# Broadcast Encryption and Some Other Primitives

Lecture 21

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  - c.f. (Ciphertext Policy) Attribute-Based Encryption: set of recipients decided dynamically

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  - Note: revoked users collude

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  - © Can use "hybrid encryption": encrypt a fresh key for a onetime encryption scheme (seed of a PRG), and use that key to encrypt the message

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    - Will settle for S such that it has at most r users revoked

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  - But can use PRG to derive keys so that each user hold only O(log²n) different keys

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    - If  $X_{uu'}$  covers a party at leaf w, it can derive  $K_{uu'}$ : Let v be the highest ancestor of u' for which w is not a descendent (i.e., v's sibling is on the u-w path). Use  $M_{u,v}$  to derive  $K_{uu'}$ .

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  - Security relies on an indistinguishability assumption involving O(n) group elements (cf. DDH has 3 group elements)

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- Useful for broadcast encryption, but also considered independently

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  - Use with subset cover based broadcast encryption? Can be used for "subset tracing", but not satisfactory if D decrypts only when, say, the subset that will be traced is large

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- Scheme with  $O(\sqrt{n})$  ciphertext, using bilinear pairing [BSW'06]

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  - May impose an upperbound on the number of colluding parties

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  - If P is a random symmetric polynomial of degree k in each variable, then the scheme is k-secure (i.e., for up to k users outside the group, the group key is perfectly random)

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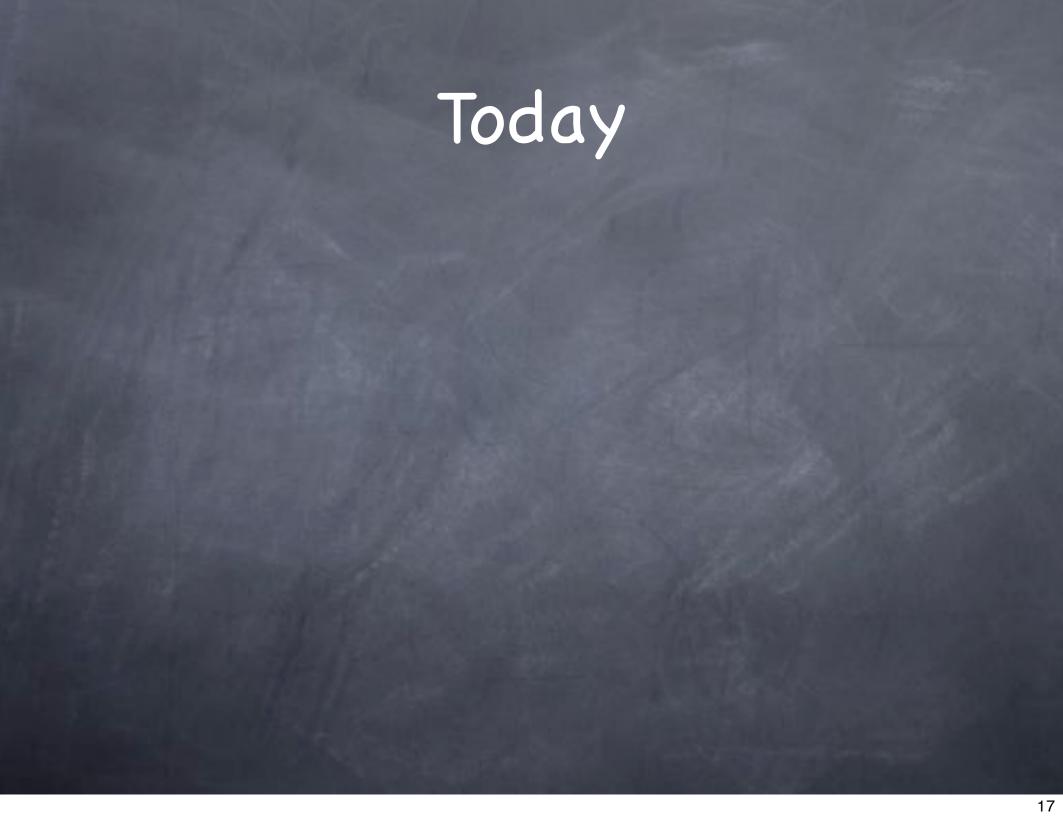
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- Can convert to authenticated group key agreement [KY'03]



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