Pairing-Based Cryptography &

Generic Groups

Lecture 19

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 - By D-BDH the key $e(g,g)^{abc} = e(g,g^{abc})$ is pseudorandom given eavesdropper's view (g^a,g^b,g^c)

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 - Trivial if only one witness. Very useful when two kinds of witnesses

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 - May exploit similar assumptions as used in the basic scheme

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 - Then, soundness will be under certain computational assumptions

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- Useful in proving statements like "these two commitments are to the same value", or "I have a signature for a message with a certain property", when appropriate commitment/signature scheme is used

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 - When e: $G_1 \times G_2 \rightarrow G_T$: DDH in G_1 and /or G_2
 - When G has composite order: Pseudorandomness of random elements from a prime order subgroup of G.

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- Useful in at least "prototyping" new primitives (e.g. IBE)

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 - In addition, if modeling a group with bilinear pairing, also provides the pairing operation and operations for the target group

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 - And an exhaustive analysis in terms of formal polynomials to show requisite security properties

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- Better practice: when possible identify simple (new) assumptions sufficient for the security of the scheme. Then prove the assumption in the generic group model

KEA-1: Given (g,g^a) for a random generator g and random a, if a PPT adversary extends it to a DDH tuple (g,g^a,g^b,g^{ab}) then it "must know" b

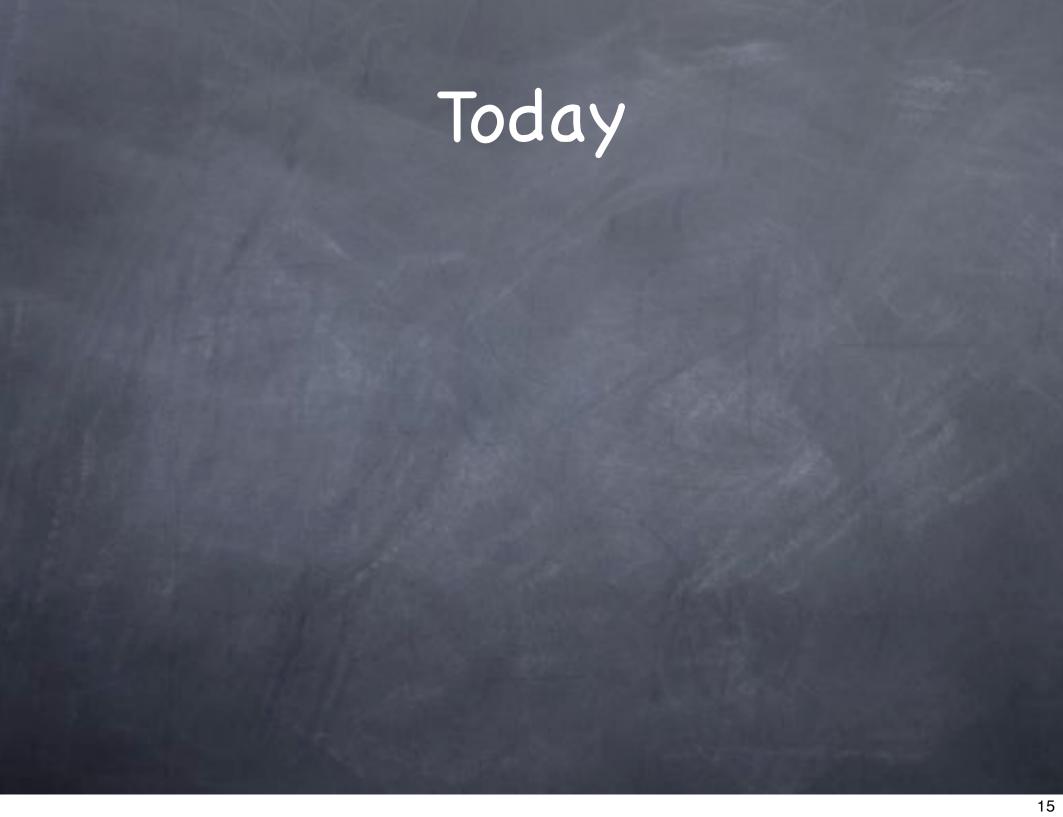
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