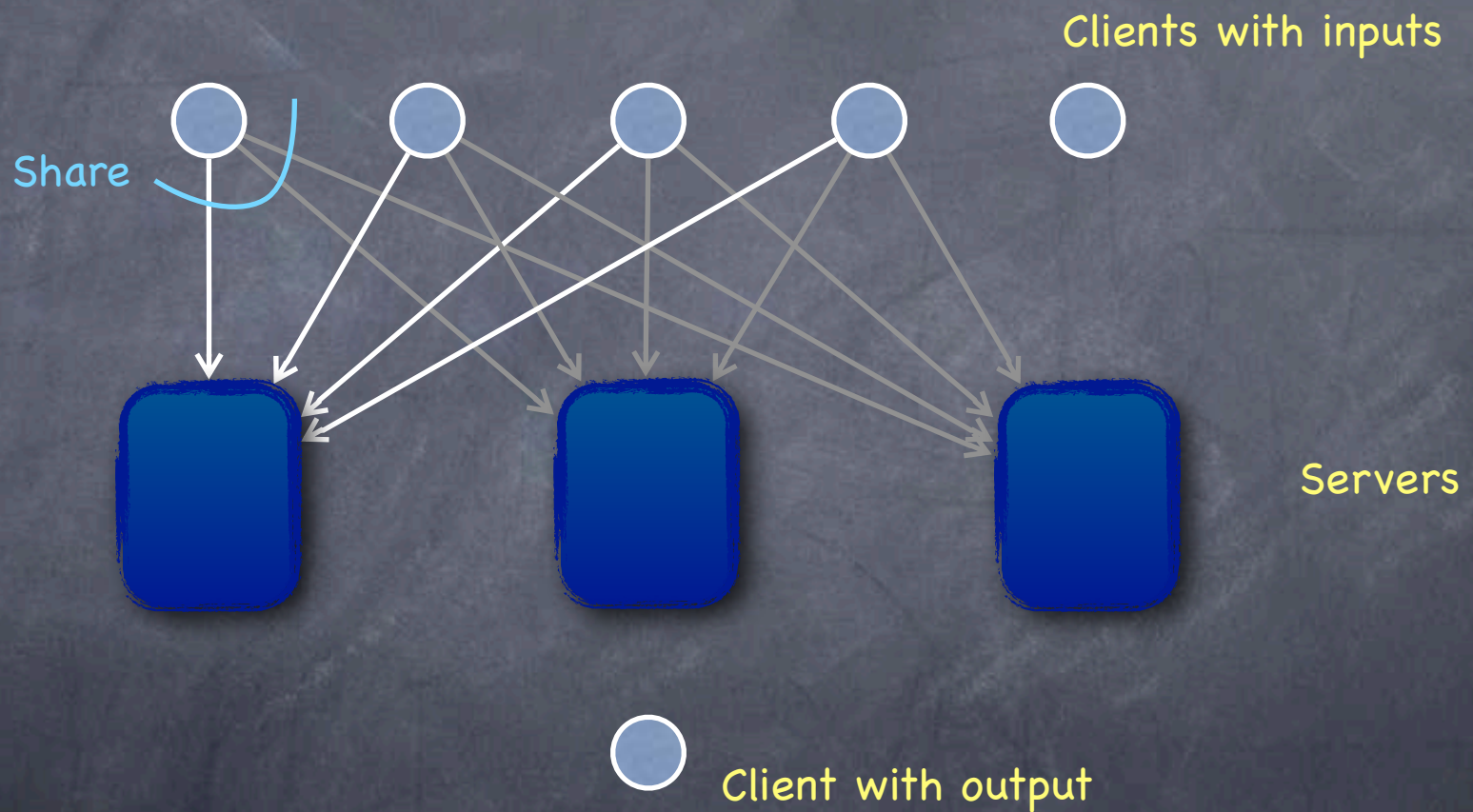

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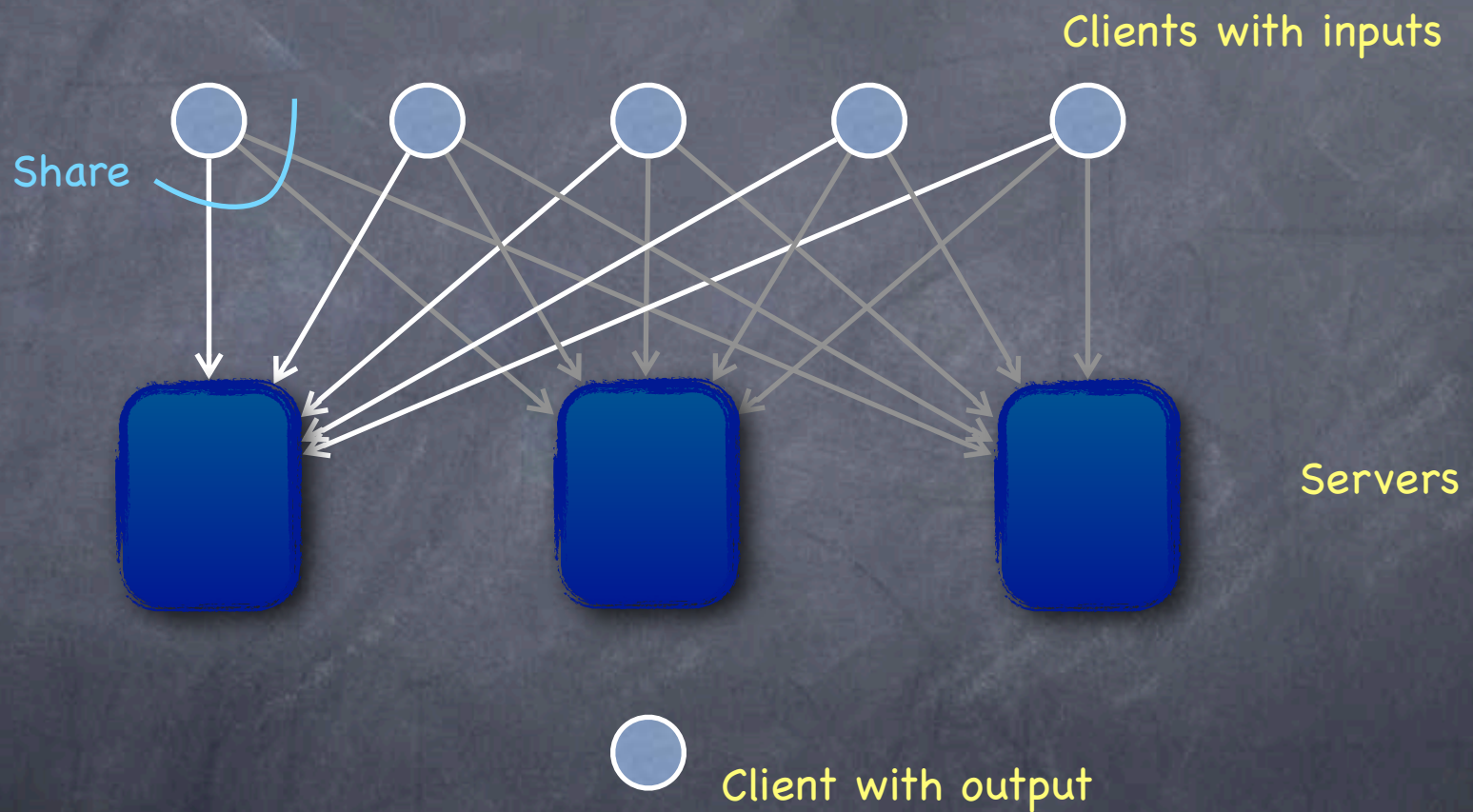

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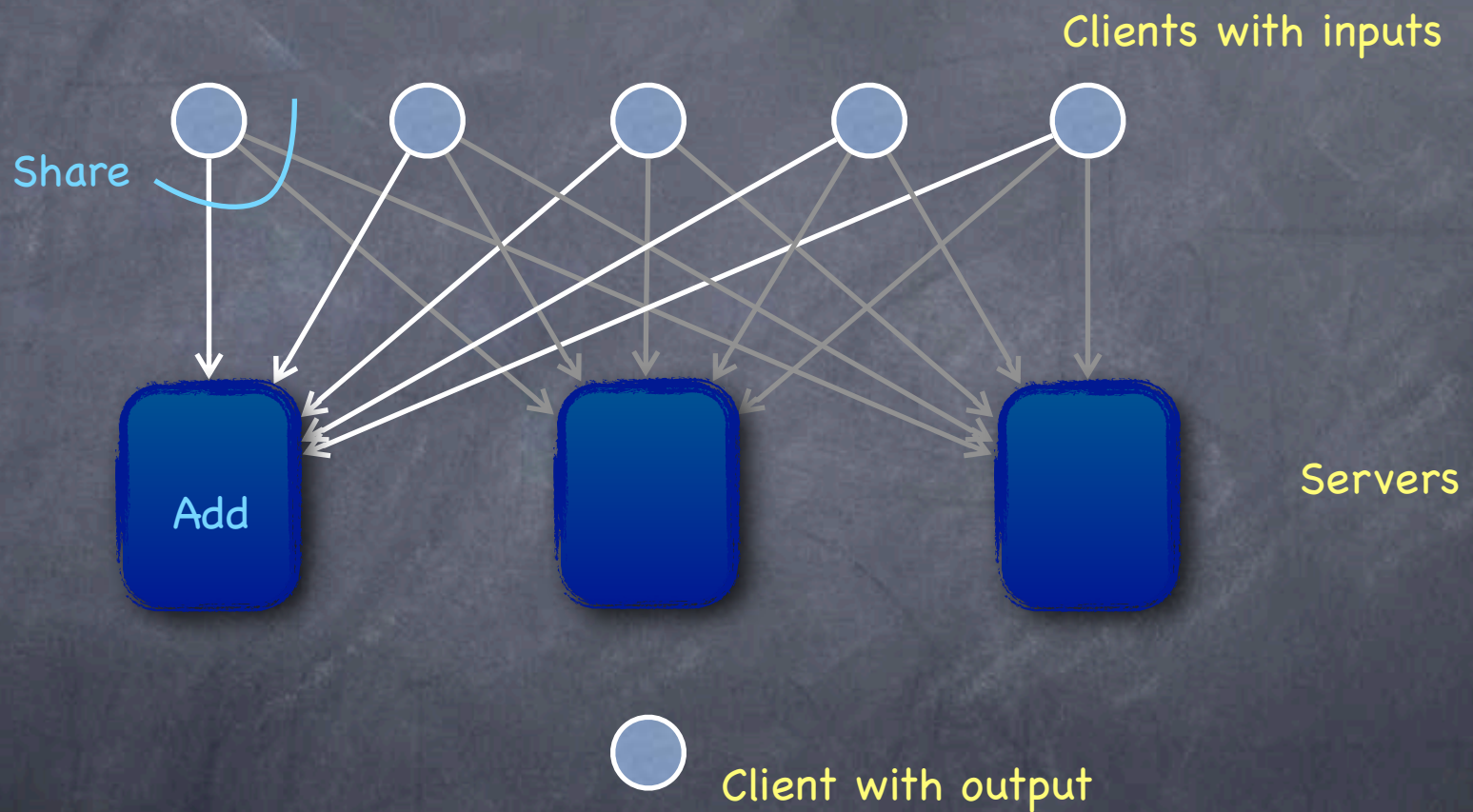

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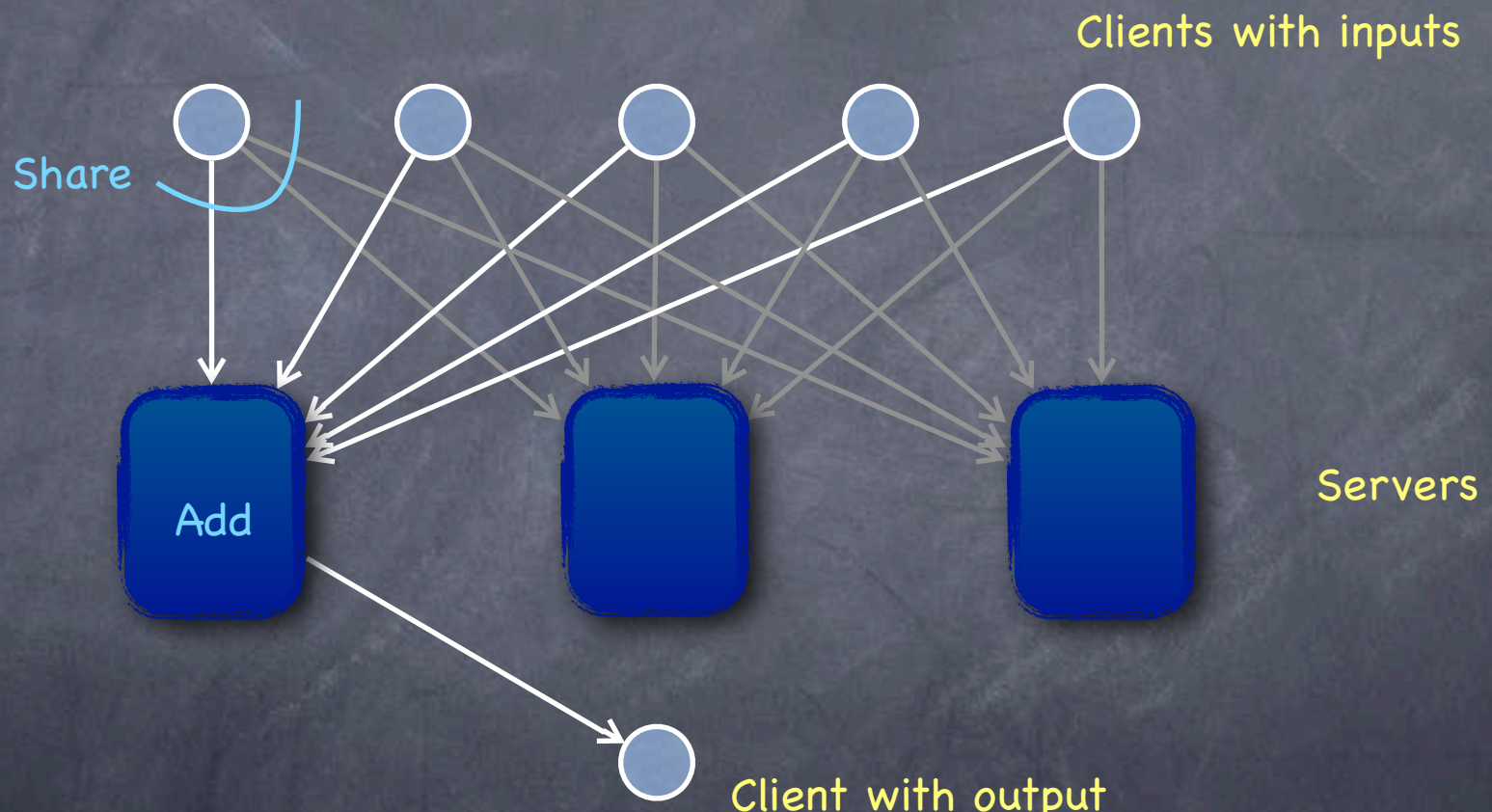

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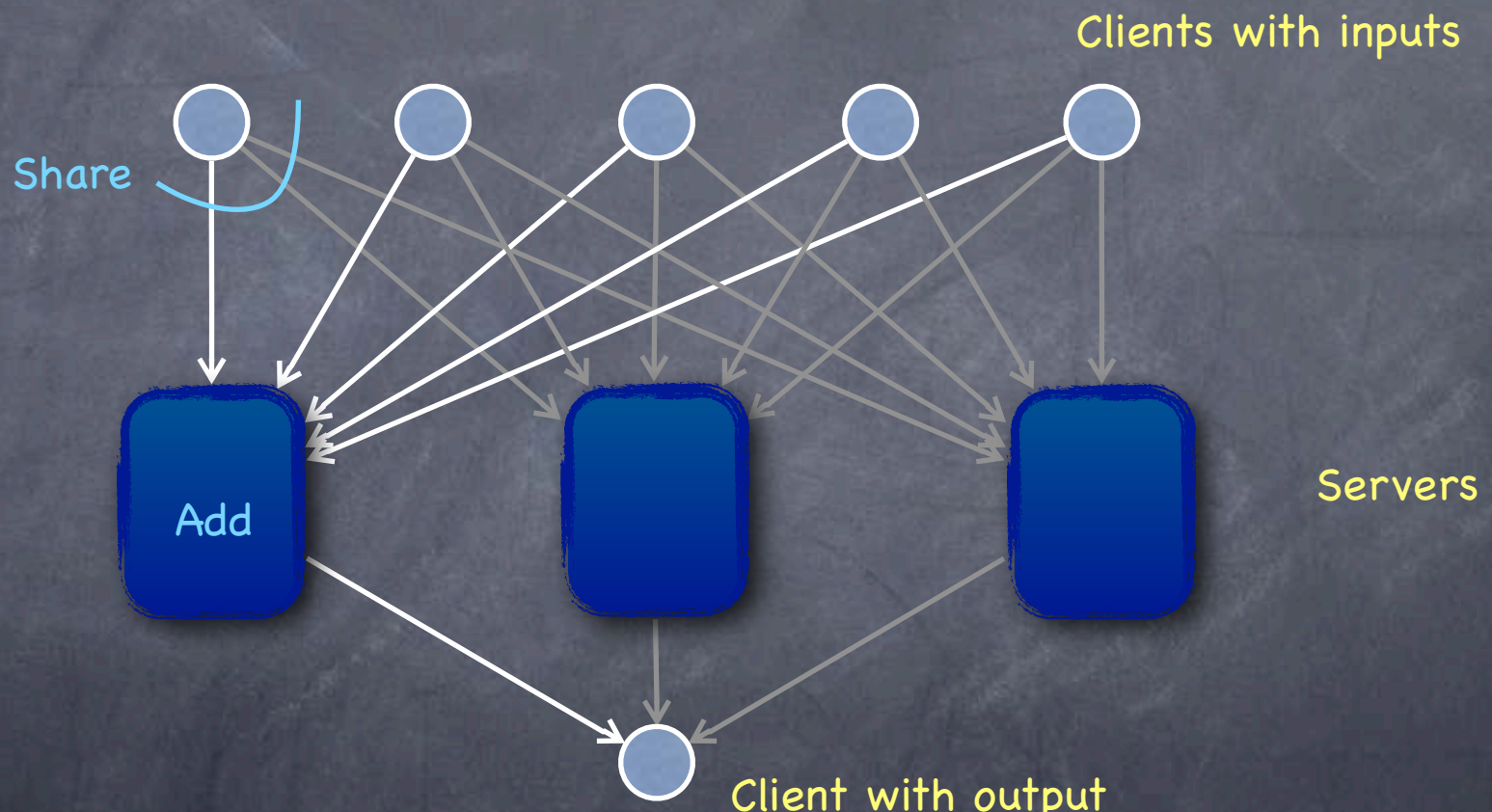

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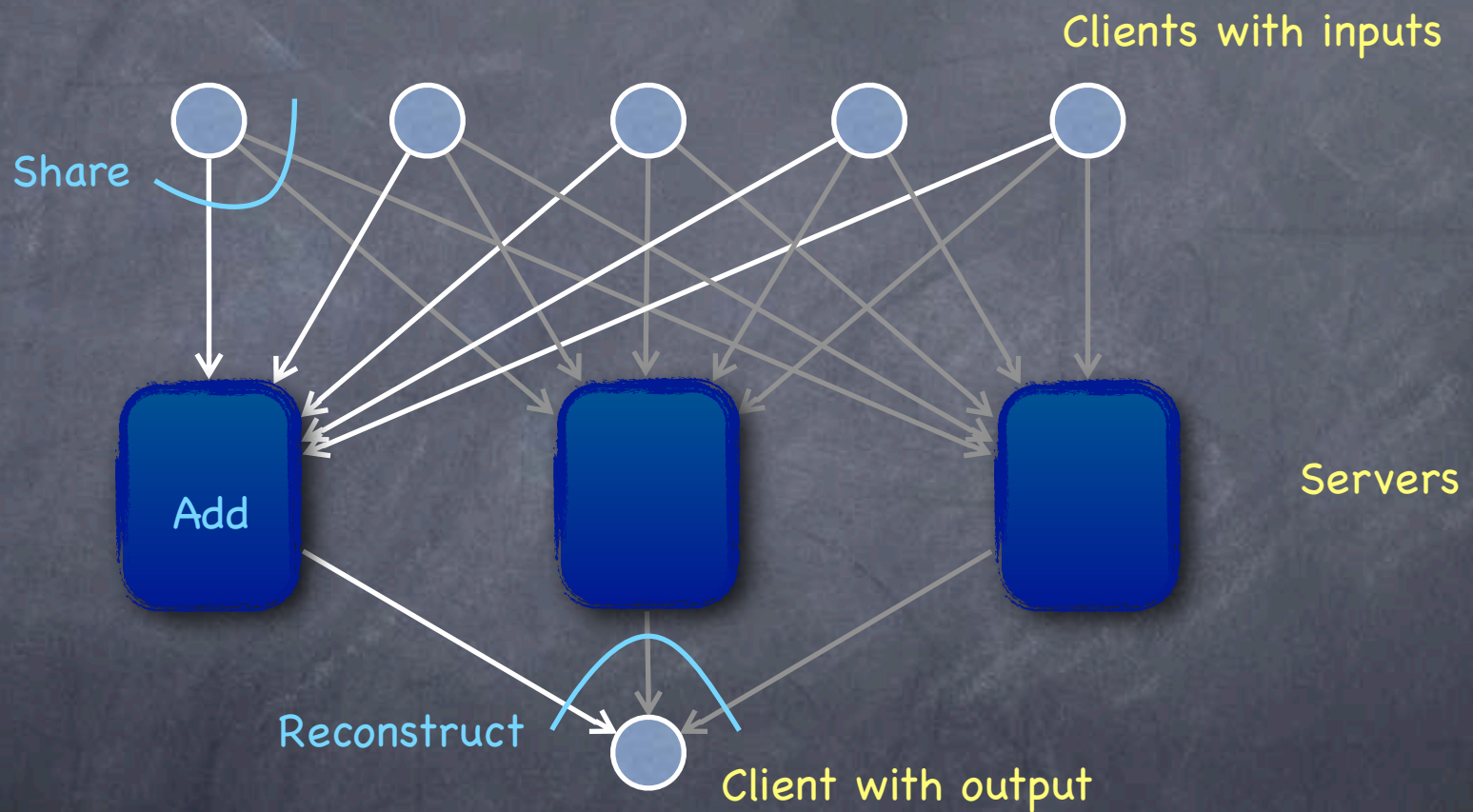

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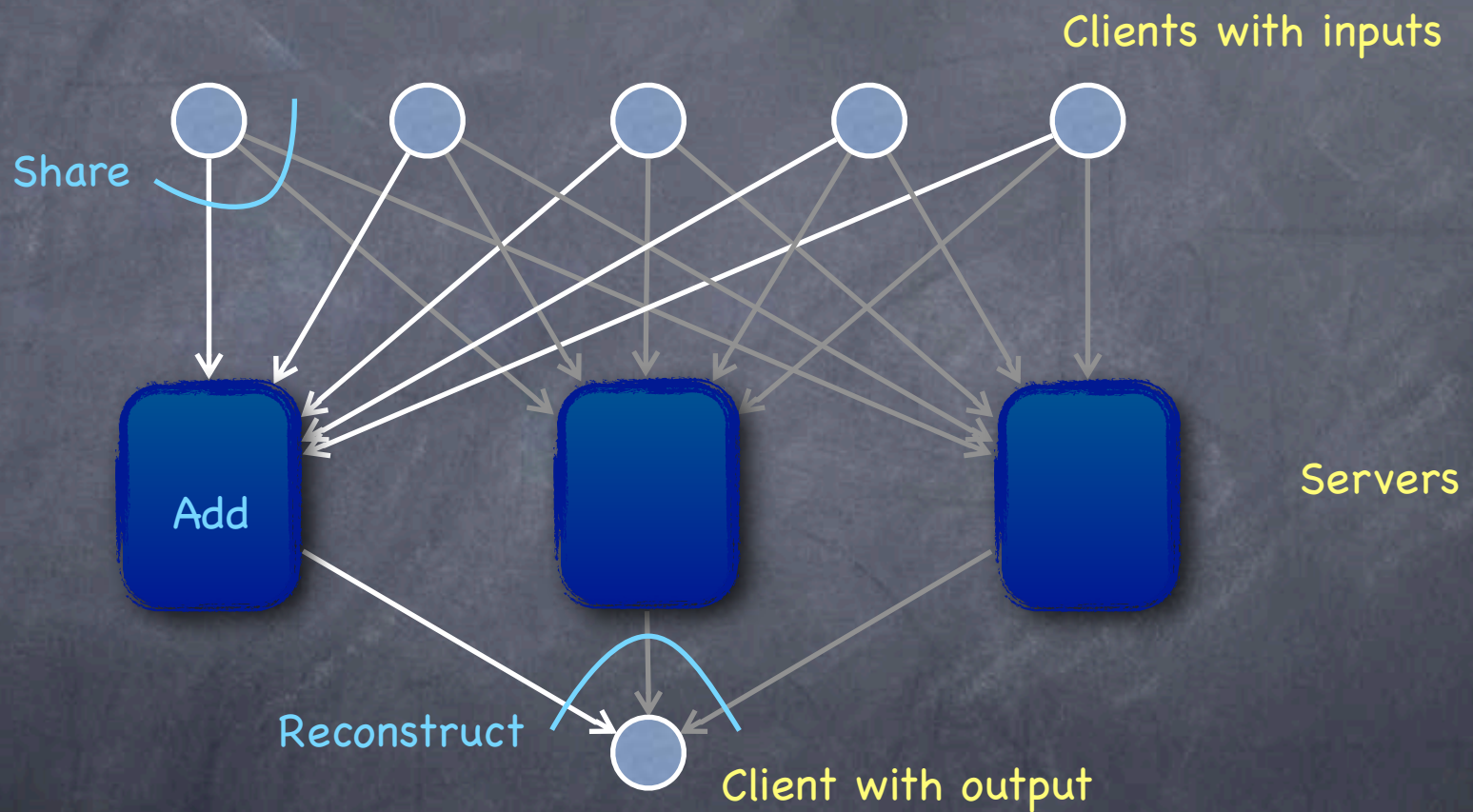

- Secure against passive corruption (no colluding set of servers/clients learn more than what they must) if at least one server stays out of the collusion

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    - Not all access structures have ideal schemes
  - Non-linear schemes can be more efficient than linear schemes

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- Next: secrecy against computationally bounded players