Hash Functions in Action

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Lecture 11

Hash Functions

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- Also often required: "unpredictability"
- Today: applications of hash functions (and what we require of them)

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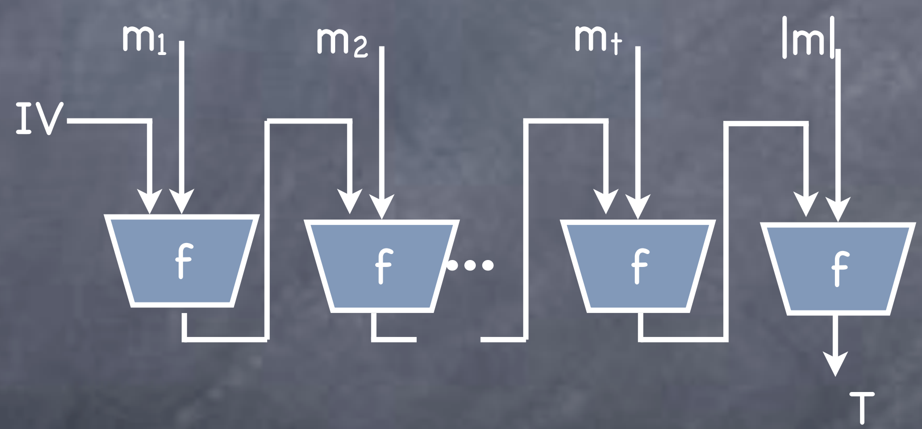

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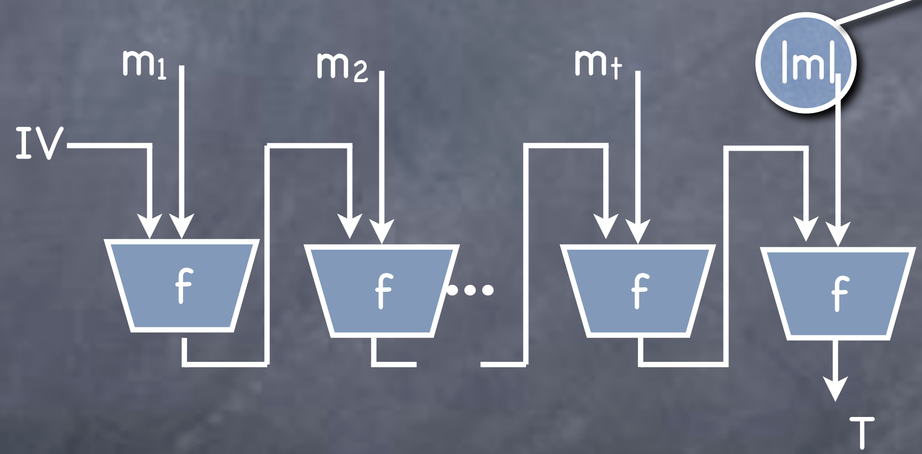
Collision resistance even with variable input-length



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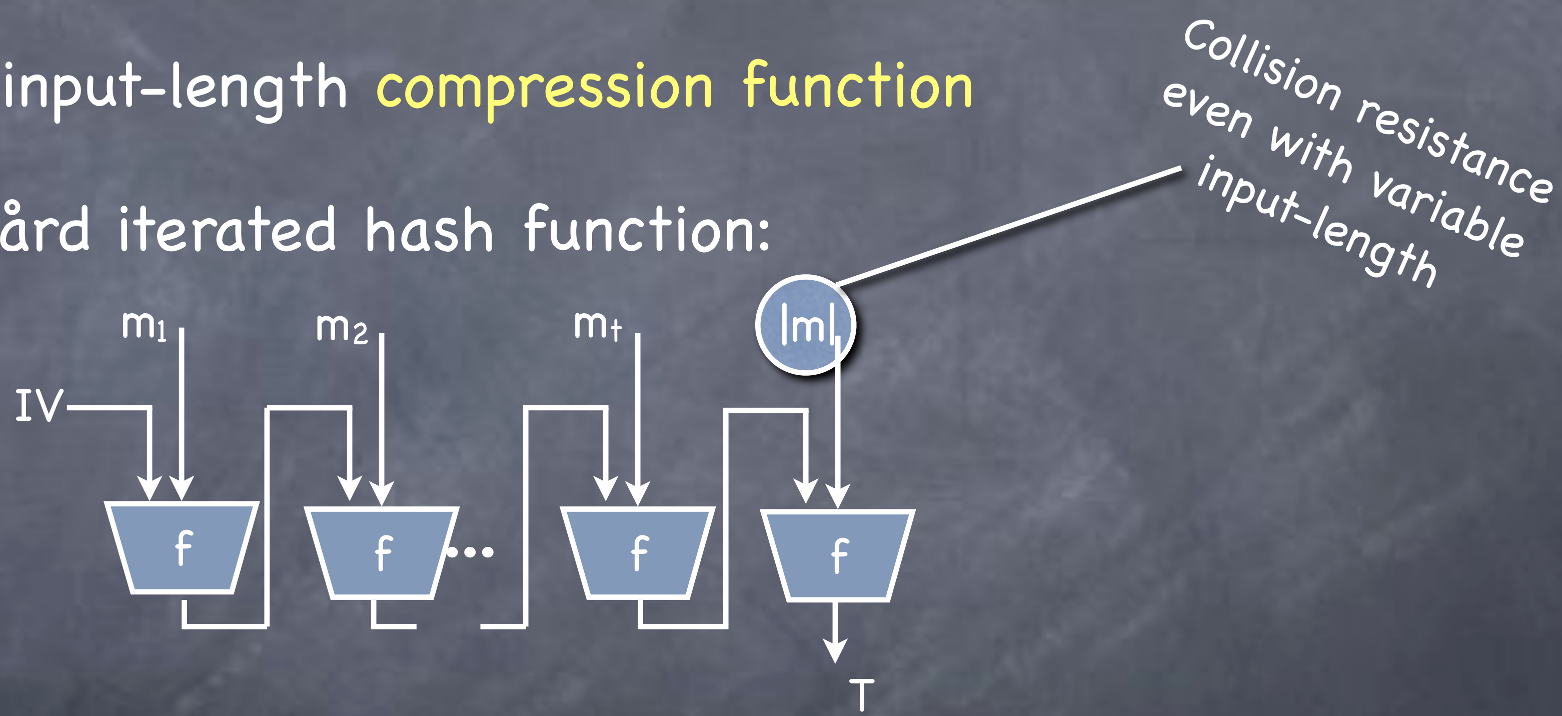

- If f "collision resistant", then so is the Merkle-Damgård iterated hash-function (for any IV)

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With 2-Universal Hash Functions

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    - Seeing hash of one input gives no information on hash of another value

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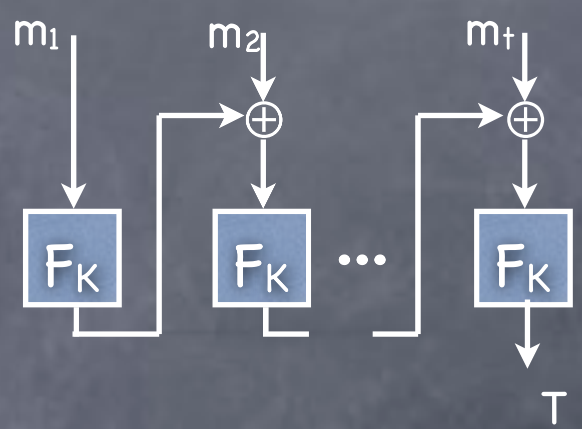

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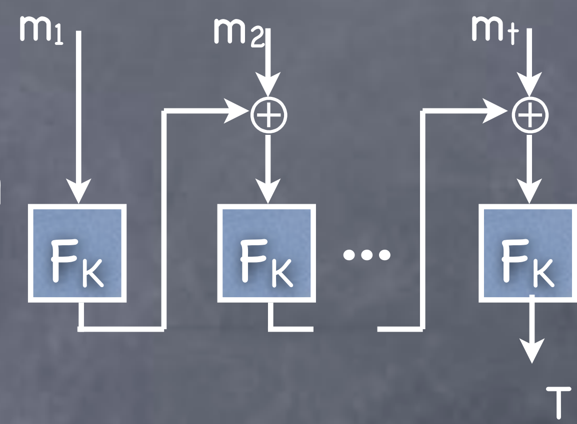

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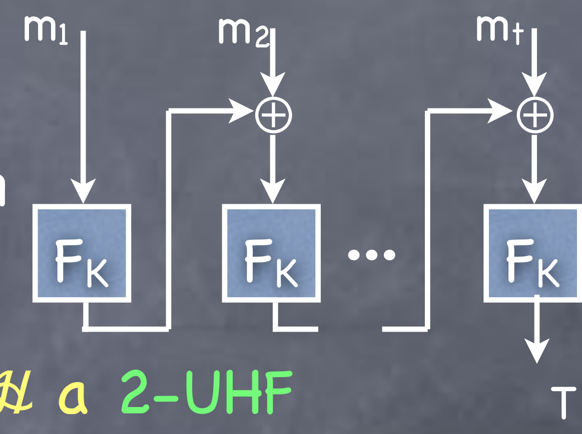

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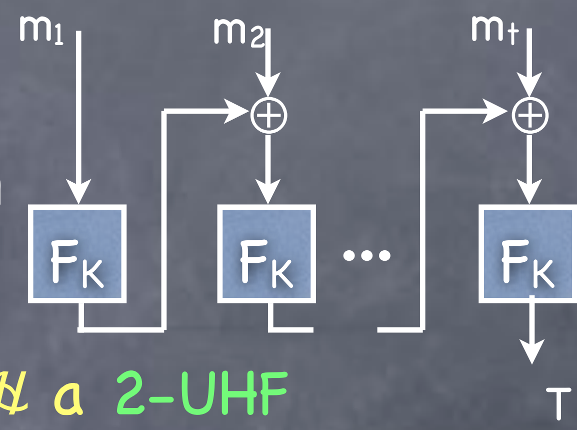

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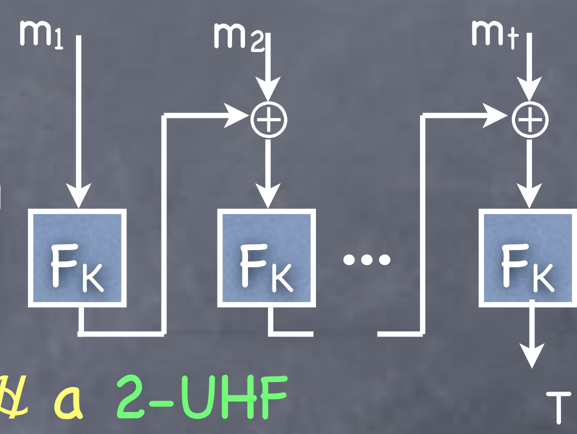

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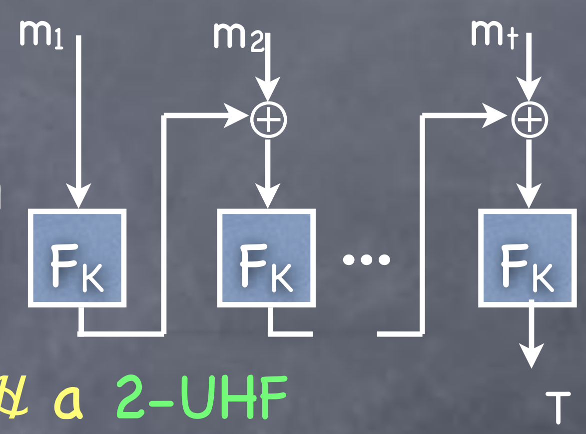

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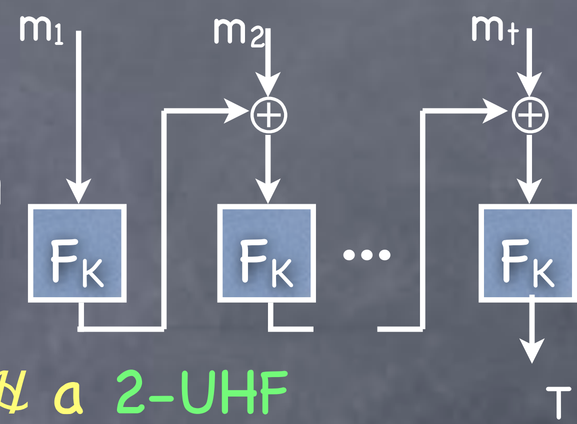

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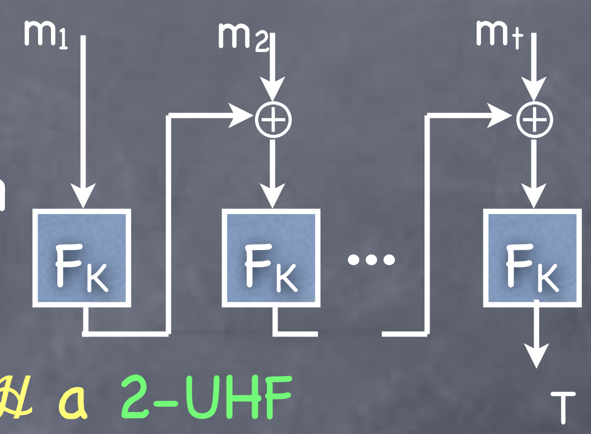

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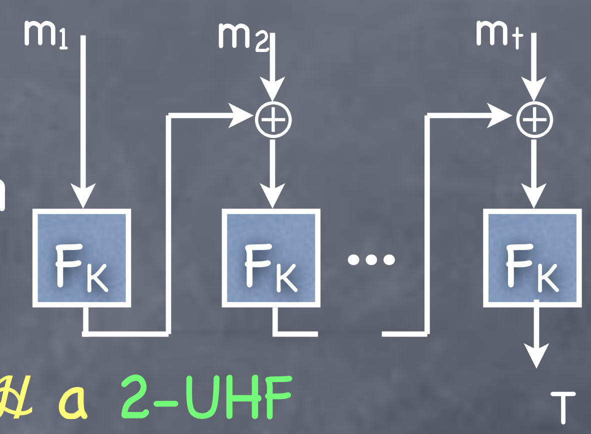

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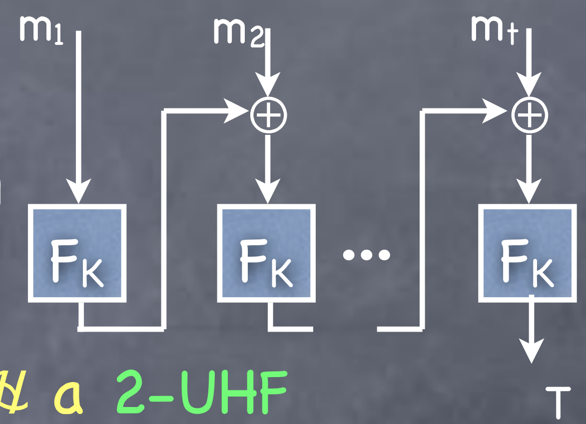

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- Leave variable input-lengths to the hash?

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  - Compression functions (with key as IV)

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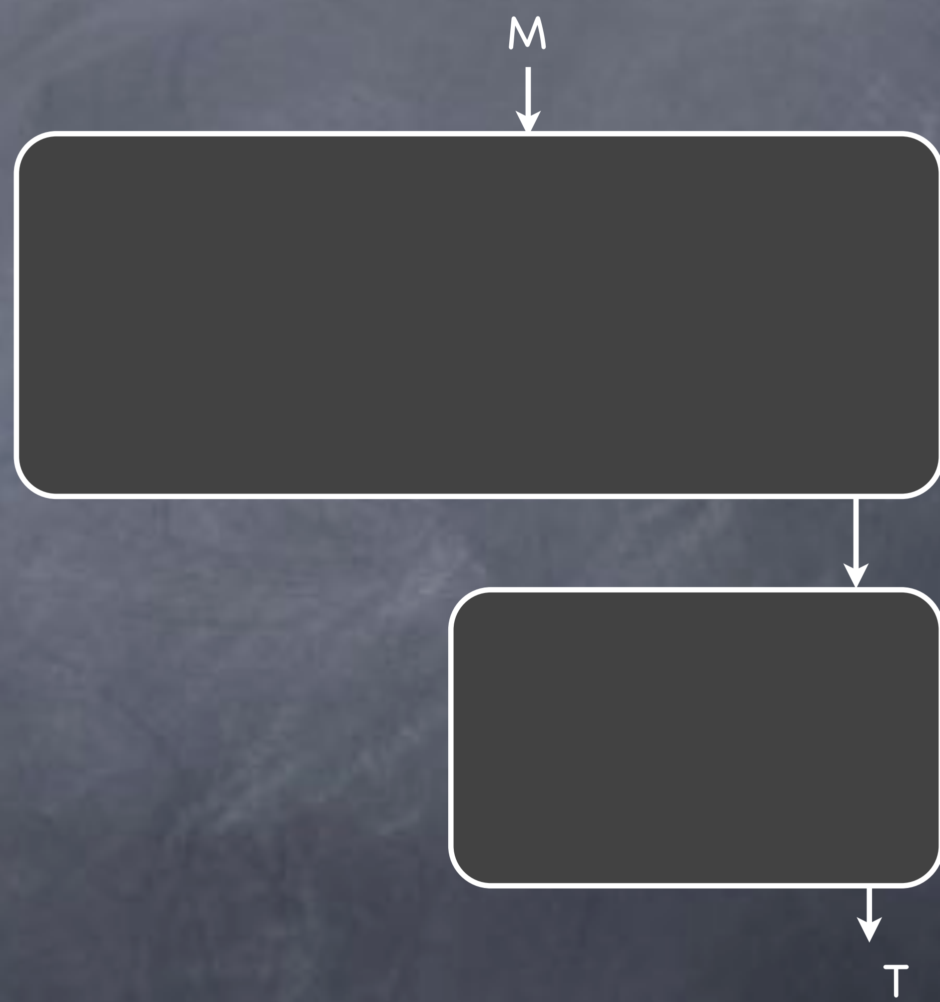

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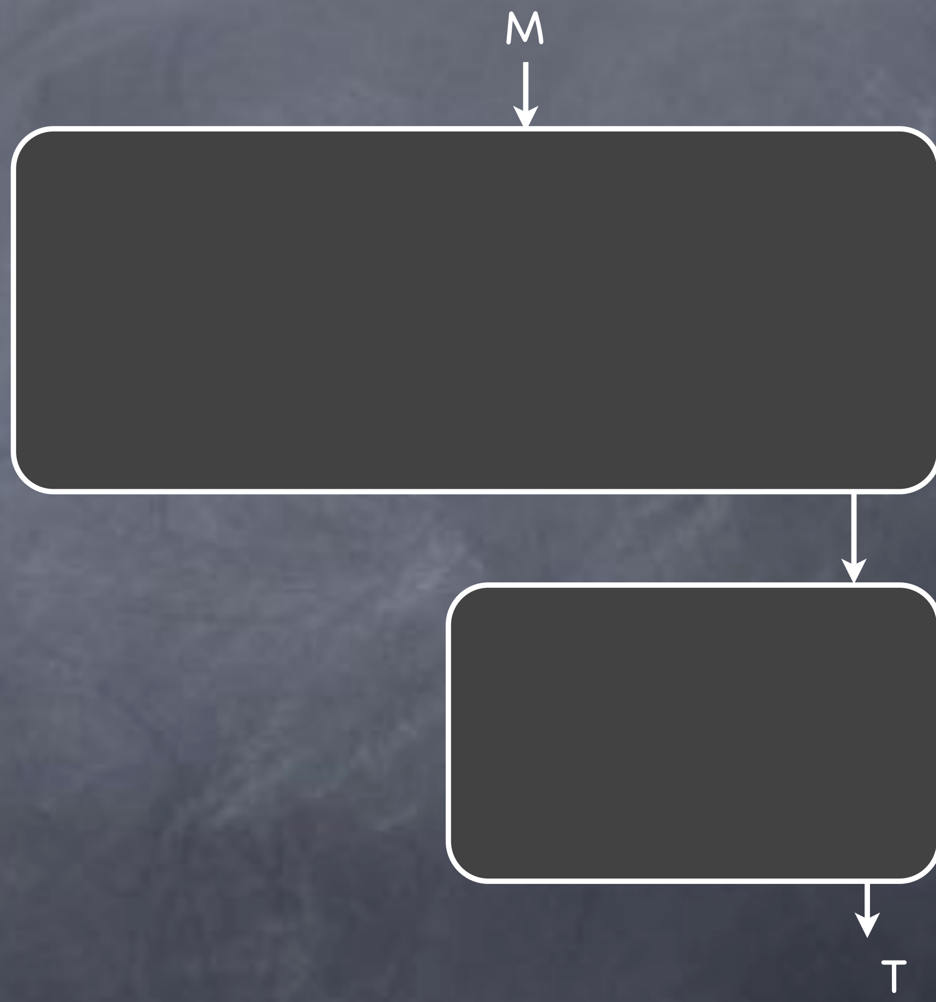

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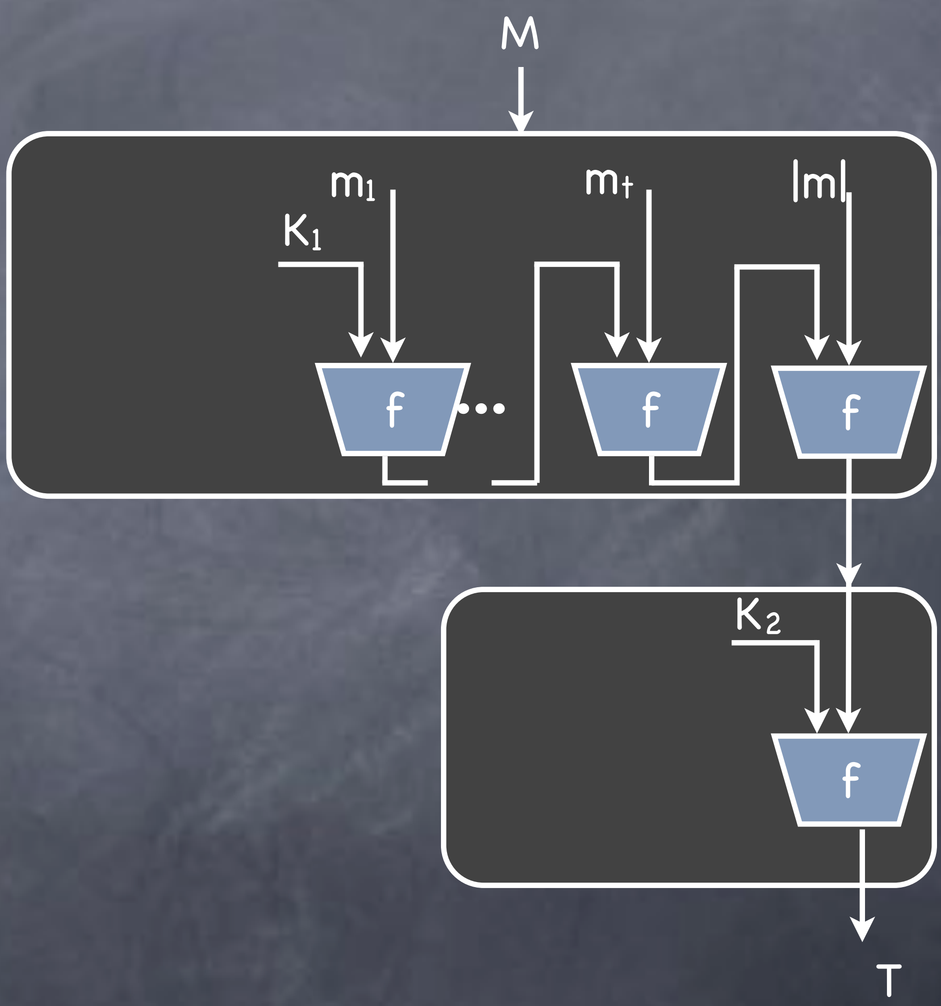

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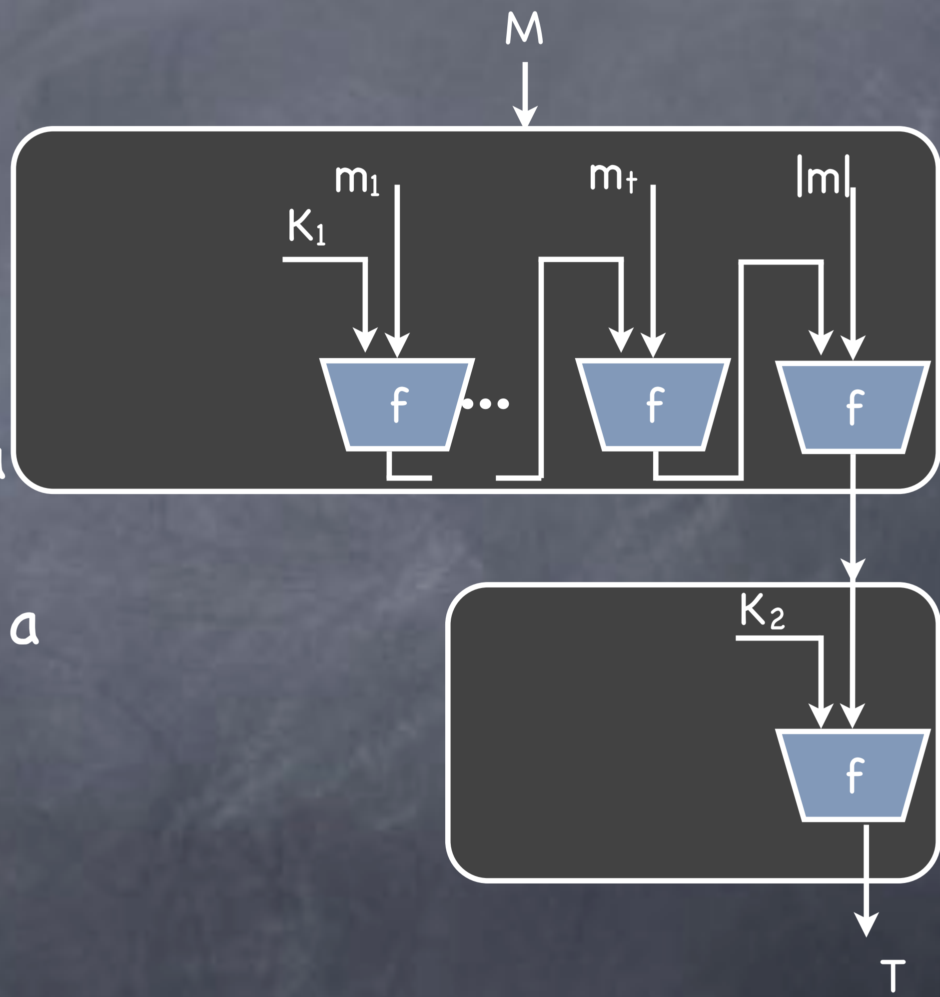

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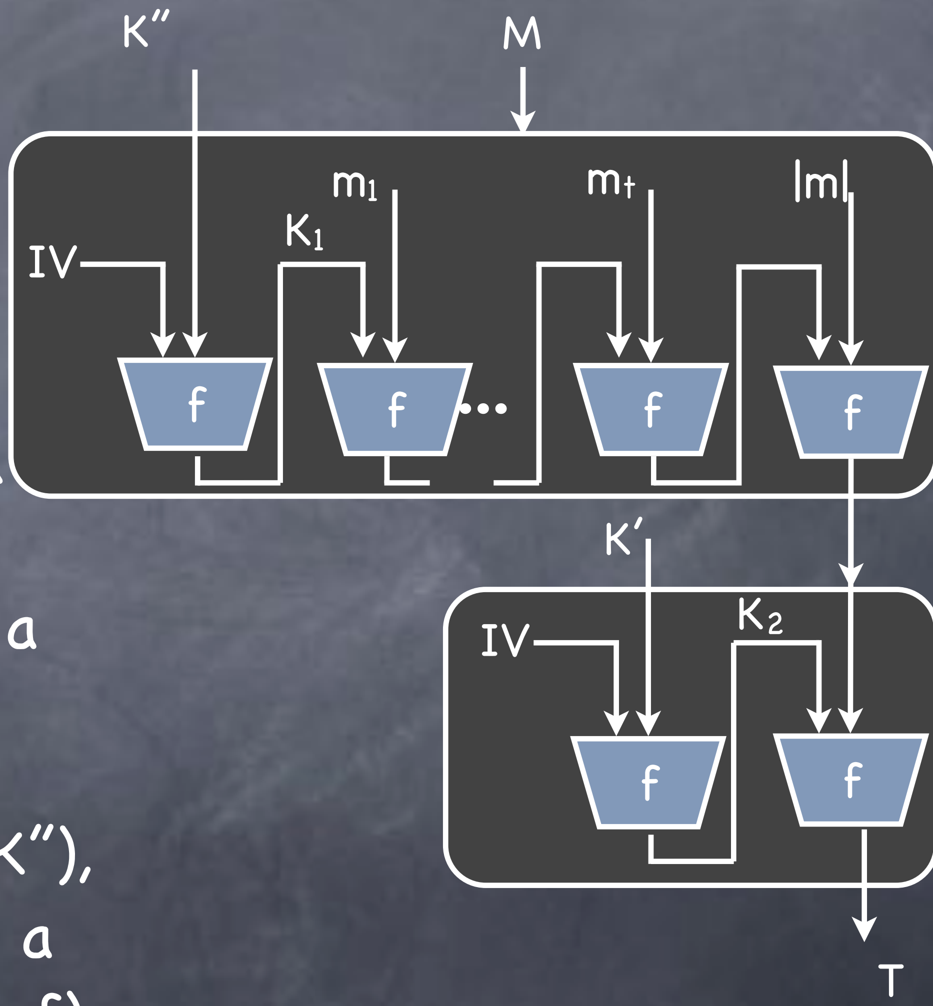
  - In HMAC (K_1,K_2) derived from (K',K''), in turn heuristically derived from a single key K. If f is a (weak kind of) PRF K_1, K_2 can be considered independent



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  - Other suggestions like SHA1(M||K), SHA1(K||M||K) all turned out to be flawed too

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    - This can then be used to build a full-fledged signature scheme starting from one-time signatures (skipped)

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    - "Standard schemes" like RSA-PSS are based on this

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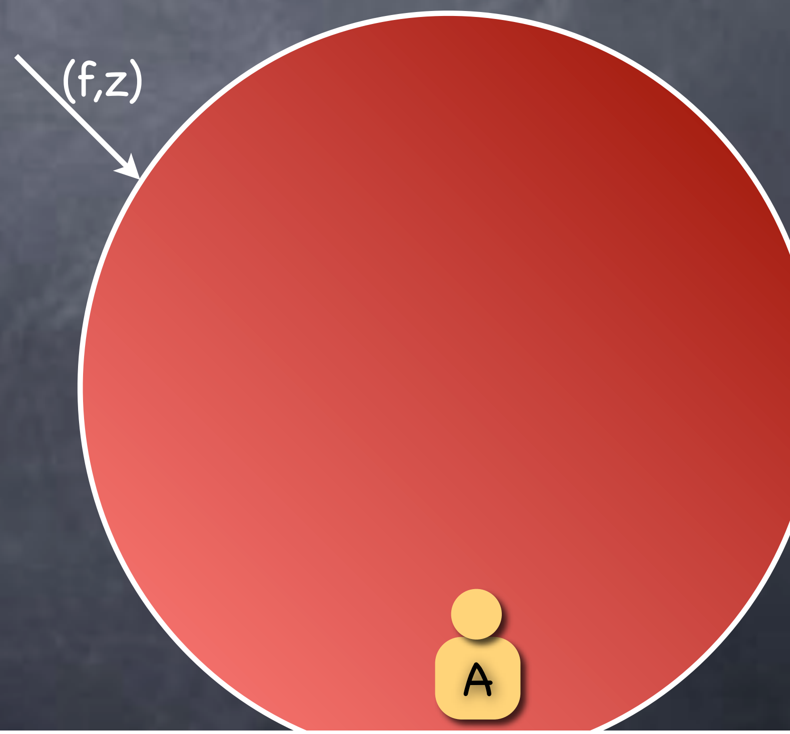

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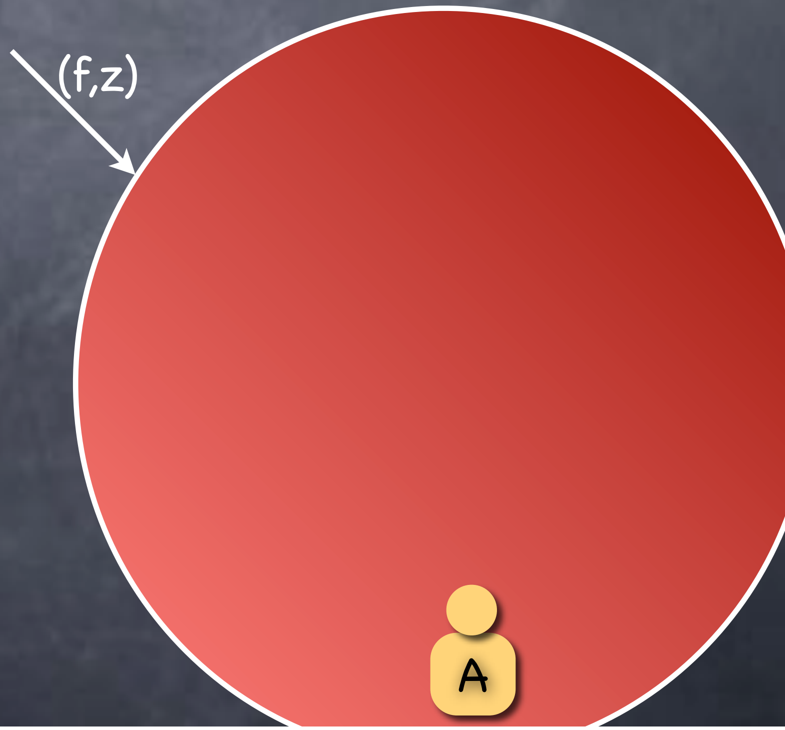

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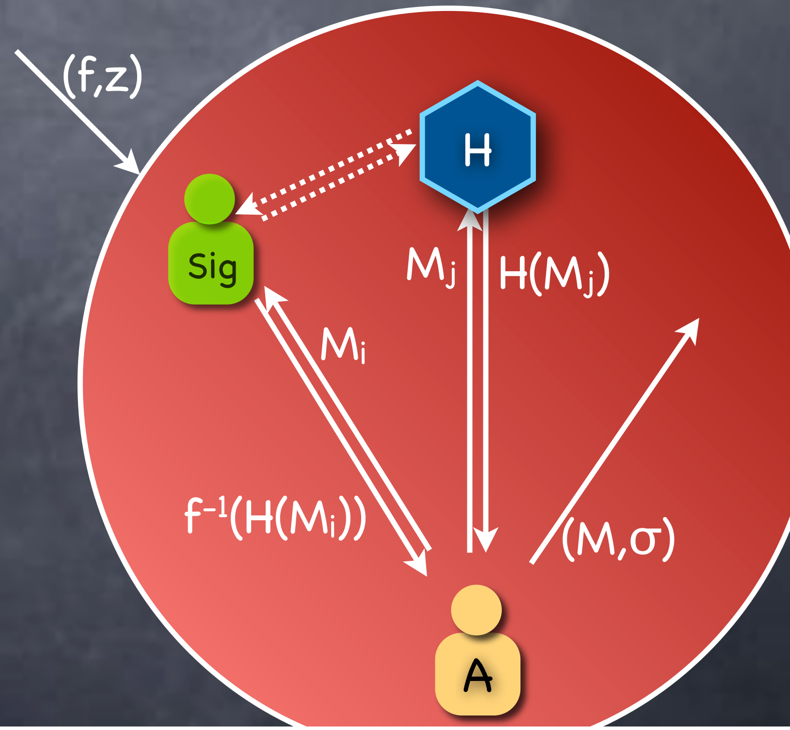

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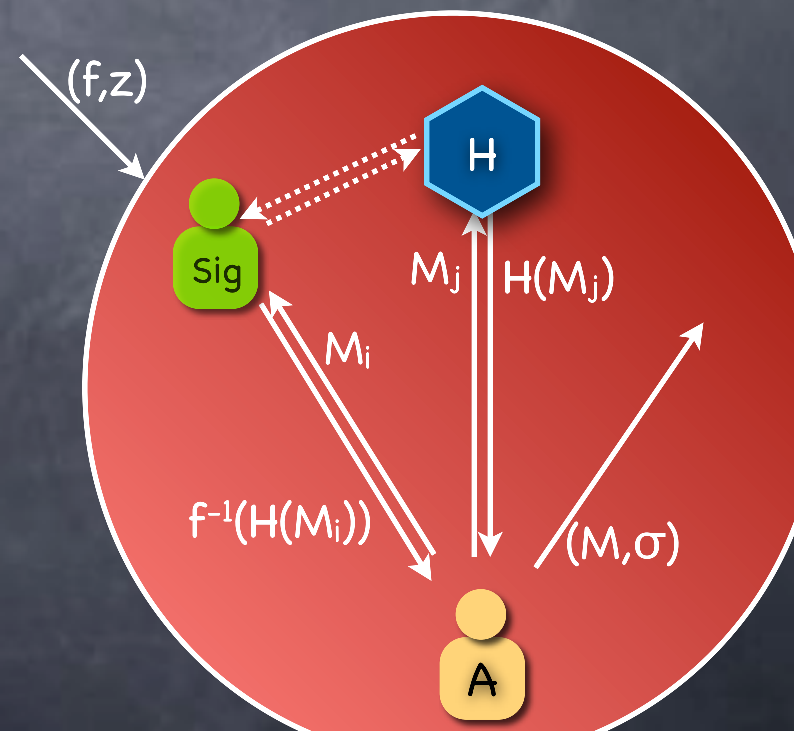

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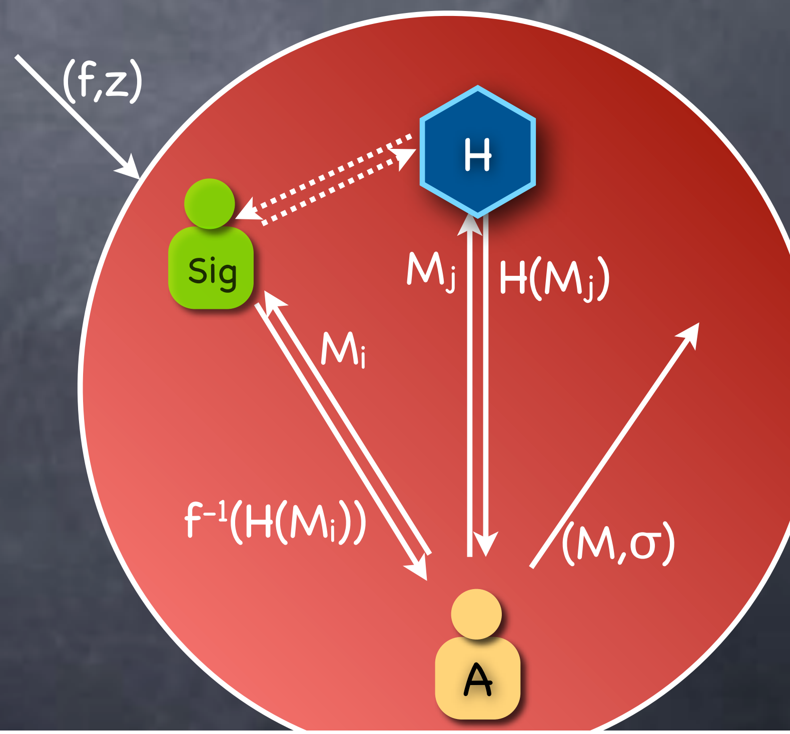

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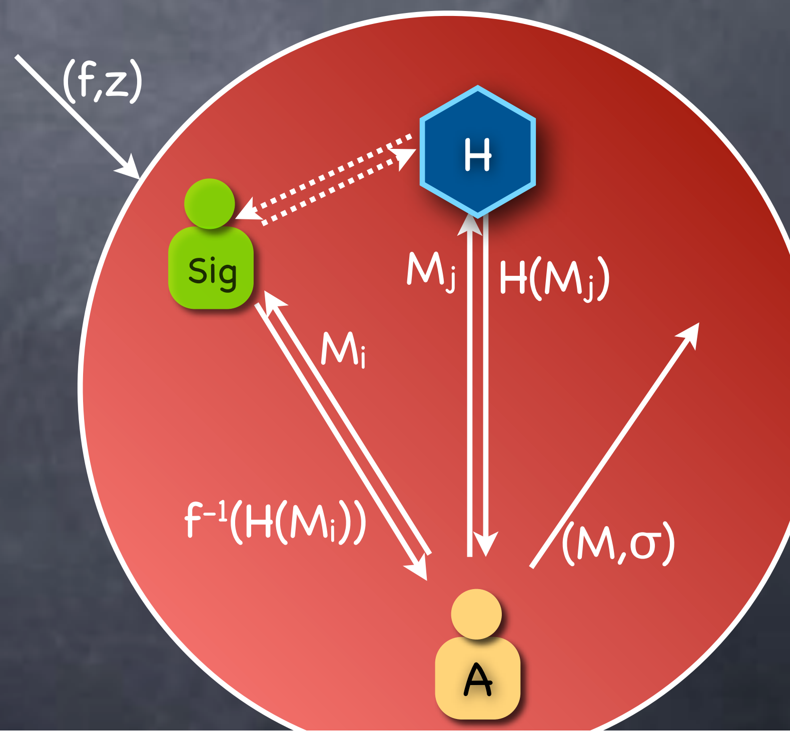

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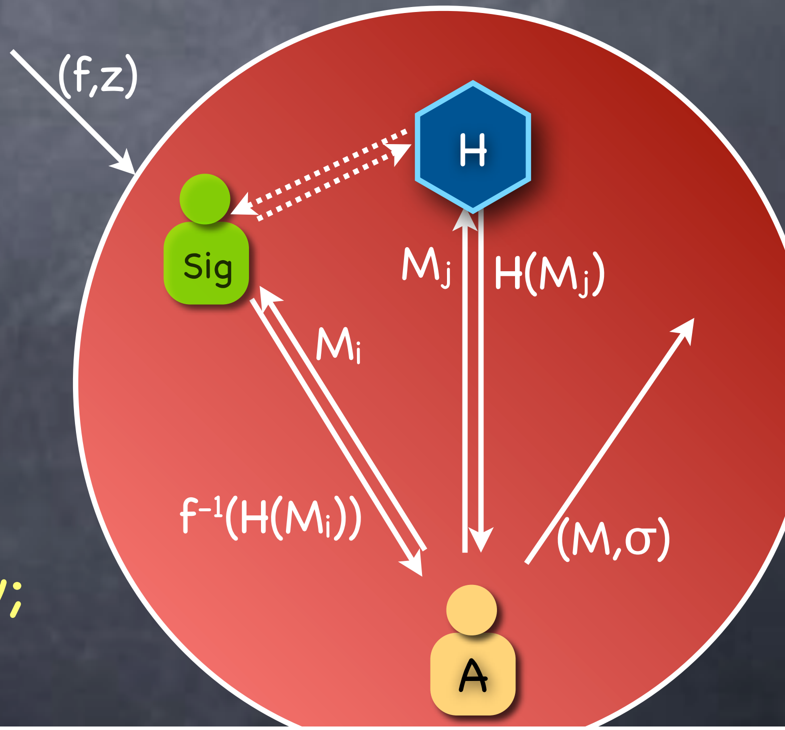

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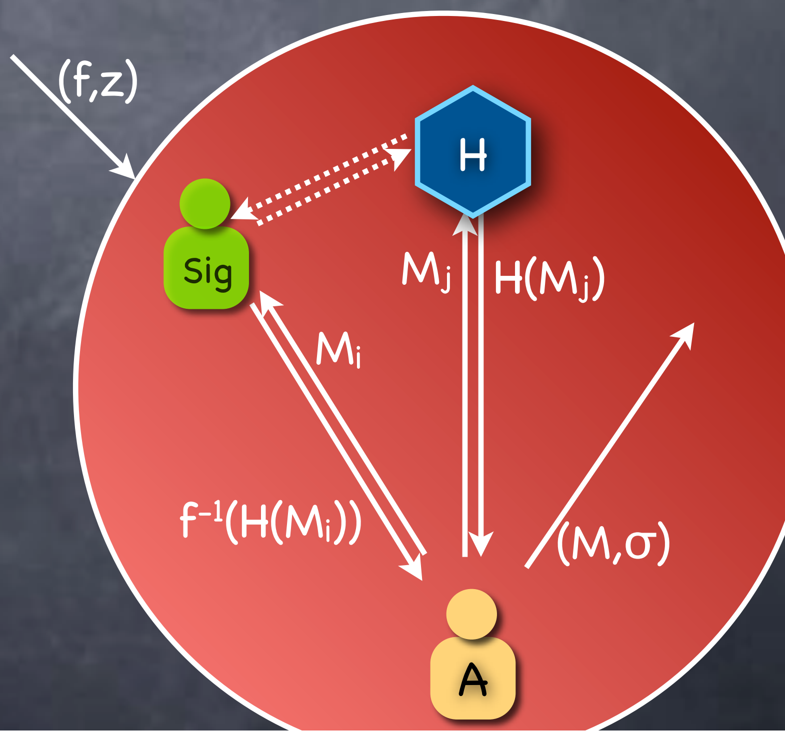

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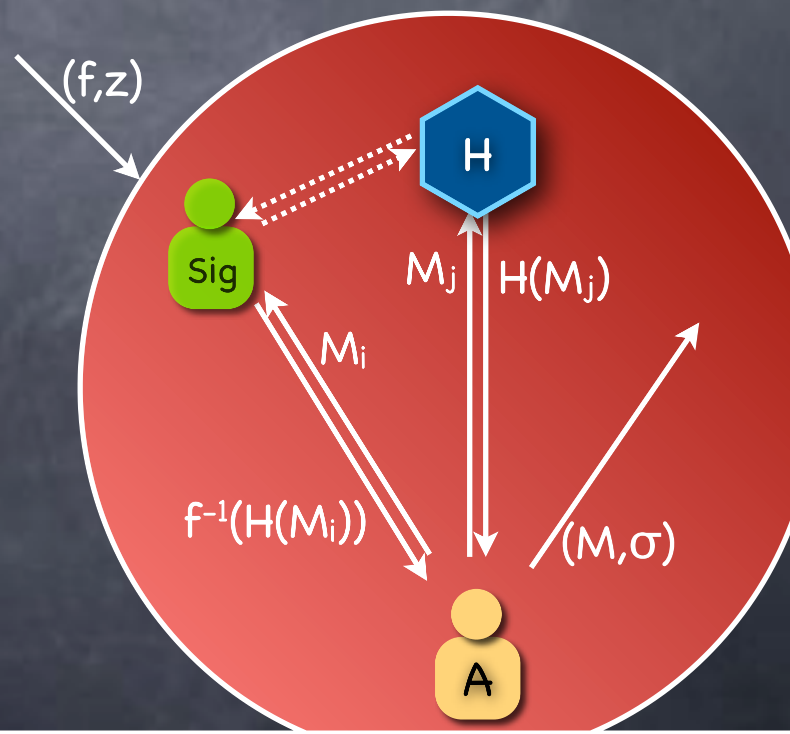

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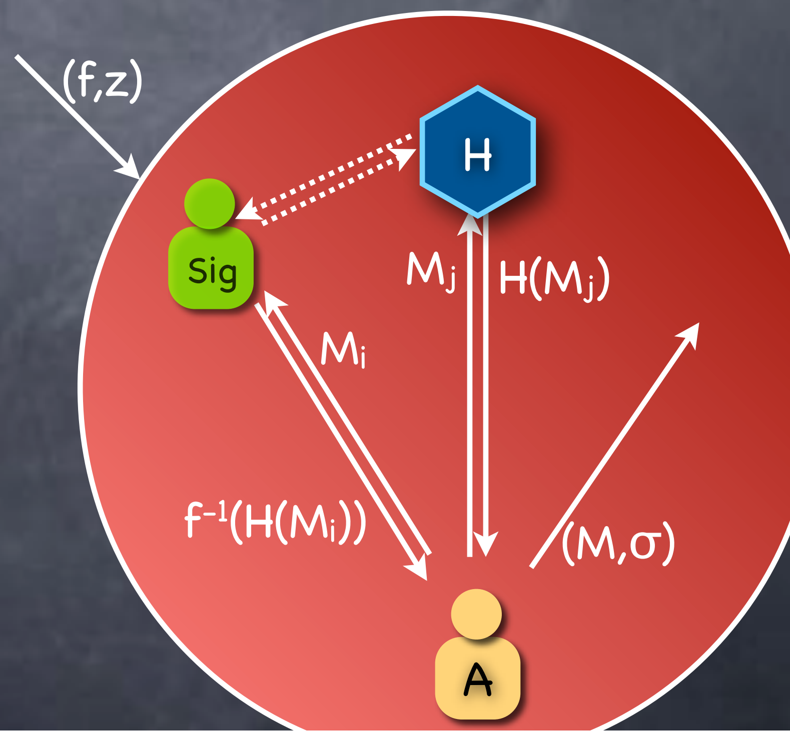

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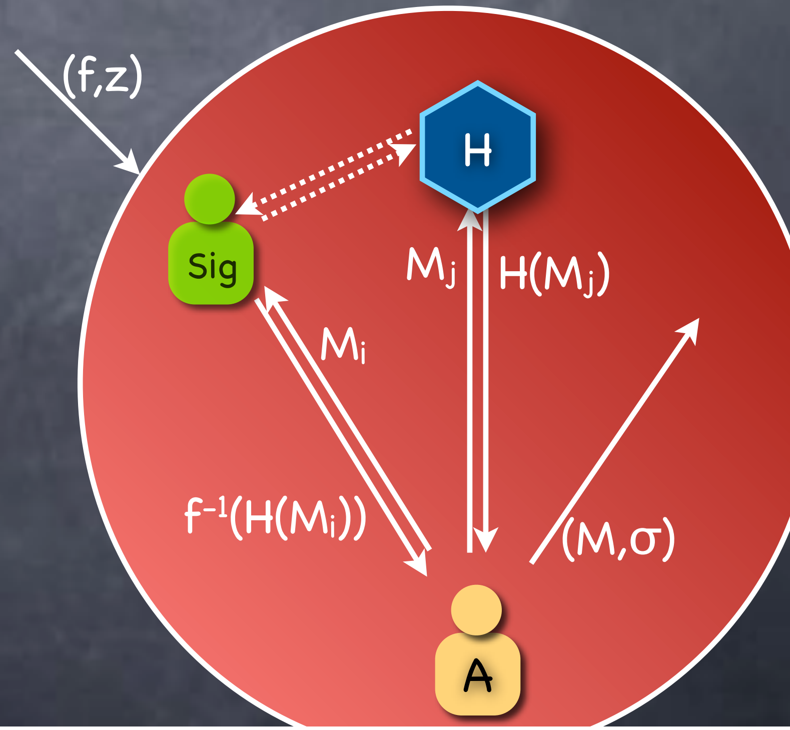

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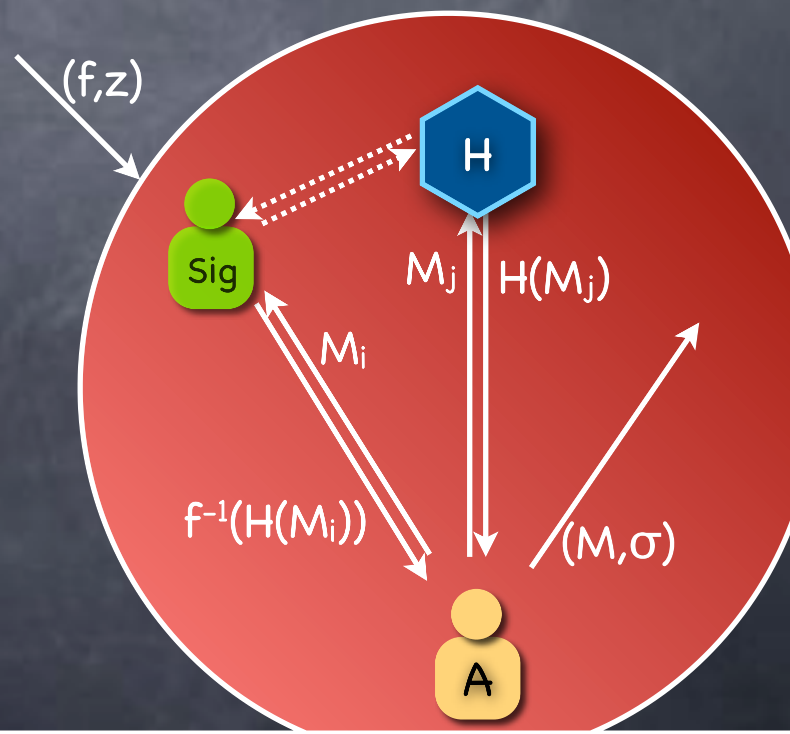

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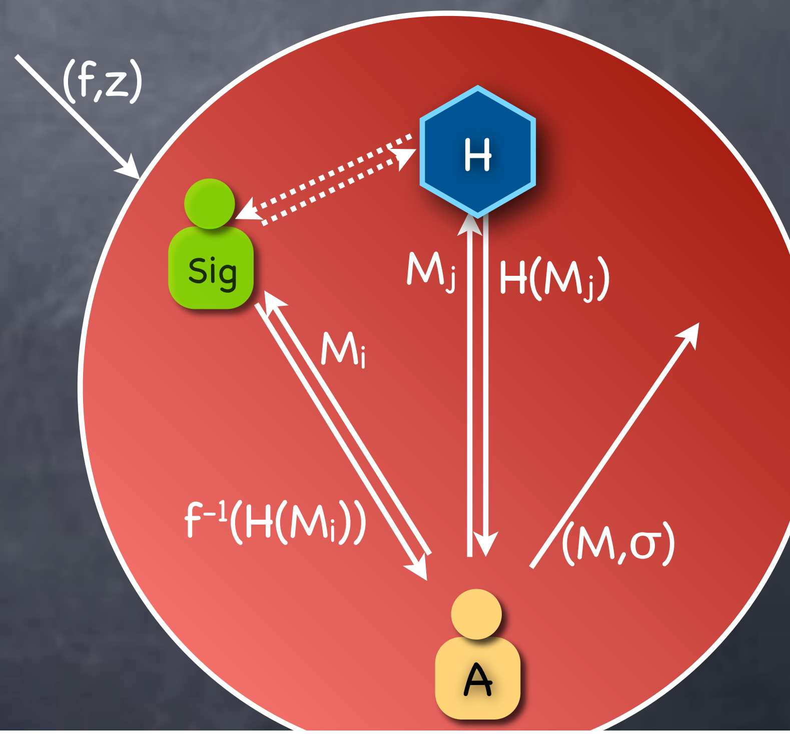

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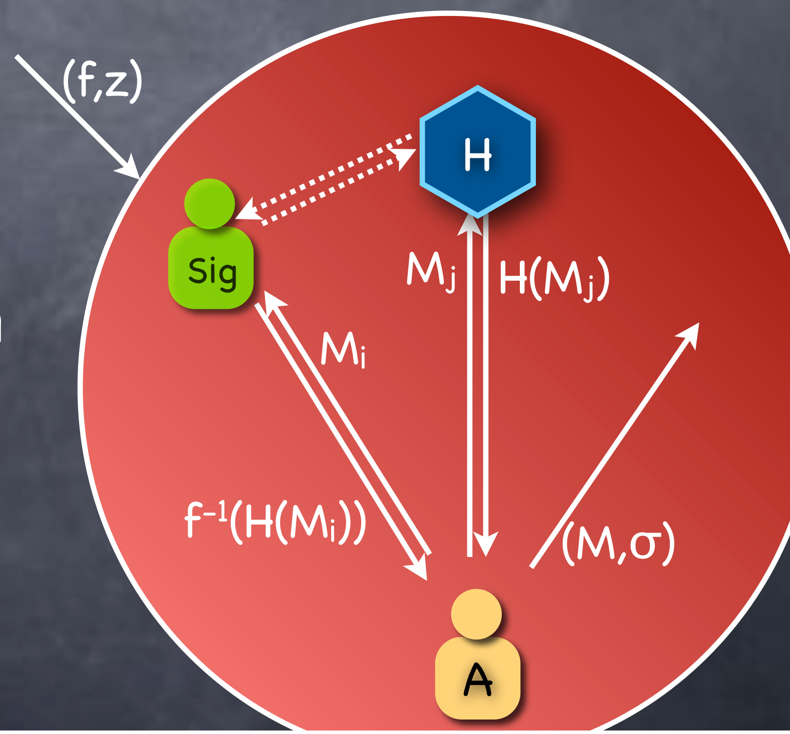

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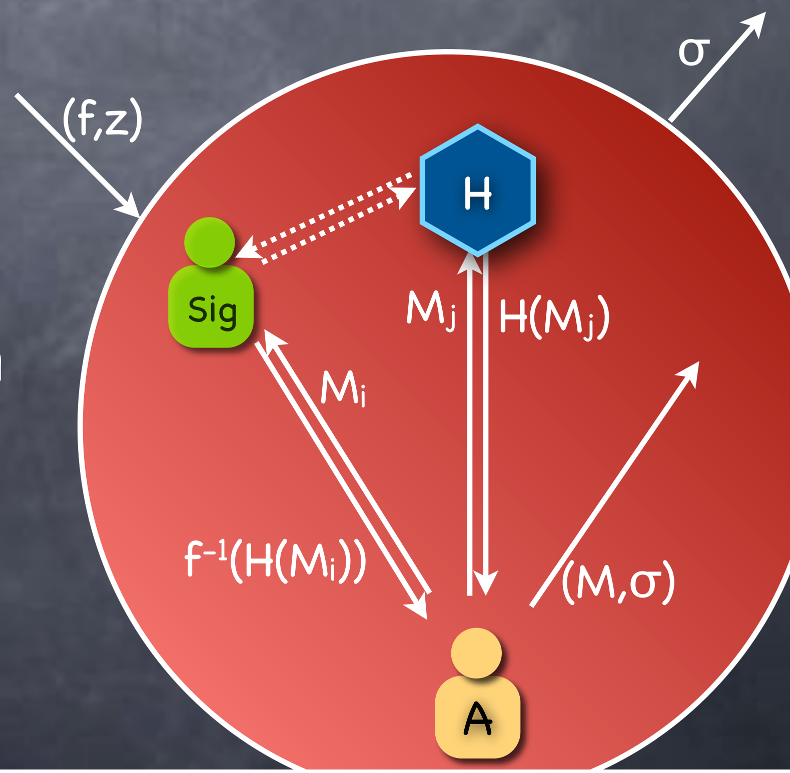

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- A general tool for purifying randomness: Randomness Extractor

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  - Statistical guarantee, if compression function/block-cipher is a random function/random permutation (not random oracle)

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  - Key derivation: Alice and Bob extract a new key, which is pseudorandom (i.e., indistinguishable from a uniform bit string)

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  - Random oracle and Trapdoor OWP for signatures