Mix-Nets

Lecture 16

Some tools for electronic-voting (and other things)

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  - (Omitted: Decryption mix-nets, which combine shuffling and decryption. Here: Re-encryption mix-nets)
- Ideal functionality: input a vector of private messages from senders, and a permutation from each mix server; output the messages permuted using the composed permutation

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- Active adversary can corrupt any subset of servers below the threshold
- Ideal: Same as for SIM-CPA, but with servers also getting the message (if the receiver decides to get it); if number of corrupted servers above threshold, adversary can block (but not substitute) output to others

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  - Proof using an Honest-Verifier ZK proof
    - Using a special purpose proof (Chaum-Pederson), rather than ZK for general NP statements

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As above, but for (r_1,r_2) s.t. $r_1v+u_1 = r_2v+u_2$ for a random v and arbit. u_1, u_2

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  - e.g. solve r from w=rv+u and w'=rv'+u (given v,w,v',w')
  - Implies soundness: for each U s.t. prover has significant probability of being able to convince, can extract r from the prover with comparable probability (using "rewinding")

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      - Removes need for interaction!

Verifiable Shuffle

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- Special soundness: given answers for $v \neq v'$ either $v_1 \neq v_1'$ or $v_2 \neq v_2'$. By special soundness, extract witness for $stmnt_1$ or $stmnt_2$

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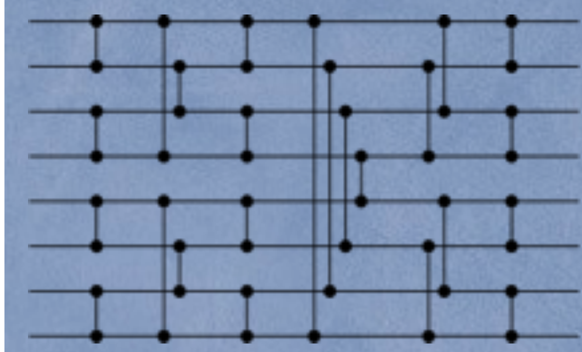

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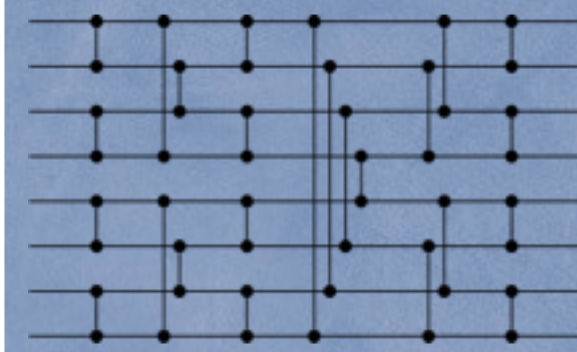

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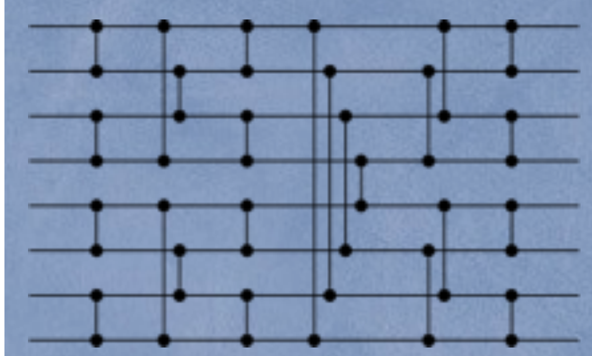

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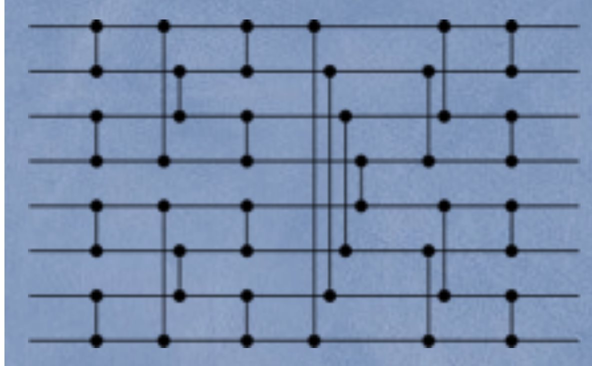

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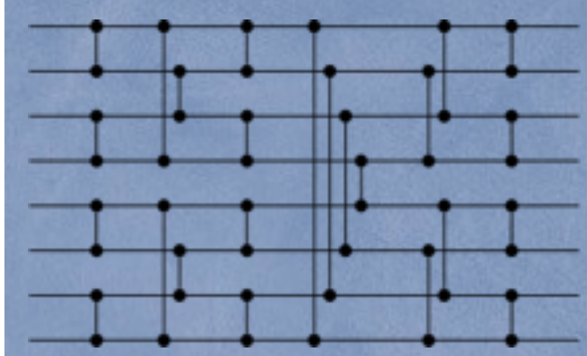

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  - 3 rounds: Parallel composition of HVZK proofs

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  - 7 rounds, using homomorphic commitments
    - Possible with sub-linear communication for the proof

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  - (Operations in respective groups)

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  - Binding, because of collision resistance when K picked at random

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  - Binding by collision-resistance: receiver picks (g,h)
  - Perfectly Hiding in a prime order group
    - If group is prime order, then all h are generators
    - Then for all x, $H_{g,h}(x,r)$ is random if r random
- Homomorphism: $Com_K(x;r) * Com_K(x';r') = Com_K(x+x';r+r')$
- HVZK PoK of (x,r): Commit to (g^{u1}, g^{u2}), and on challenge v, send $(xv+u_1)$ and $(rv+u_2)$

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- Improved efficiency: $H_{g1,..,gn,h}(x_1,\ldots,x_n,r) = g_1^{x1} \ldots g_n^{xn}\, h^r$

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- Use homomorphic commitments to carry out the polynomial evaluation and check equality (details omitted)

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    - Use homomorphic properties of the commitments to carry out equality proofs w.r.t committed permutation (omitted)

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