e-Cash

Lecture 25

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- Users should not be able to cheat honest merchants. In particular, users should not be able to double-spend

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- Merchant contacts the Bank (online) who ensures that the coin with that serial number has not been used before (i.e., no double spending) and the signature is valid. If so adds the coin to the spent-coin list

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 - Unforgeability: After t sessions, User cannot output signatures on t+1 distinct messages

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- Efficient variants (under suitable assumptions) using Groth-Sahai NIZK (or NIWI) scheme and compatible primitives

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 - Idea: verification in two sessions of the spending protocol with the same coin exposes the user's identity

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 - Merchant needs to <u>transfer</u> the User's proof to Bank (i.e., Bank should be convinced that the merchant didn't fake)

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- Verification is interactive (but can be made transferable using Fiat-Shamir heuristics in the RO model)

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 - @ e.g. (Weak) Boneh-Boyen signature: Sign_{SK}(x) = $g^{1/(SK+x)}$

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 - Compact e-Cash: Remove linking multiple spending

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 - Need a PRF that supports efficient proofs

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- Efficient (but interactive) HVZK proofs known for requisite statements. Used to get compact e-cash in the Random Oracle Model [CHLO6]
- Alternately, working in groups with bilinear pairings, can use Groth-Sahai NIZK (under appropriate assumptions)

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- Security/Efficiency/Usability issues: need to cancel stolen electronic wallet; need to recharge electronic wallet (cellphone?) online, but protect it from malware; efficient multiple denomination coins; allow transferability; tracing may not deter double-spending

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 - Bob and Carol cannot link the persons who proved credentials to the persons who obtained credentials

- Introduced by Chaum in 1985
- Similar to e-cash, but must allow multiple uses (double-spending not an issue)
- Alice should be able to prove to Bob that she has a credential from Carol (cf. Alice withdraws a coin from Carol and spends it with Bob)
 - Bob and Carol cannot link the persons who proved credentials to the persons who obtained credentials
 - And they cannot link together multiple proofs coming from the same user

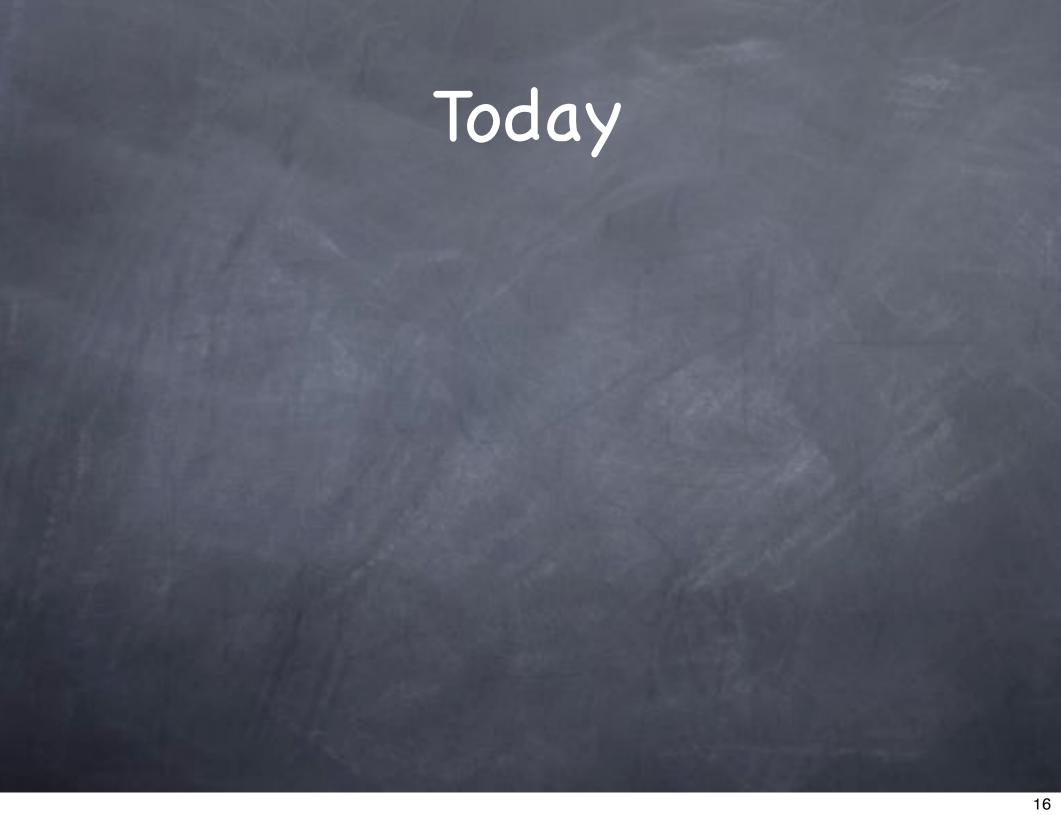
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- Obtaining credential: Carol signs SK_A using the P-Signature scheme using A_C (without learning SK_A). If Carol is a root authority, she requires a proof that A_C is a valid commitment of SK_A that corresponds to PK_A (not anonymous). Else Carol verifies that A_C has a credential from the root authority (as below)

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- Proving: Alice wants to prove to Carol that owner of A_C has a credential from Bob. She commits SK_A again to get A' and shows that she has a signature from Bob on the message in A'. She also proves that A' and A_C have the same message



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