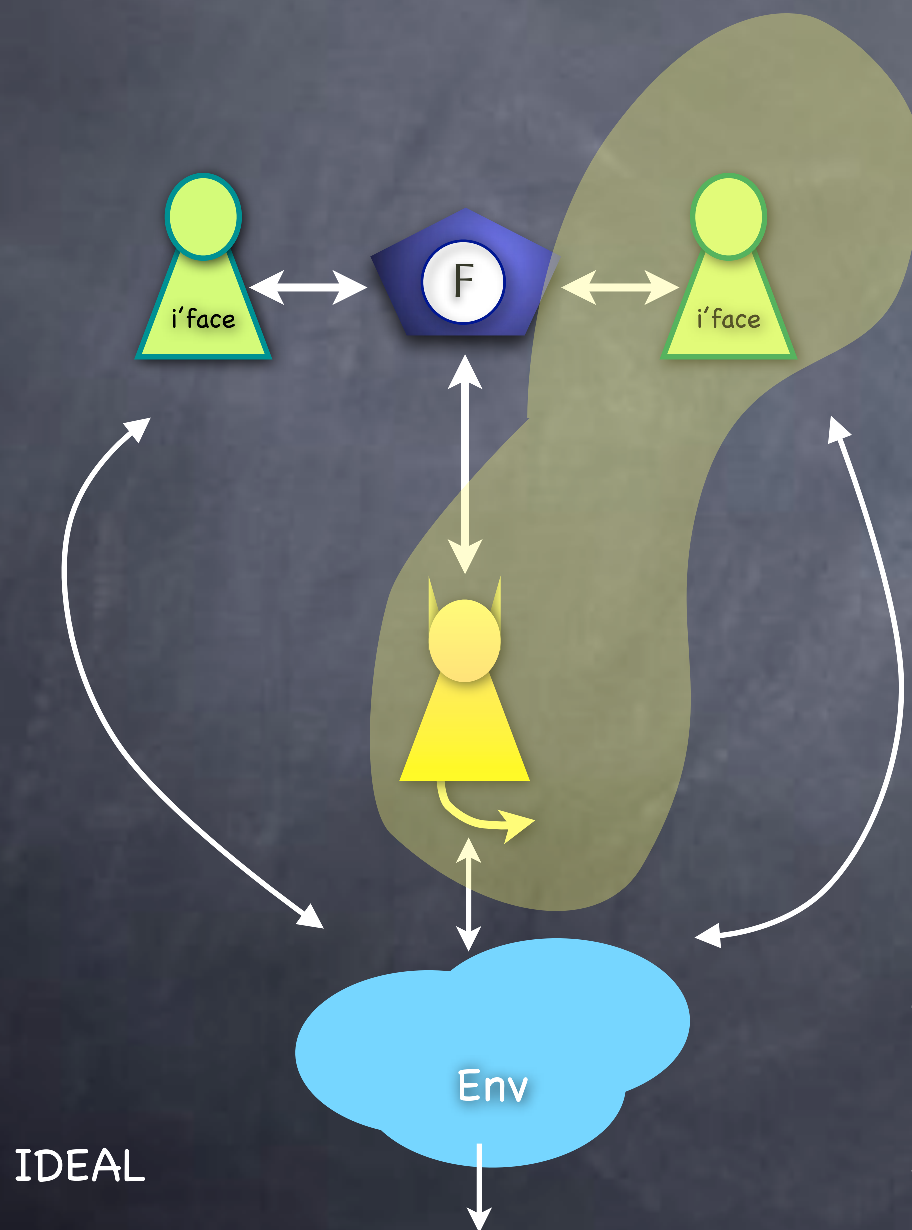
Secure 2-Party Computation

Lecture 12

Yao's Garbled Circuit

RECALL

SIM-Secure MPC



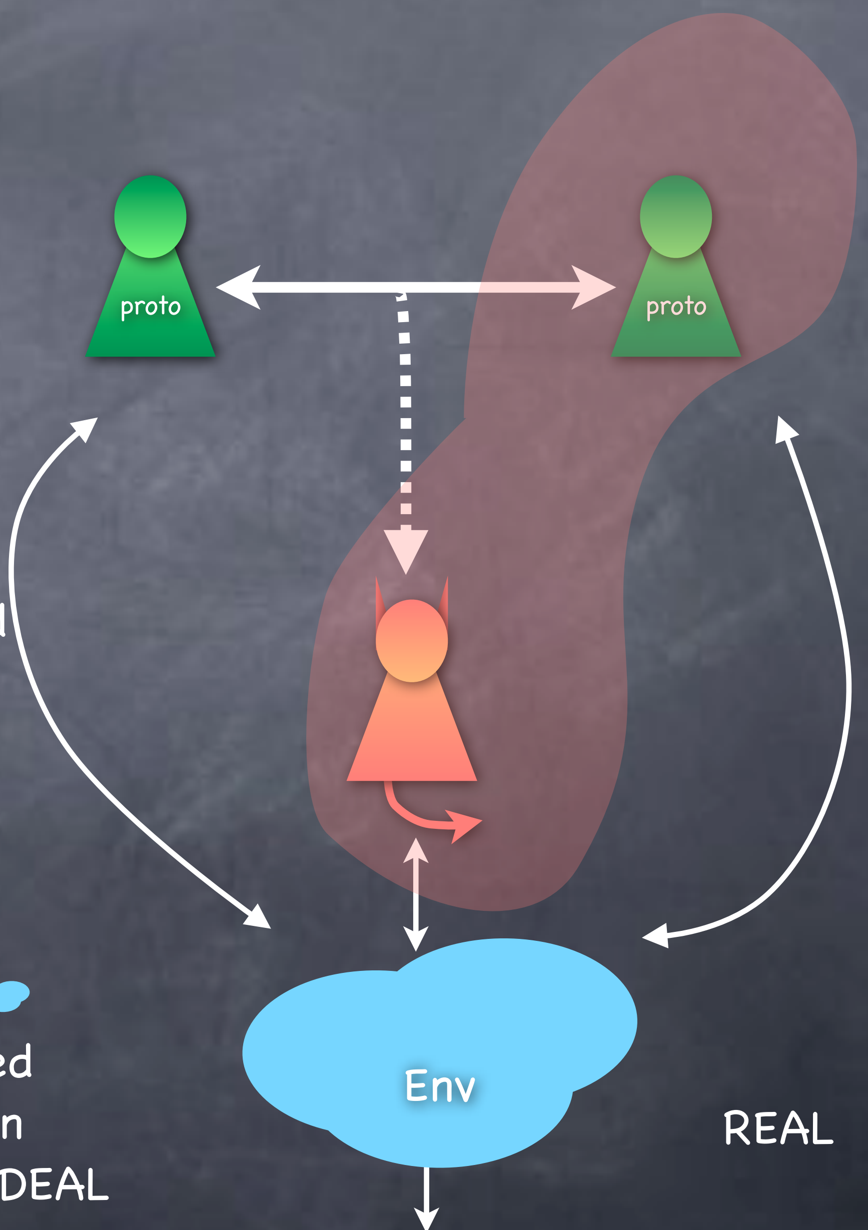
Secure (and correct) if:

∀

∃ s.t.

∀

output of is distributed identically in REAL and IDEAL



RECALL

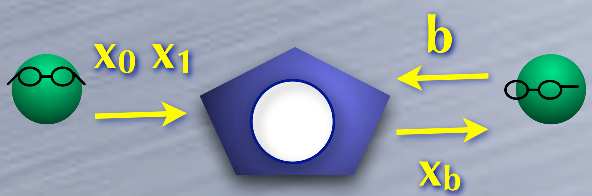
Passive Adversary

- Gets only read access to the internal state of the corrupted players (and can use that information in talking to environment)
  - Also called "Honest-But-Curious" adversary
  - Will require that simulator also corrupts passively
- Simplifies several cases
  - e.g. coin-tossing [why?], commitment [coming up]
- Oddly, sometimes security against a passive adversary is more demanding than against an active adversary
  - Active adversary: too pessimistic about what guarantee is available even in the IDEAL world
  - e.g. 2-party SFE for OR, with output going to only one party (trivial against active adversary; impossible without computational assumptions against passive adversary)

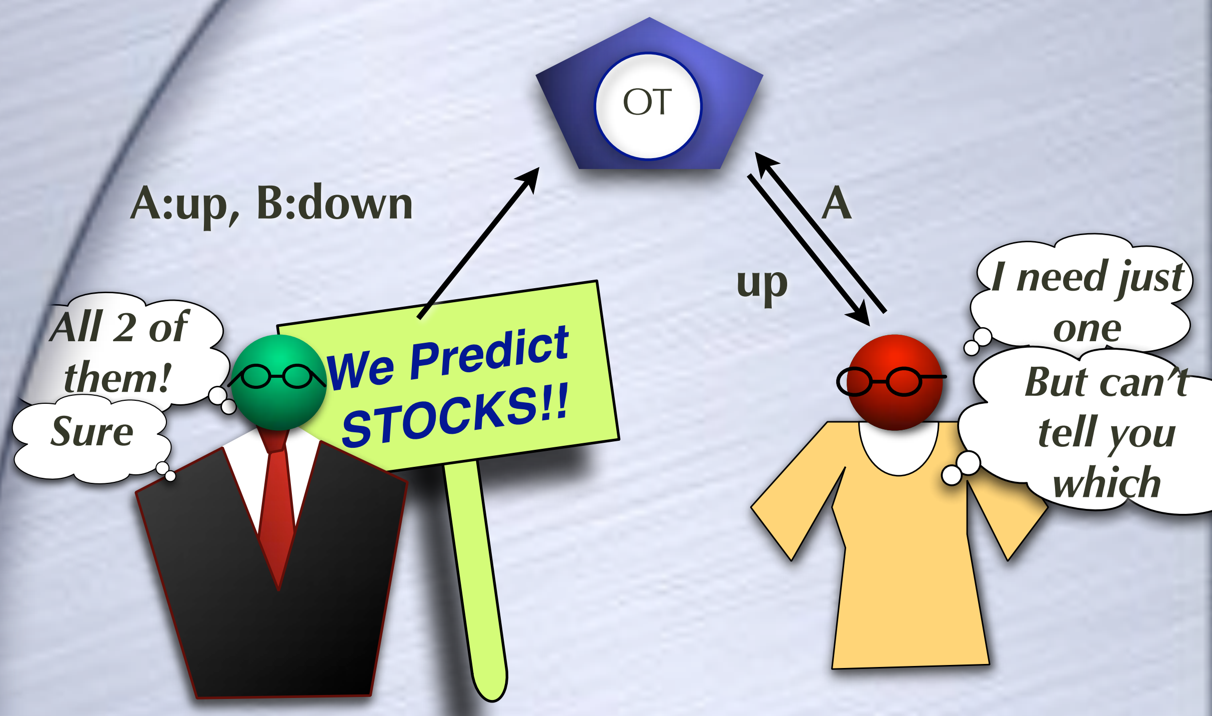
RECALL

Oblivious Transfer

- Pick one out of two, without revealing which
  - Intuitive property: transfer partial information "obliviously"



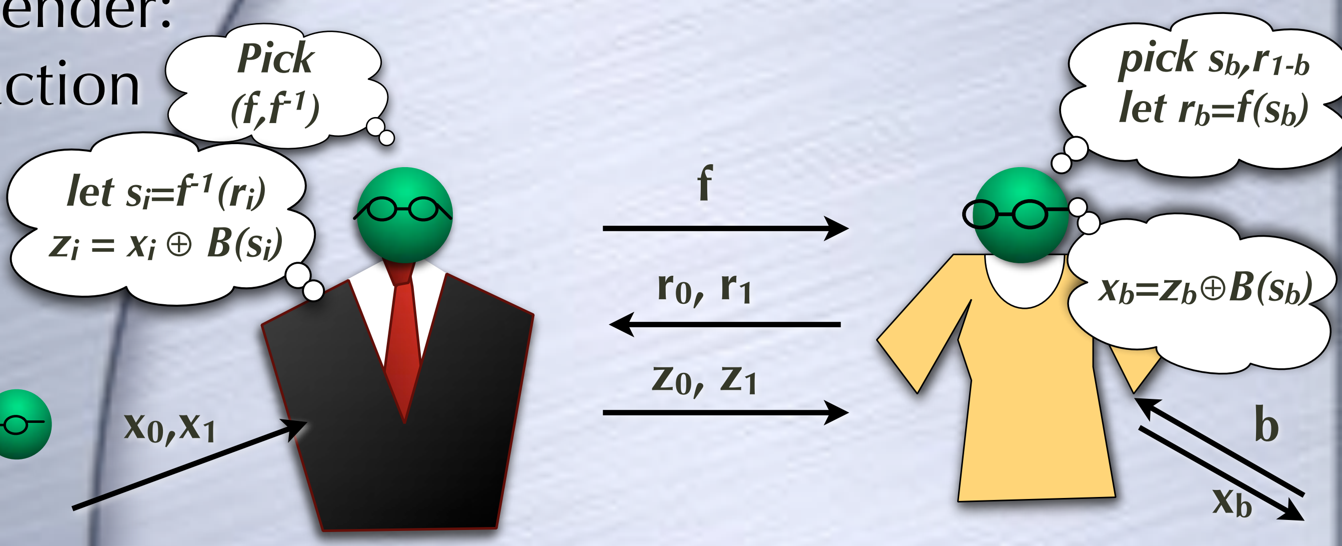
IDEAL World



RECALL

An OT Protocol (passive receiver corruption)

- Using a TOWP
  - Depends on receiver to pick x_0, x_1 as prescribed
- Simulation for passive corrupt receiver: simulate z_0,z_1 knowing only x_b (use random z_{1-b})
- Simulation for corrupt sender: Extract x_0,x_1 from interaction (pick s_{1-b} also)



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- So, for passive security, enough to consider one-sided SFE

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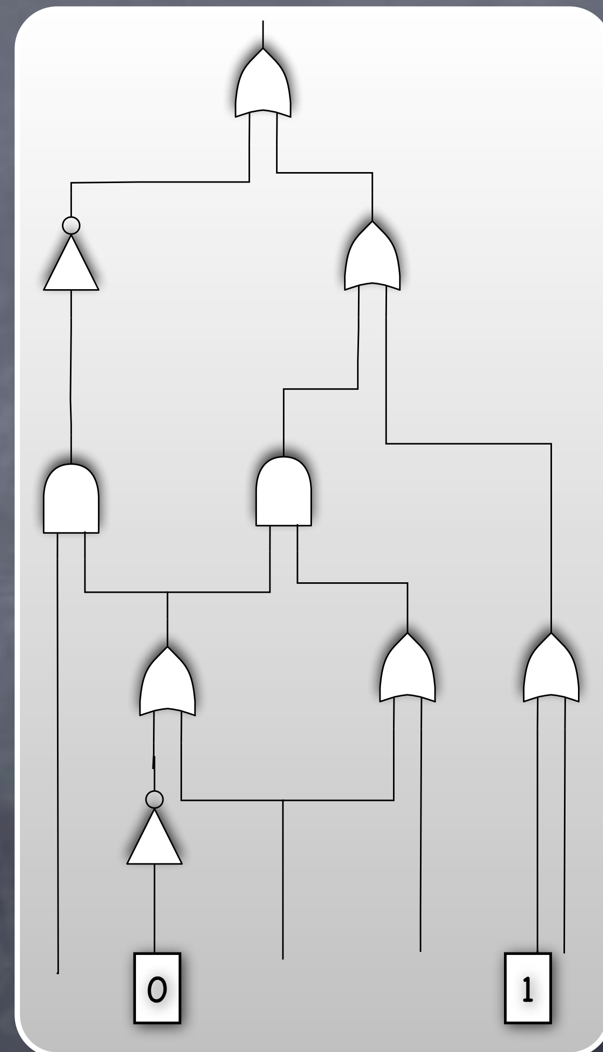
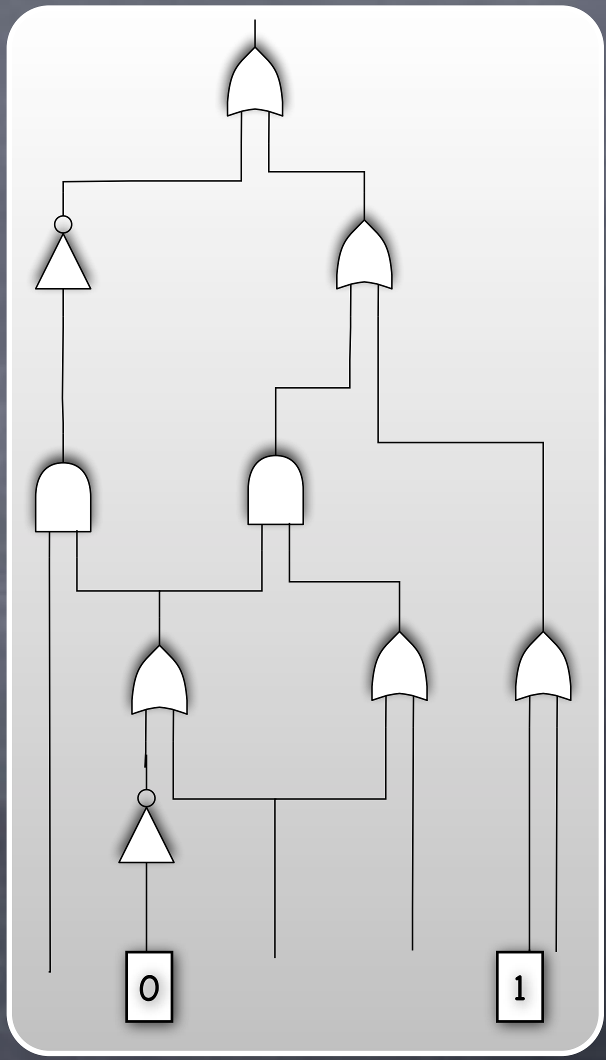
- Can we do "general" deterministic, one-sided SFE (i.e., for all functions)?

Boolean Circuits



Boolean Circuits

- Directed acyclic graph



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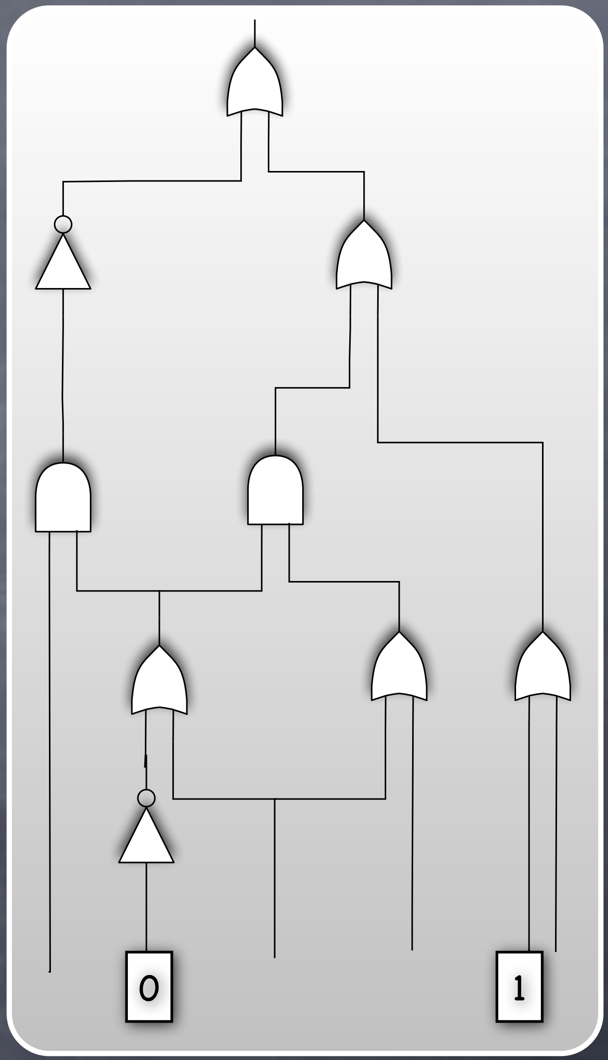

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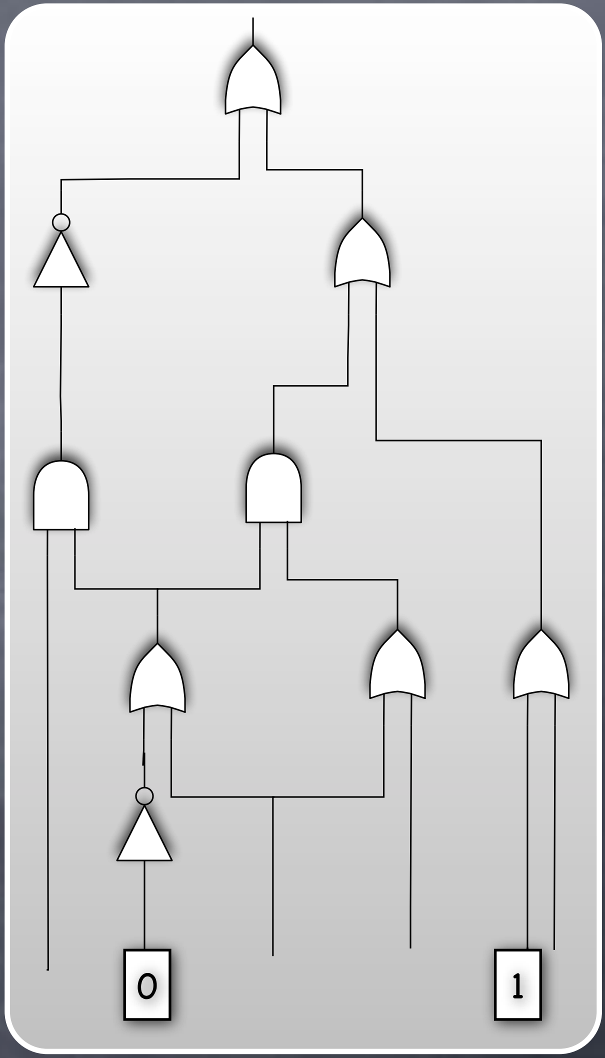

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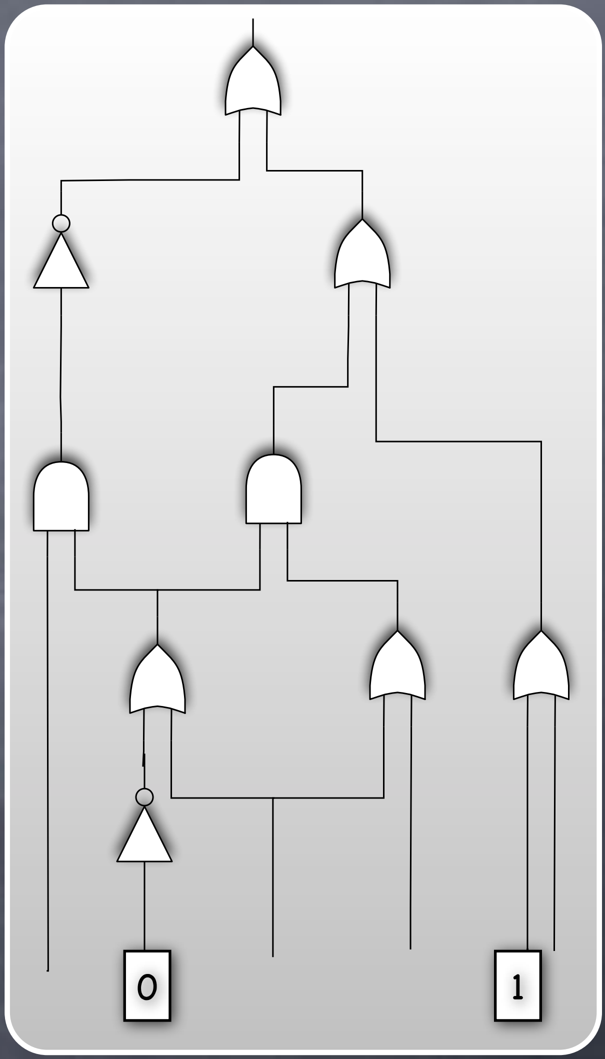

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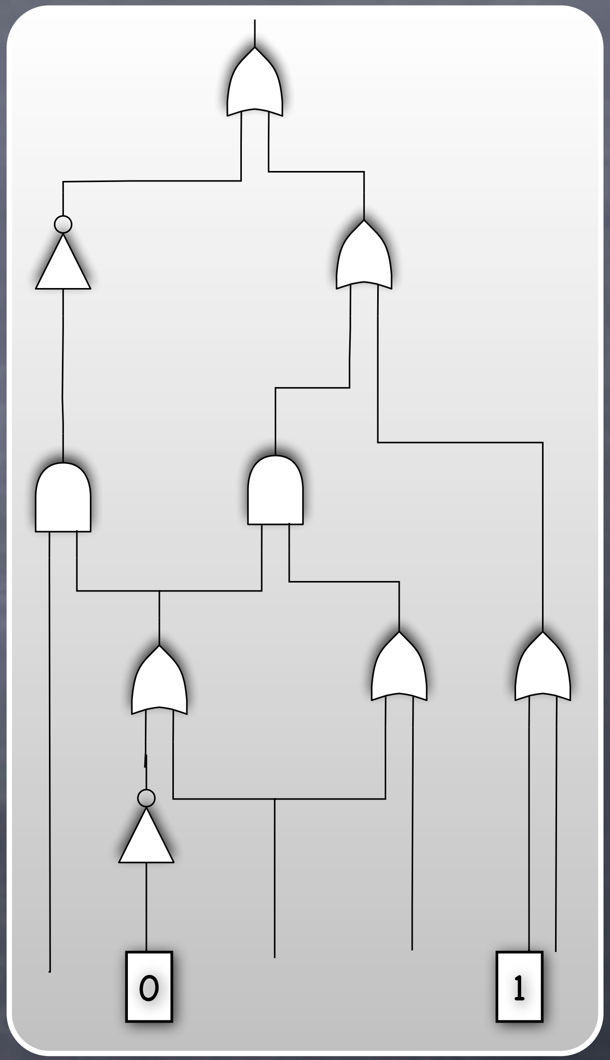

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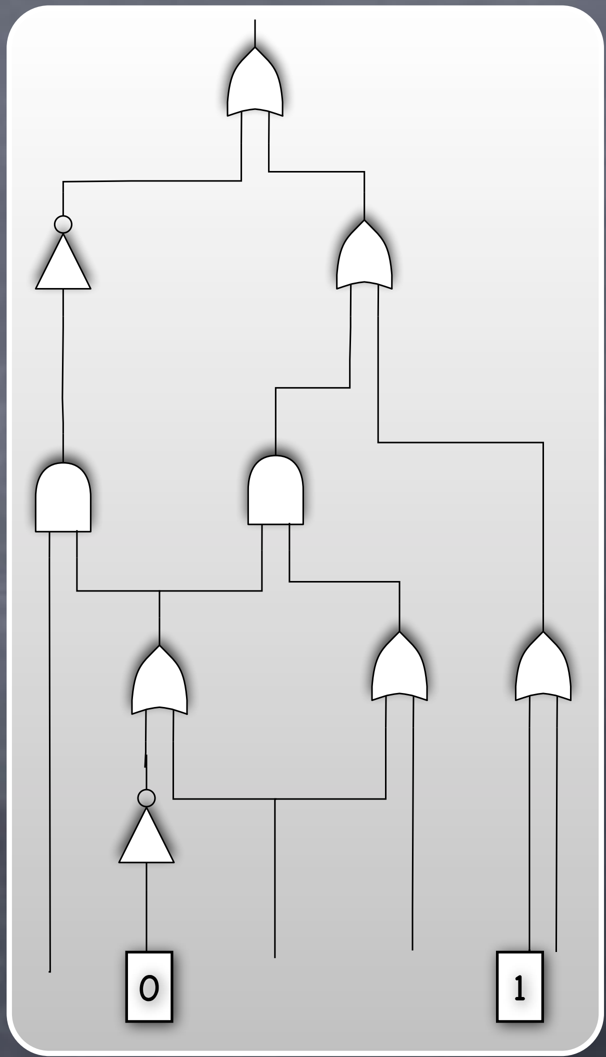

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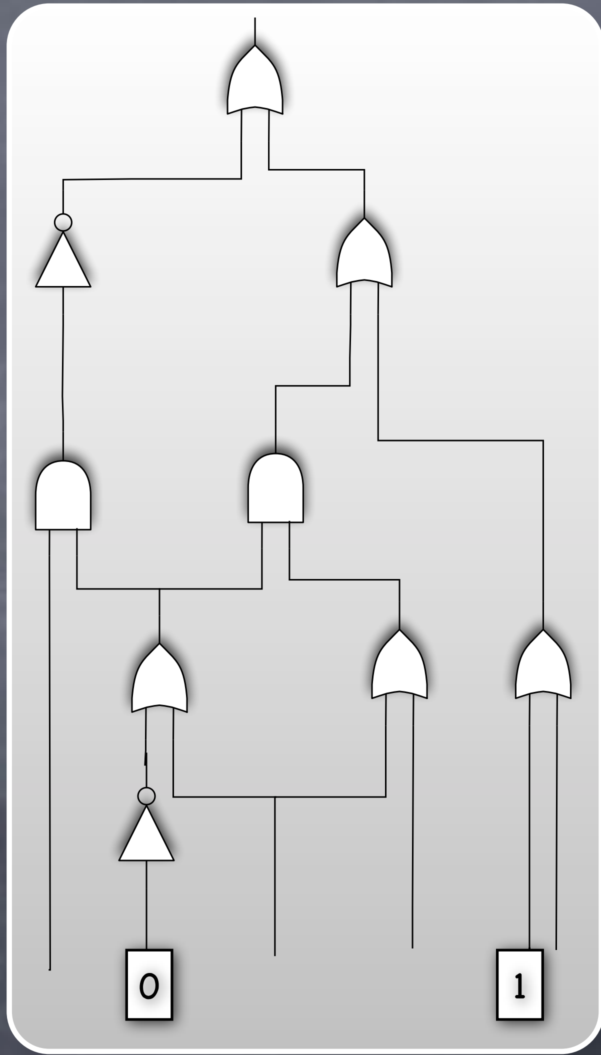
    - Note: no memory gates



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  - Often problems already described as succinct programs/circuits

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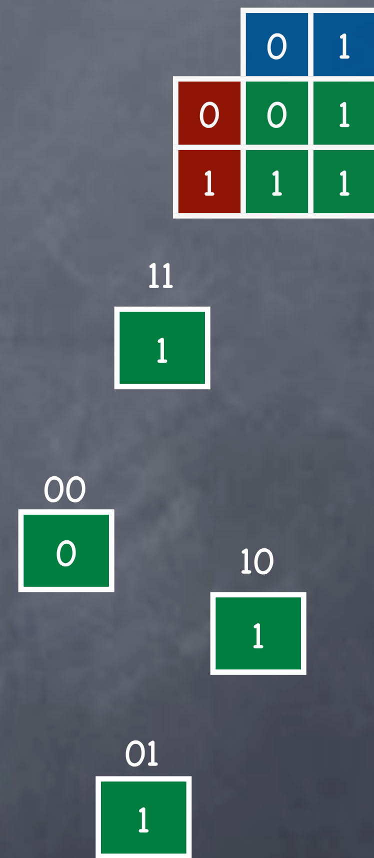

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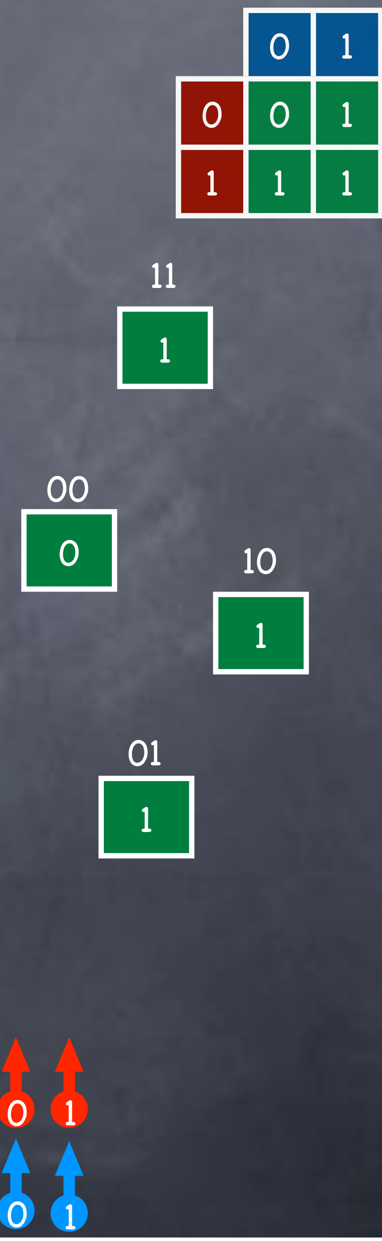

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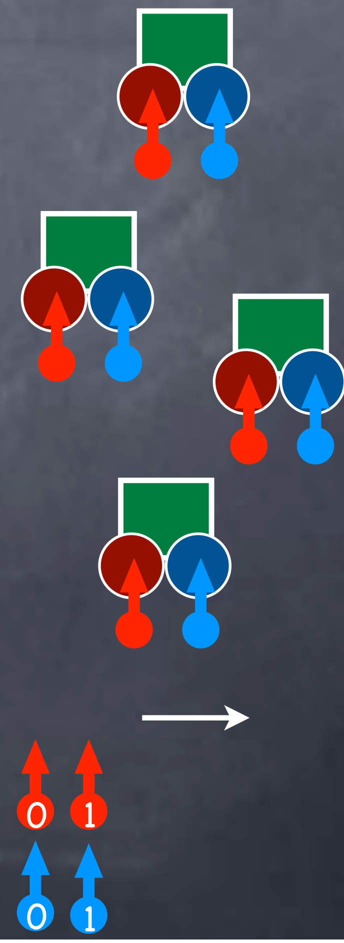

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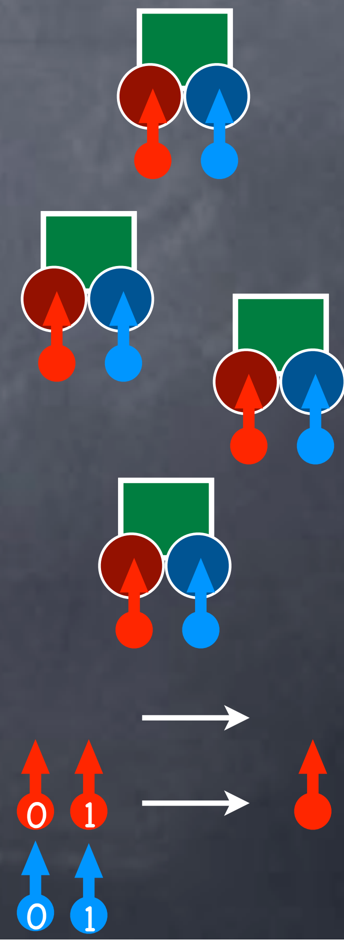

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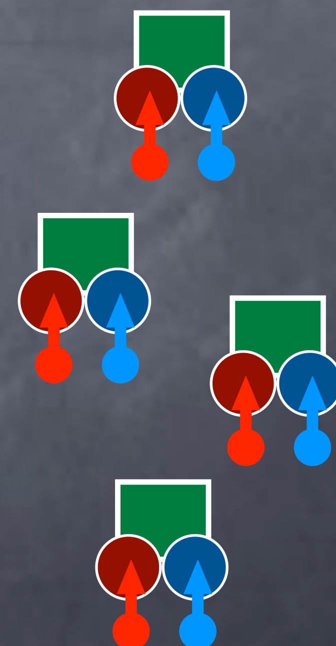

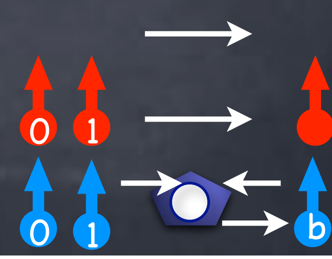

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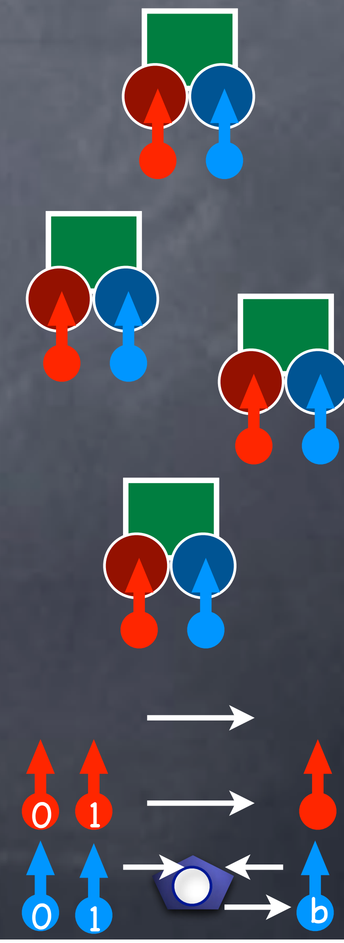
- Secure?

	0	1
0	0	1
1	1	1



A Physical Protocol

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- For curious Alice: only influence from Bob is when he picks up his key $K_{y=b}$

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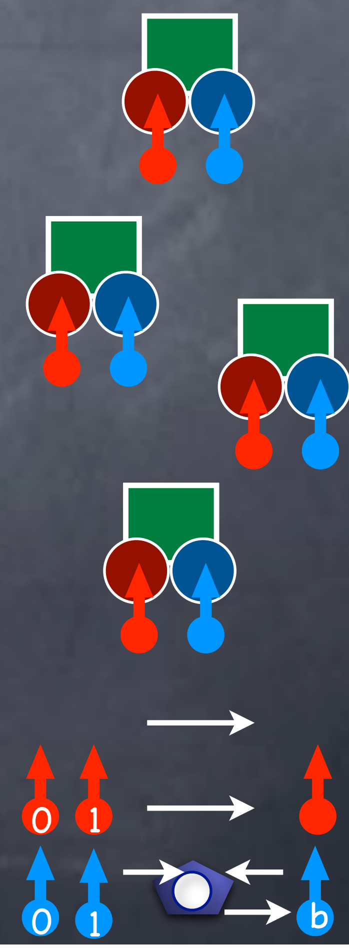

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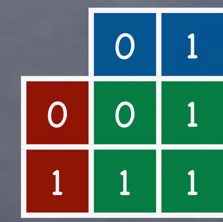

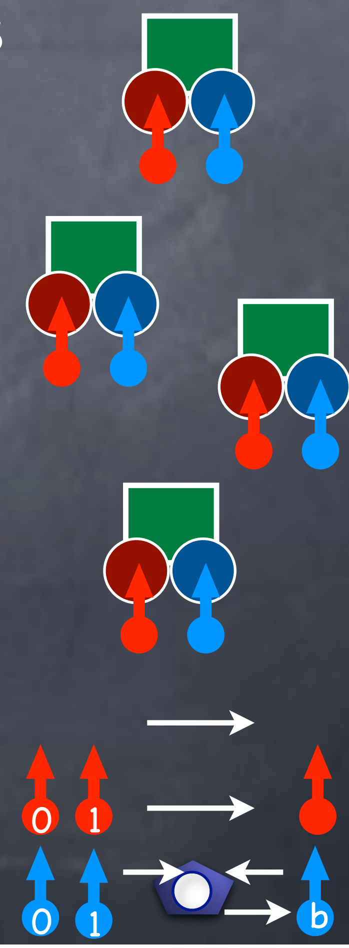

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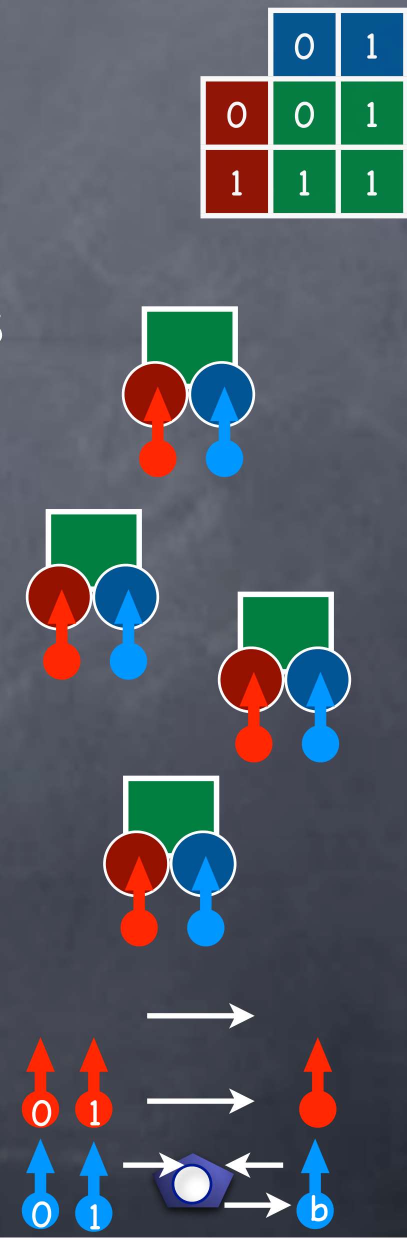

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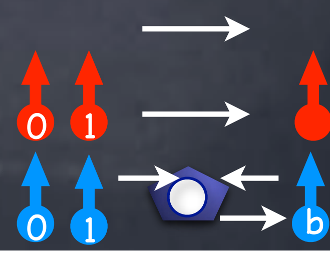

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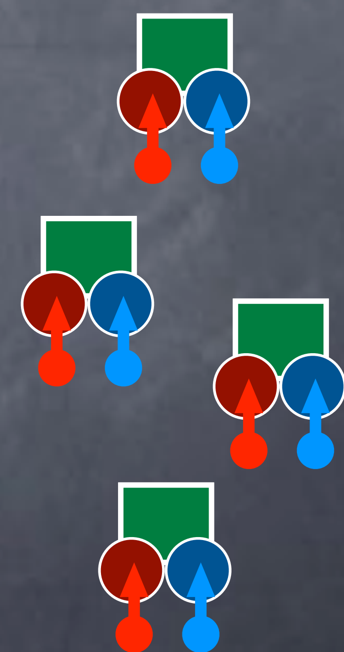

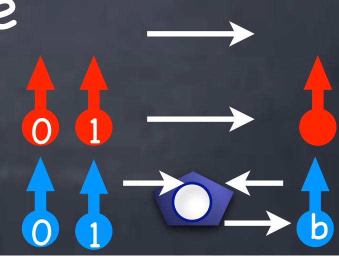

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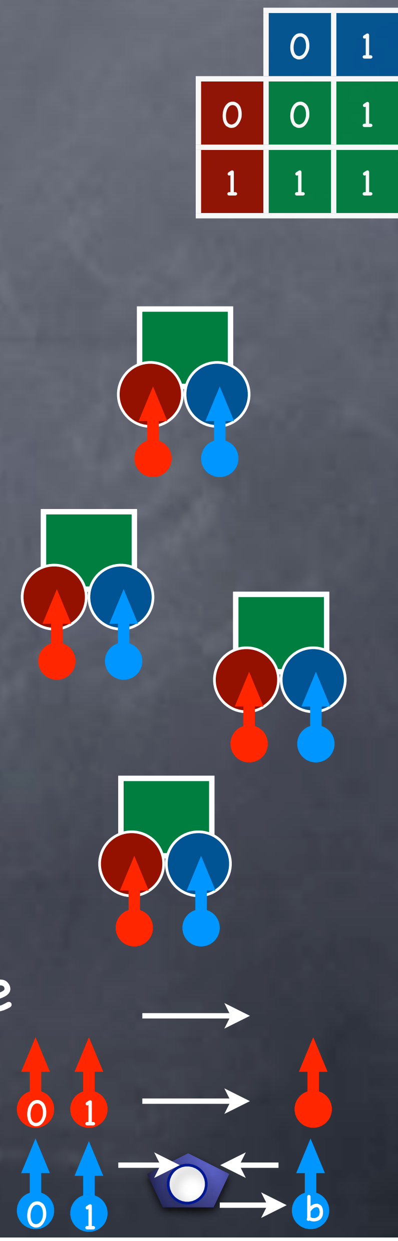
  - Formally, easy to simulate (can stuff unopenable boxes randomly)



Larger Circuits

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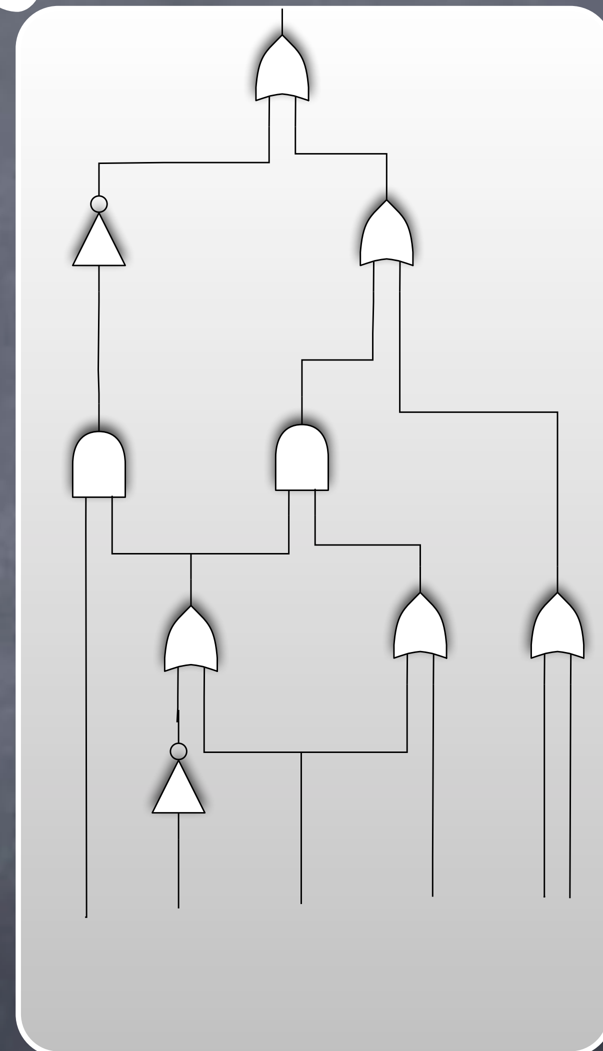

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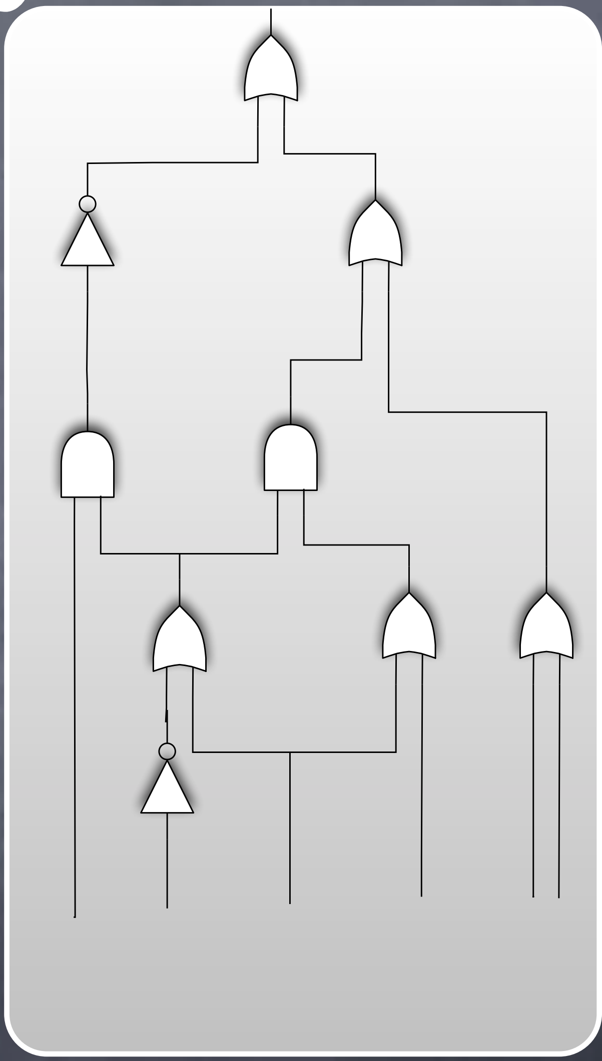

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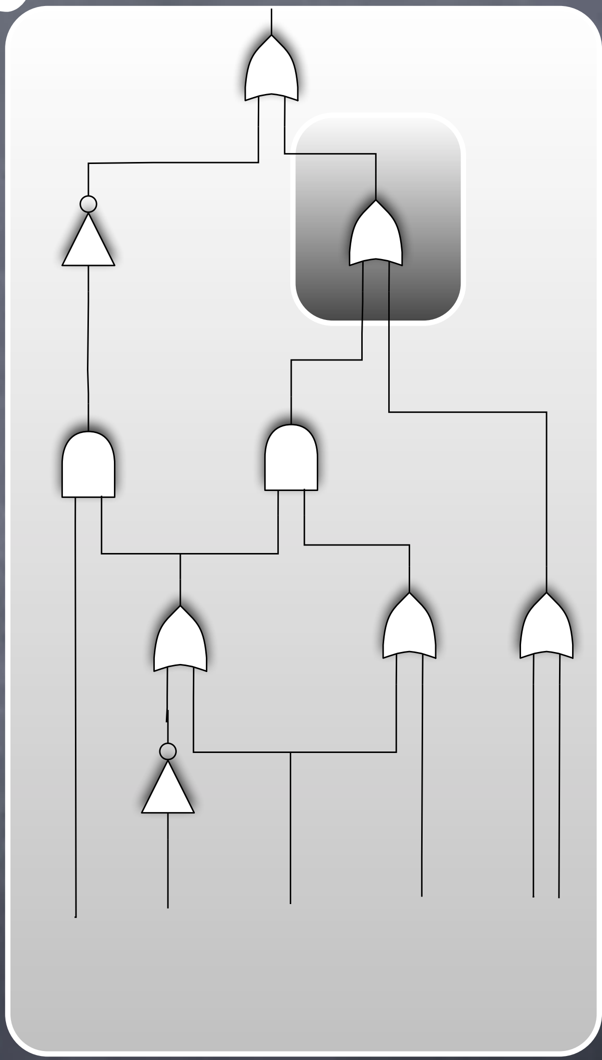

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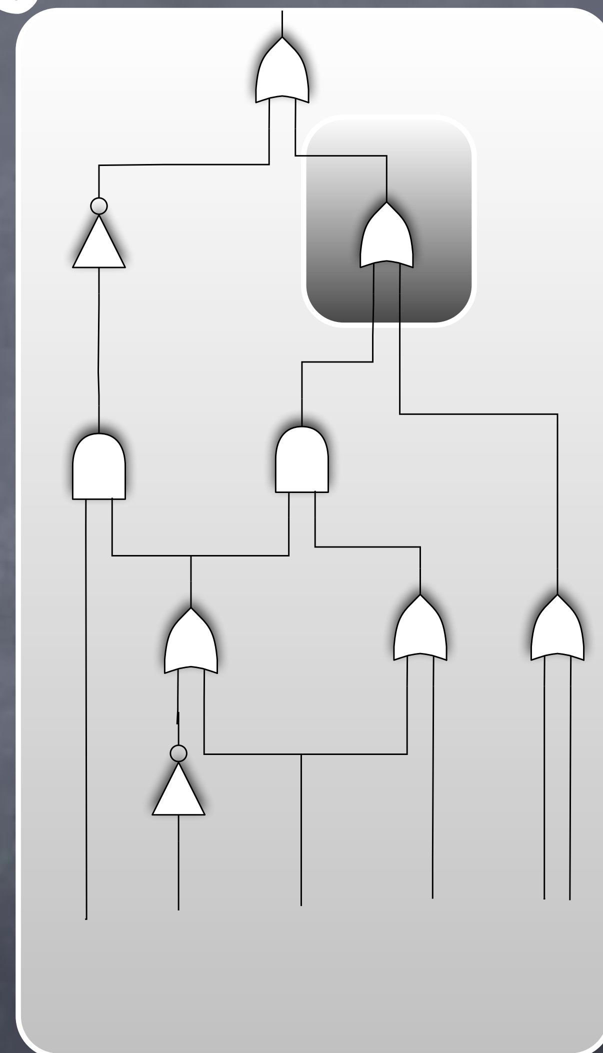

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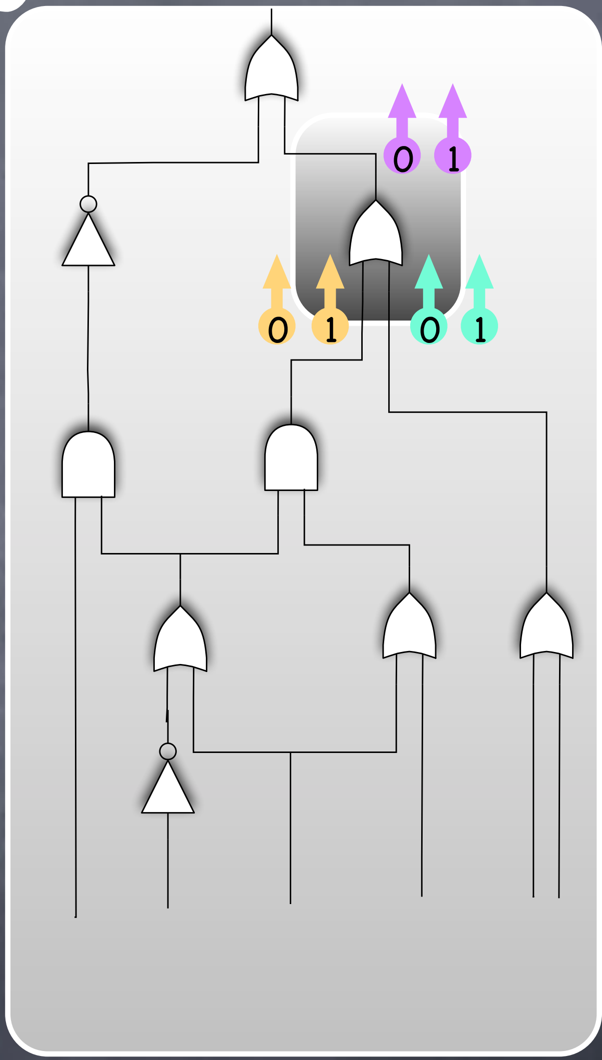

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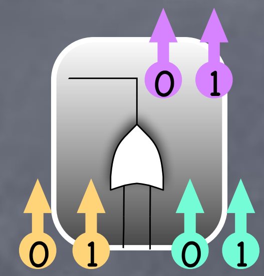

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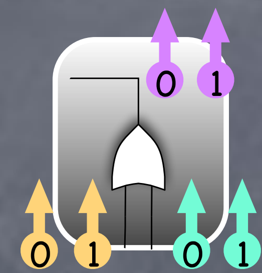

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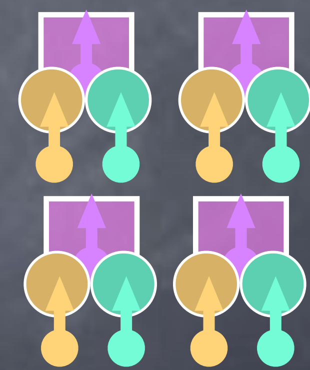

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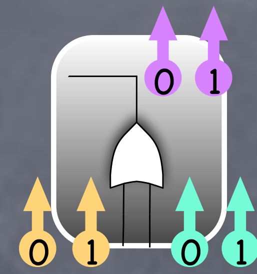

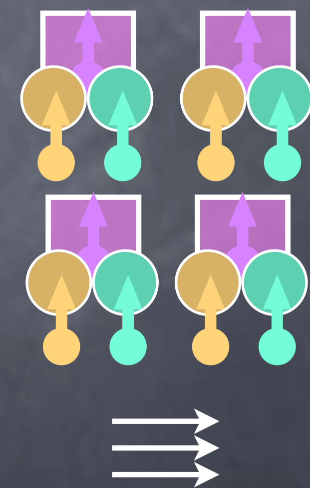

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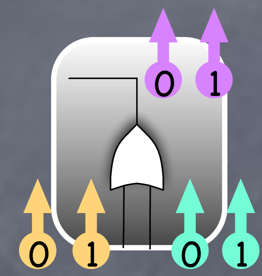

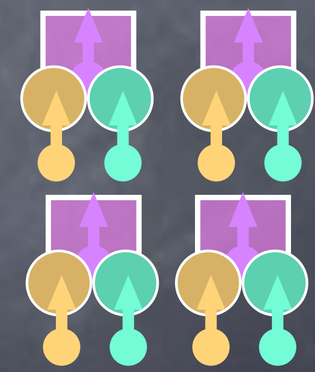

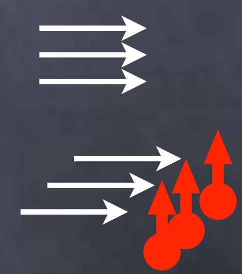

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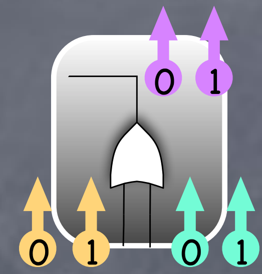

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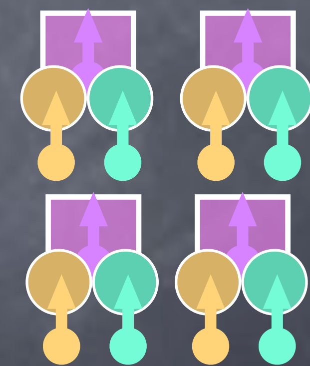

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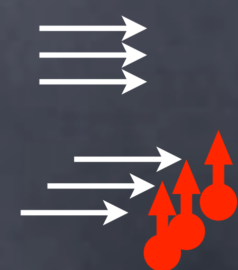

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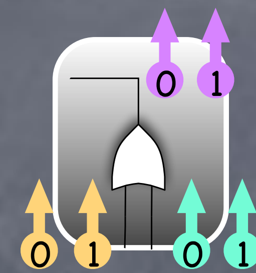

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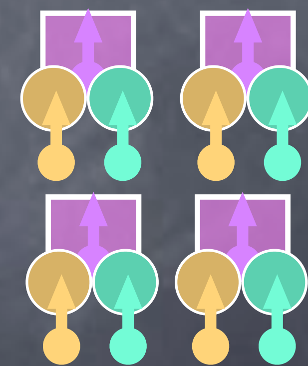

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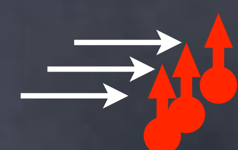

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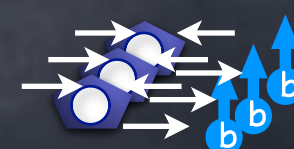

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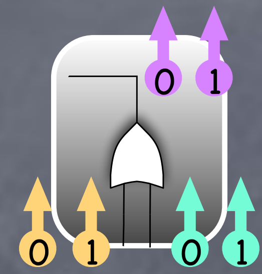

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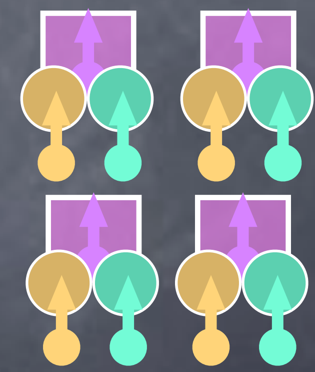

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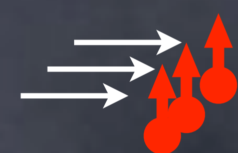

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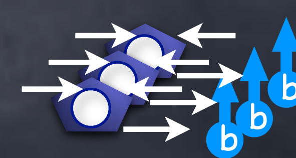

  - Boxes for output gates have values instead of keys

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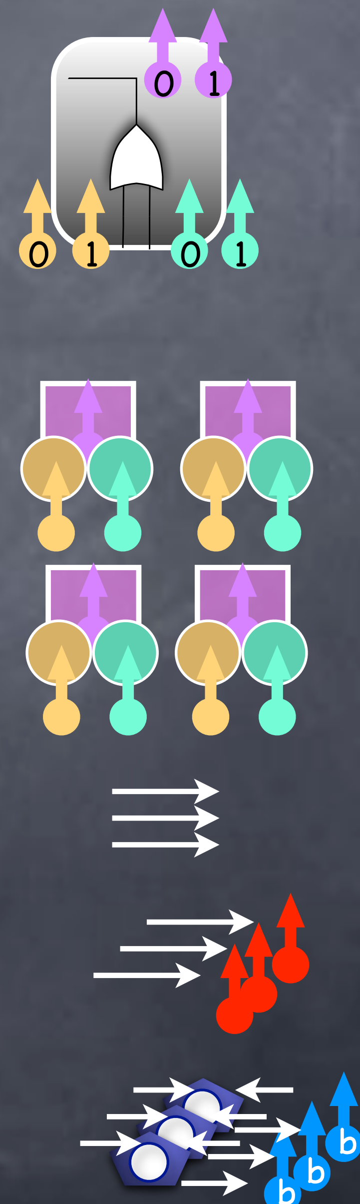

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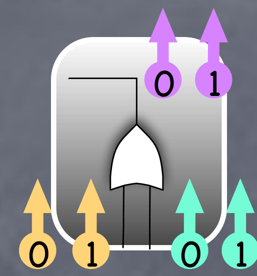
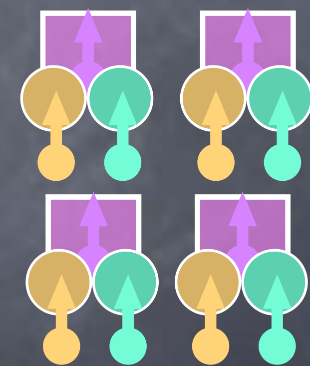
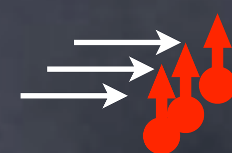
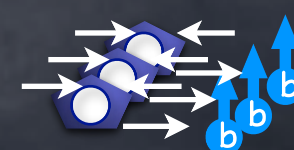
- Evaluation: Bob gets one key for each input wire of a gate, opens one box for the gate, gets one key for the output wire, and proceeds









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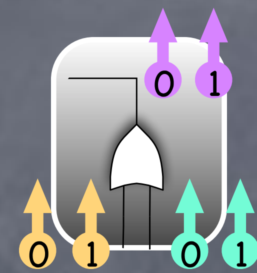

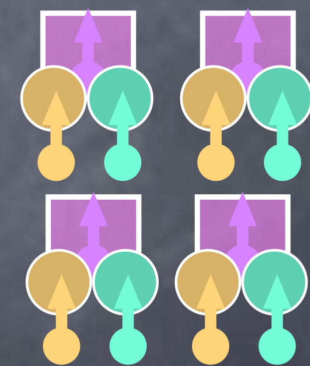

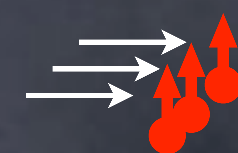

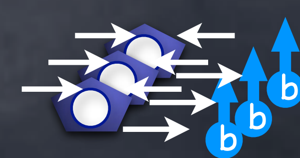

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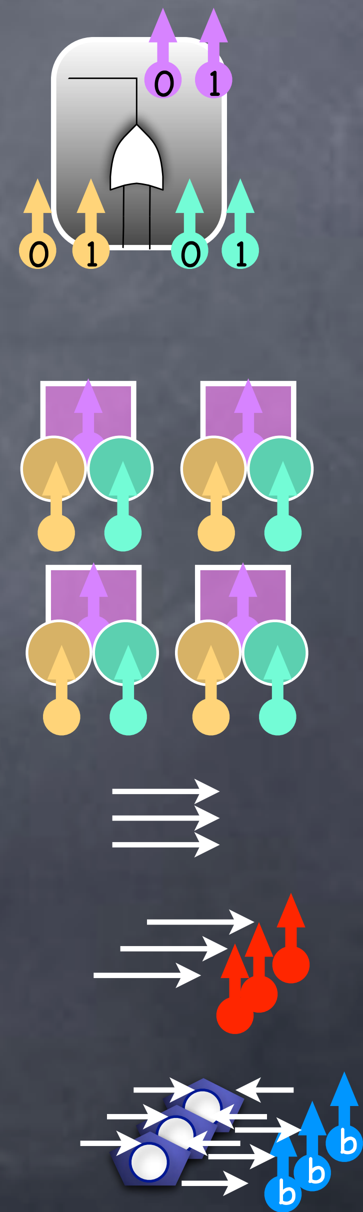

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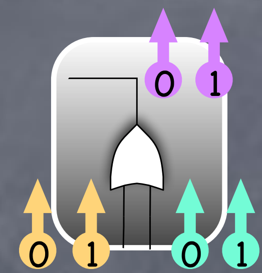

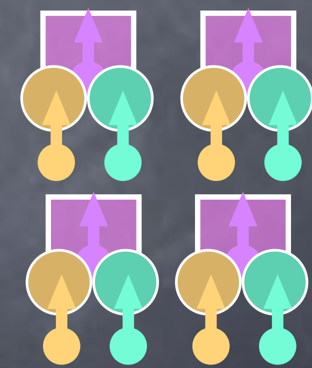

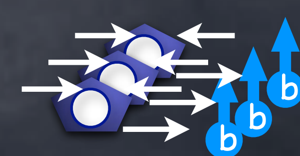

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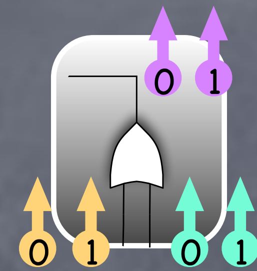

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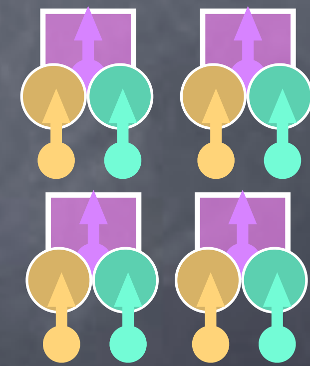

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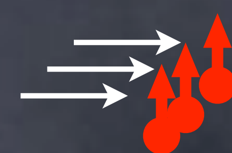
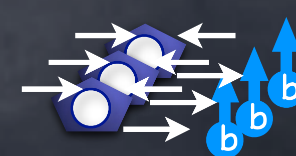
- Everything is simulatable for curious Bob given final output: Bob could prepare boxes and keys (stuffing unopenable boxes arbitrarily); for an output gate, place the output bit in the box that opens





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  - Can we really compose? Yes, for passive security.

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- Coming up: Zero-Knowledge proofs and general multi-party computation, more protocols (for different settings). Universal Composition