Public-Key Cryptography

Public-Key Cryptography

Lecture 7

Public-Key Encryption

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CCA Security

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- But in PKE, Bob wants to receive messages from Eve as well
  - Only if it is indeed Eve's message: she should know her own message!

Chosen Ciphertext Attack

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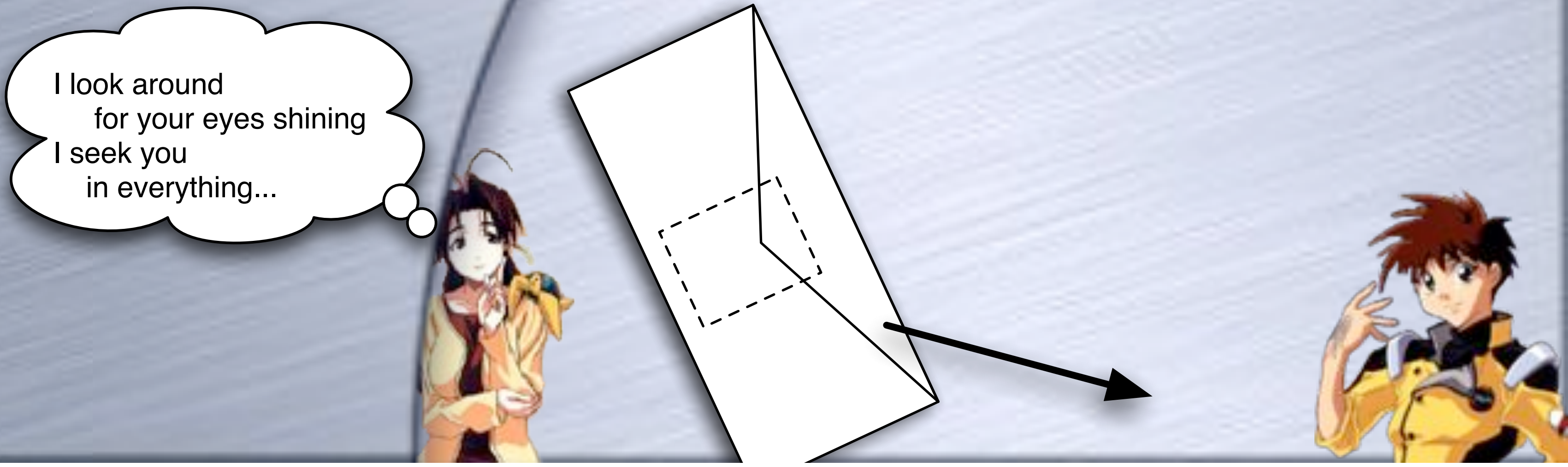

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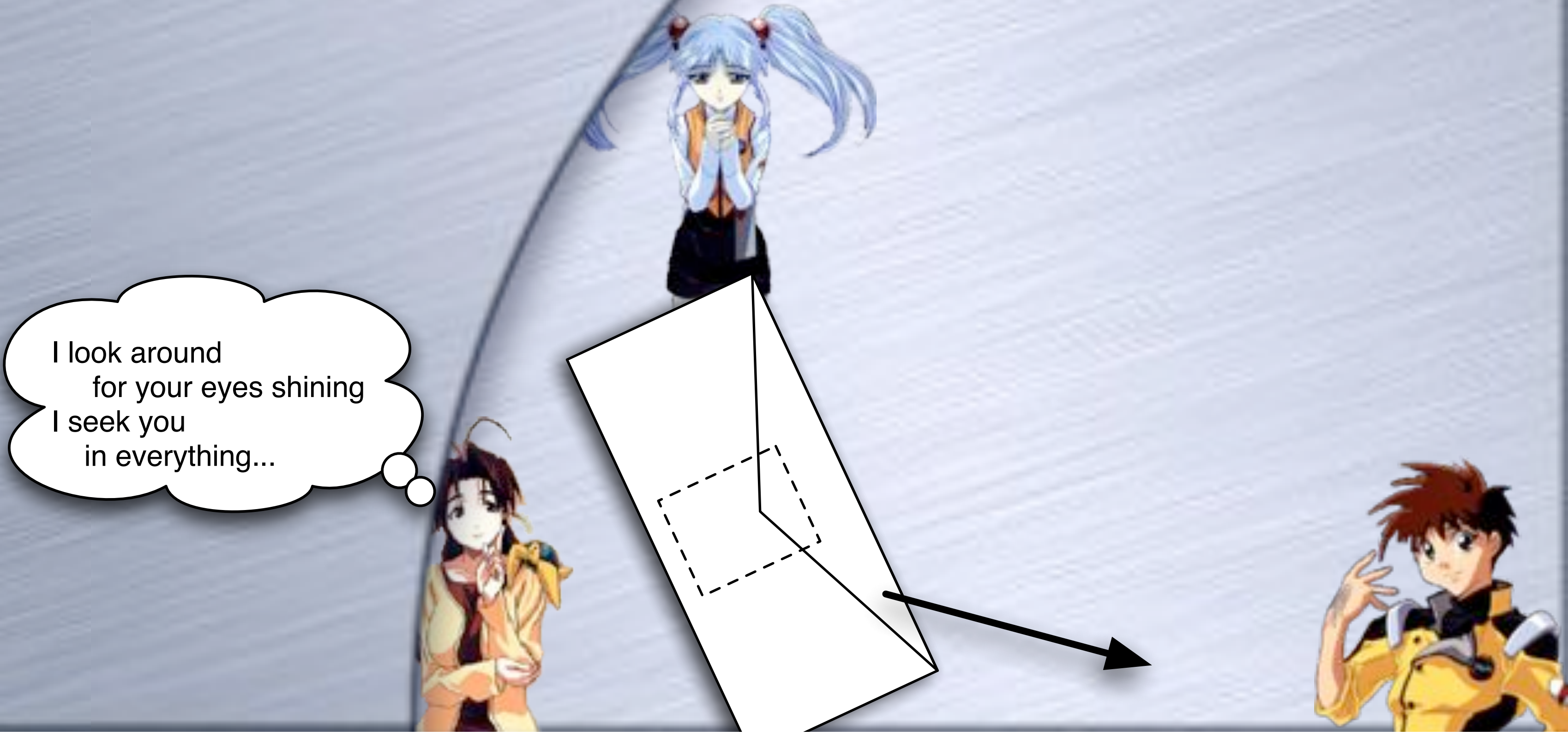

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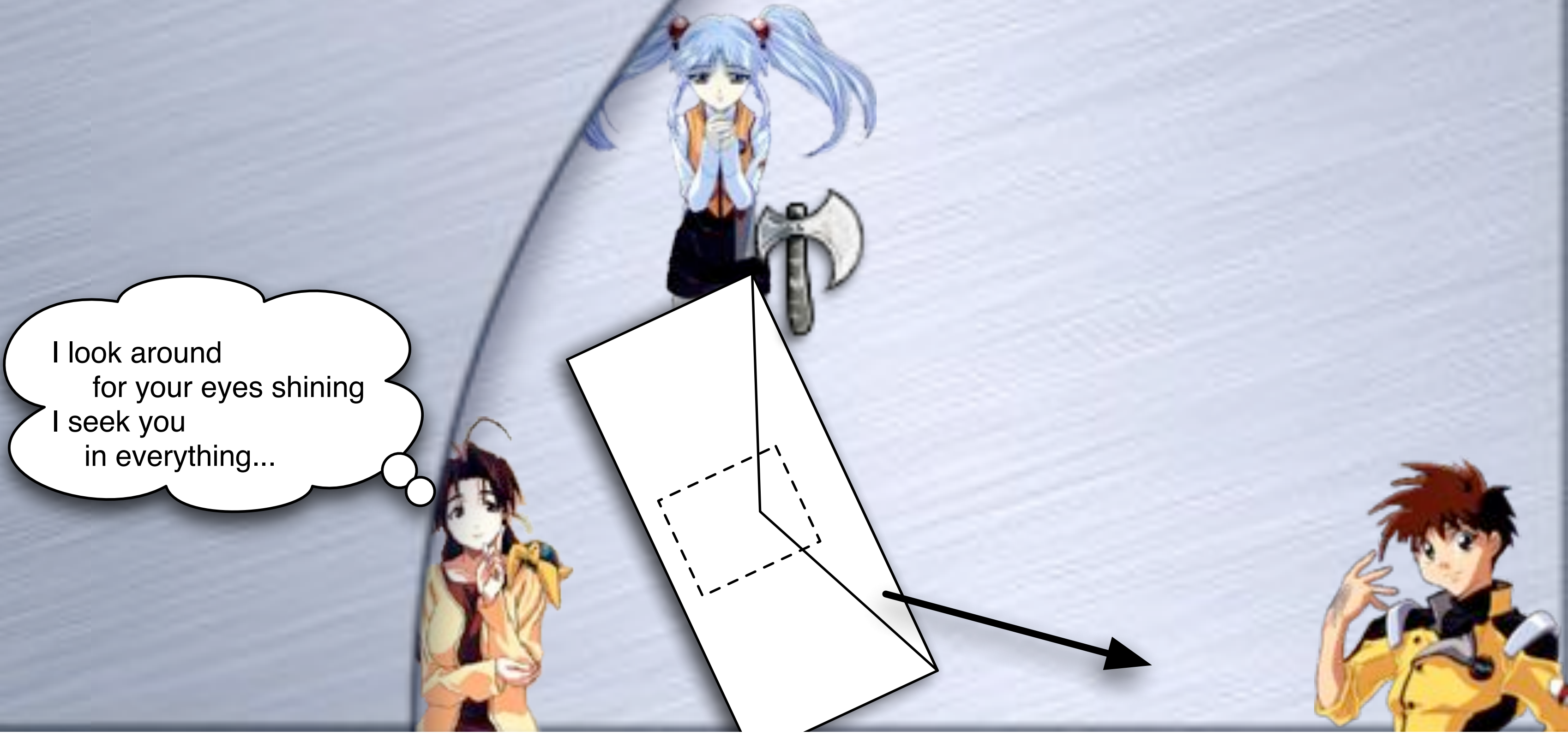

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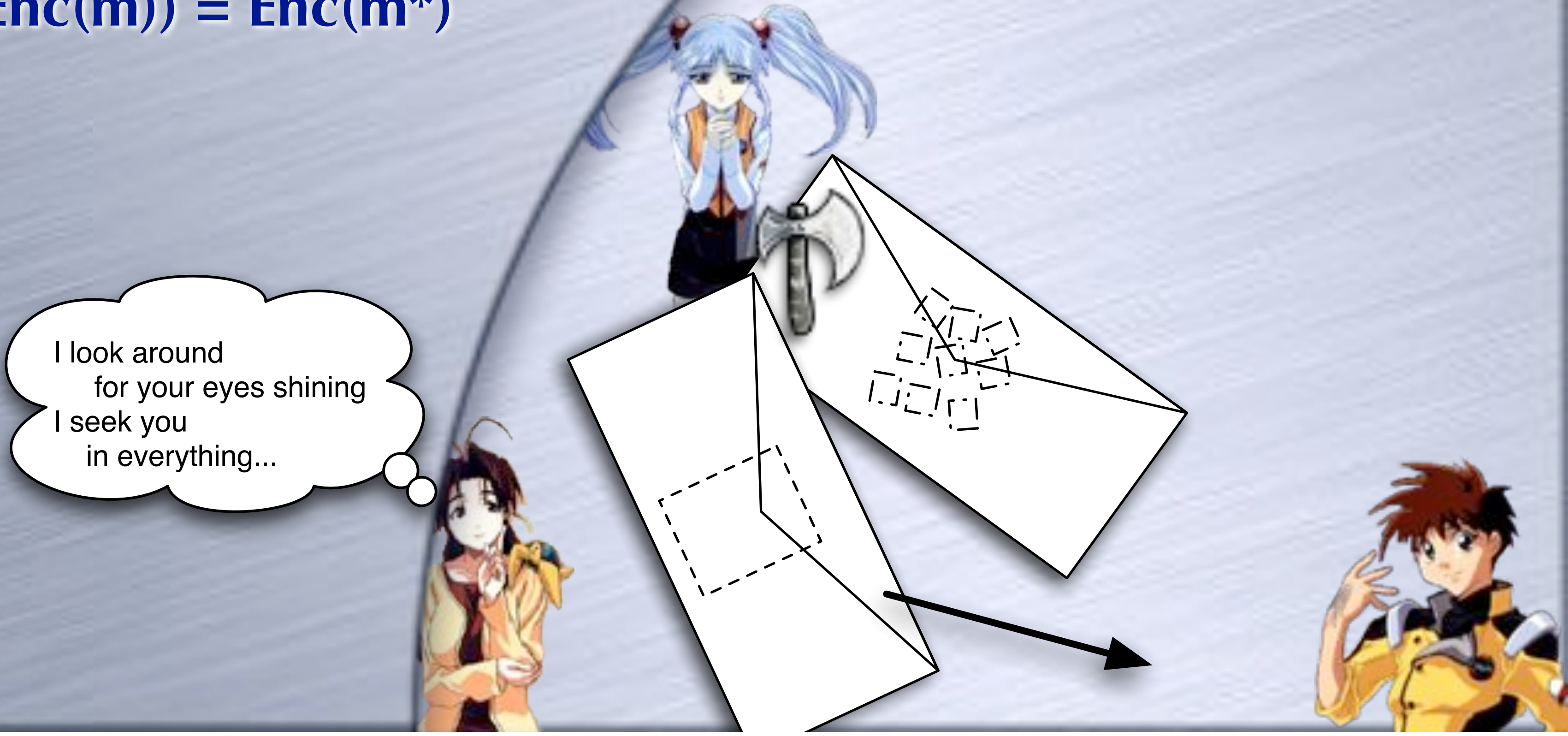

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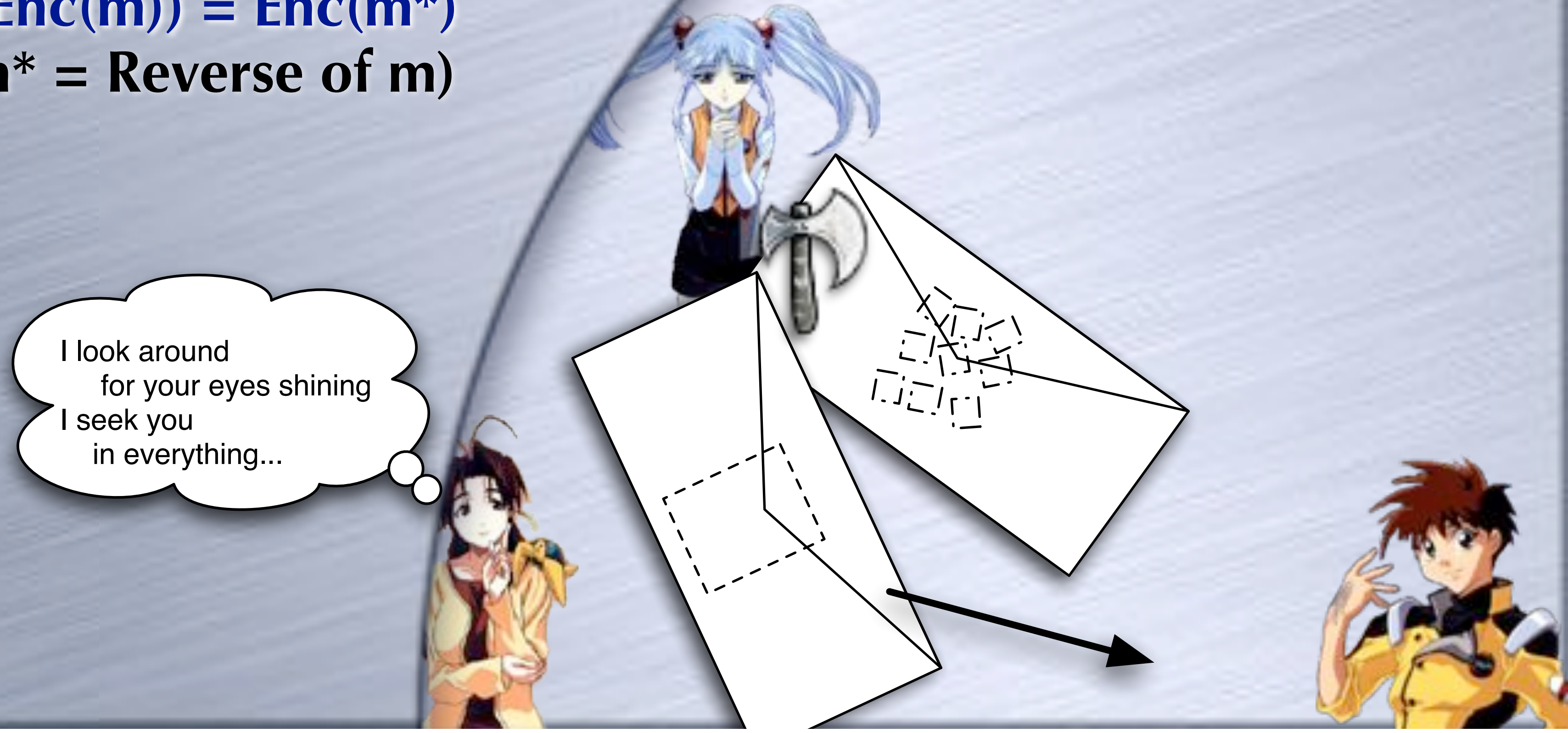

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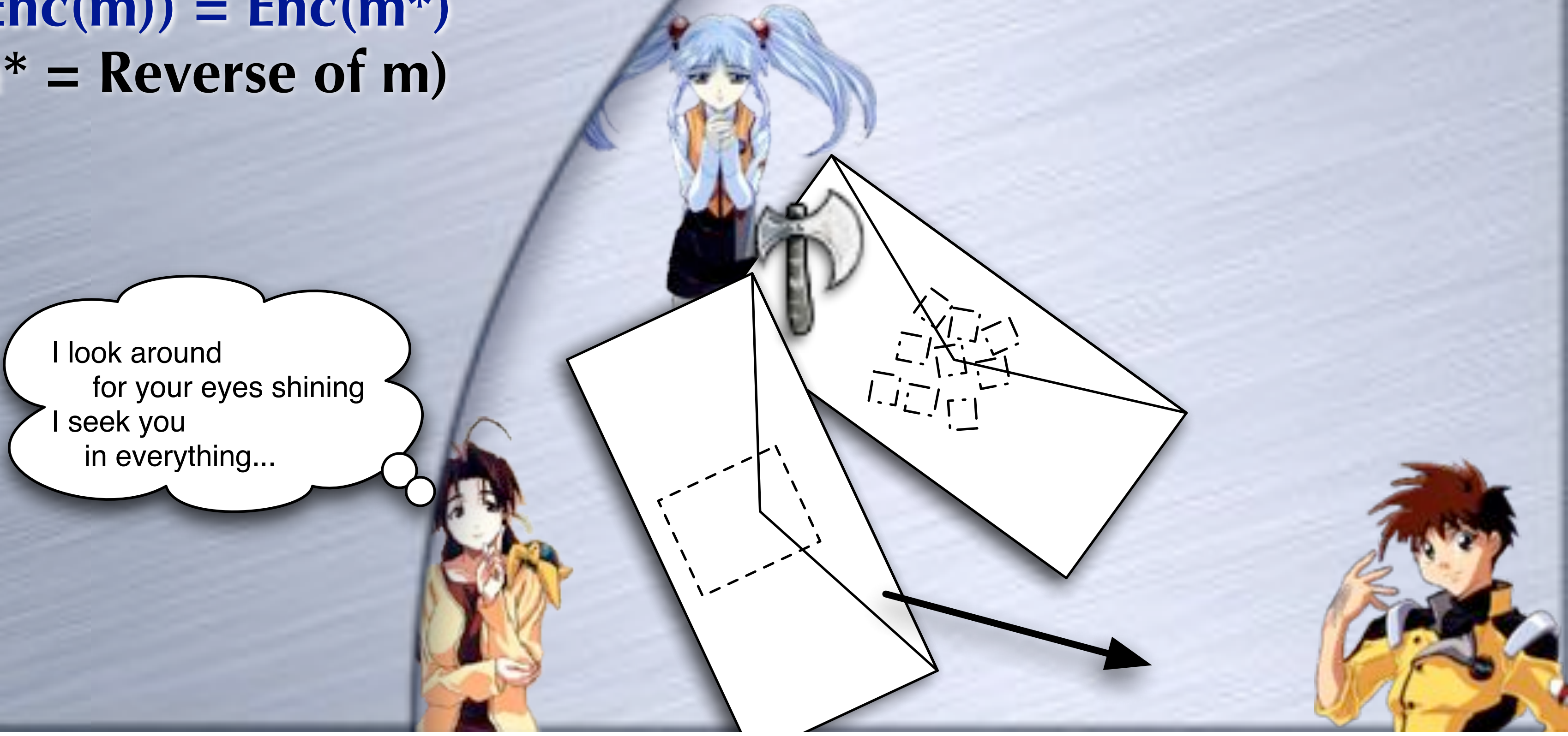

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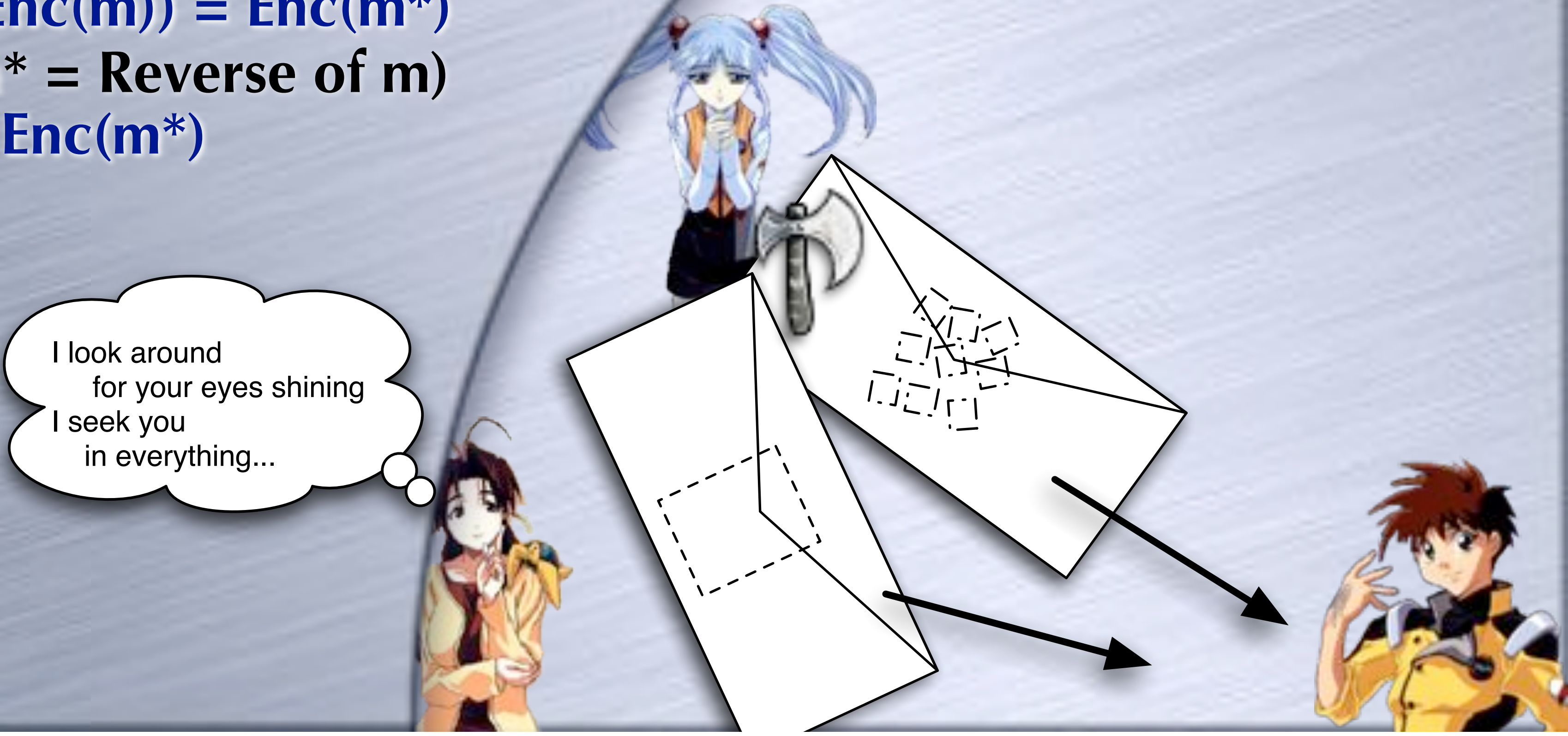

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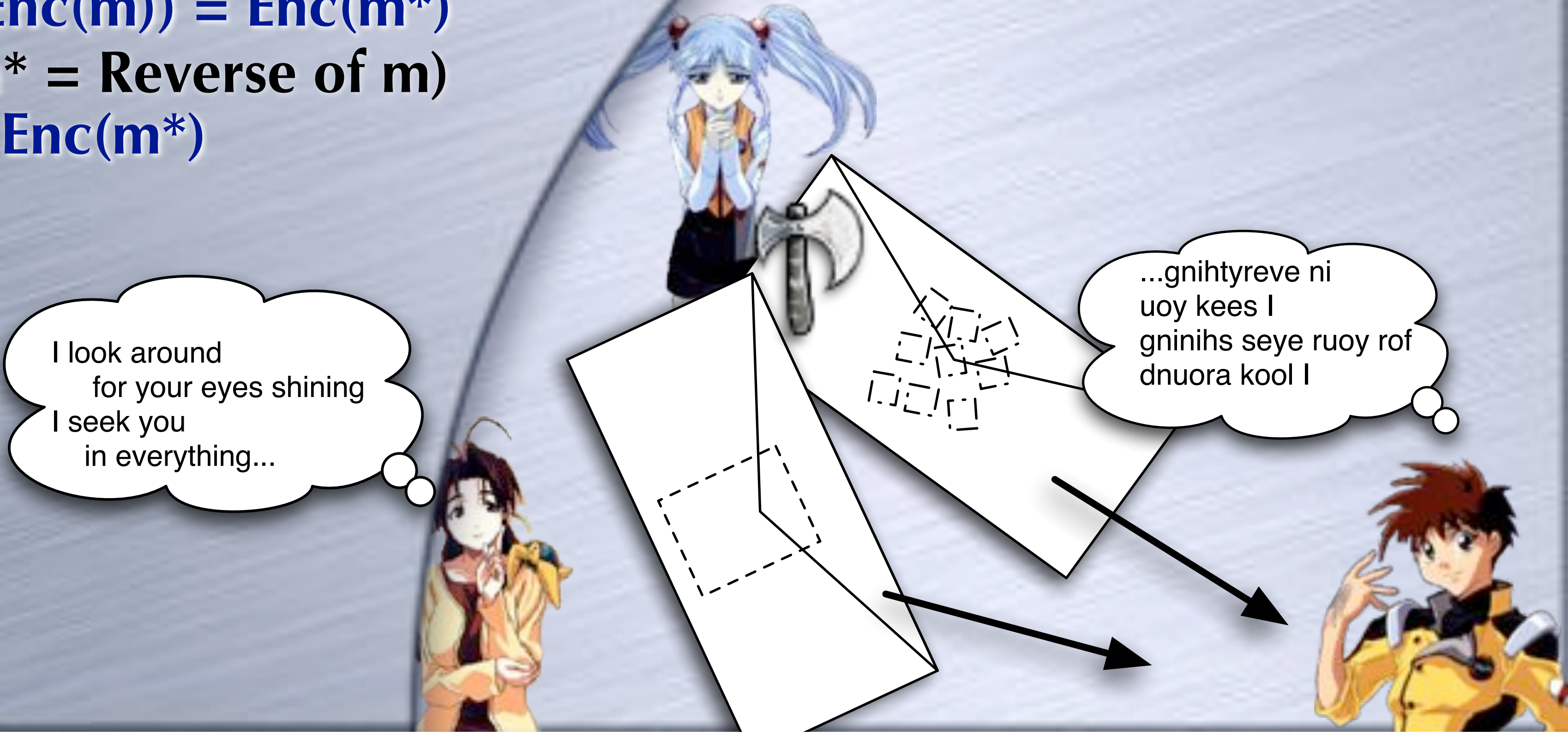

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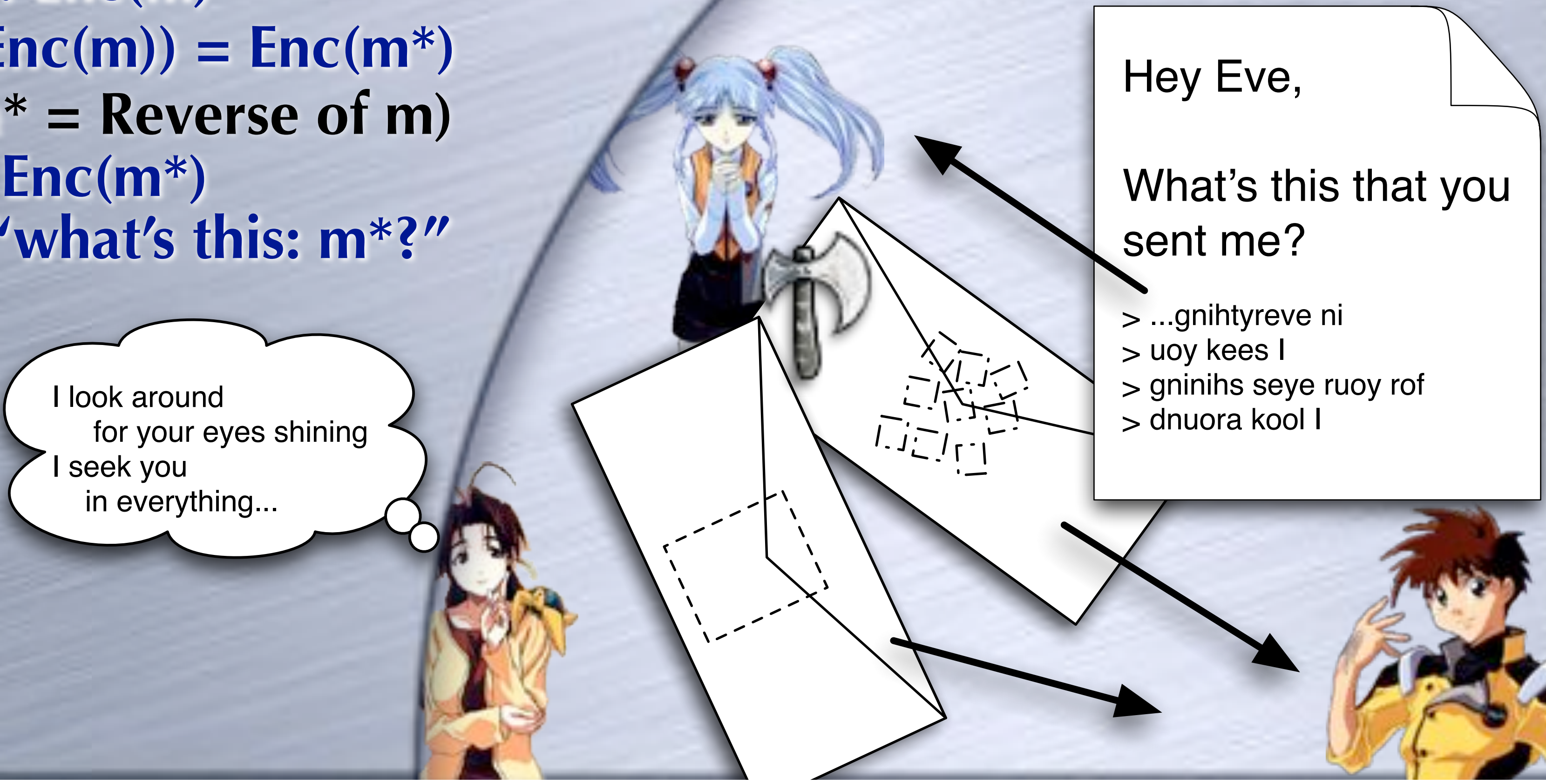

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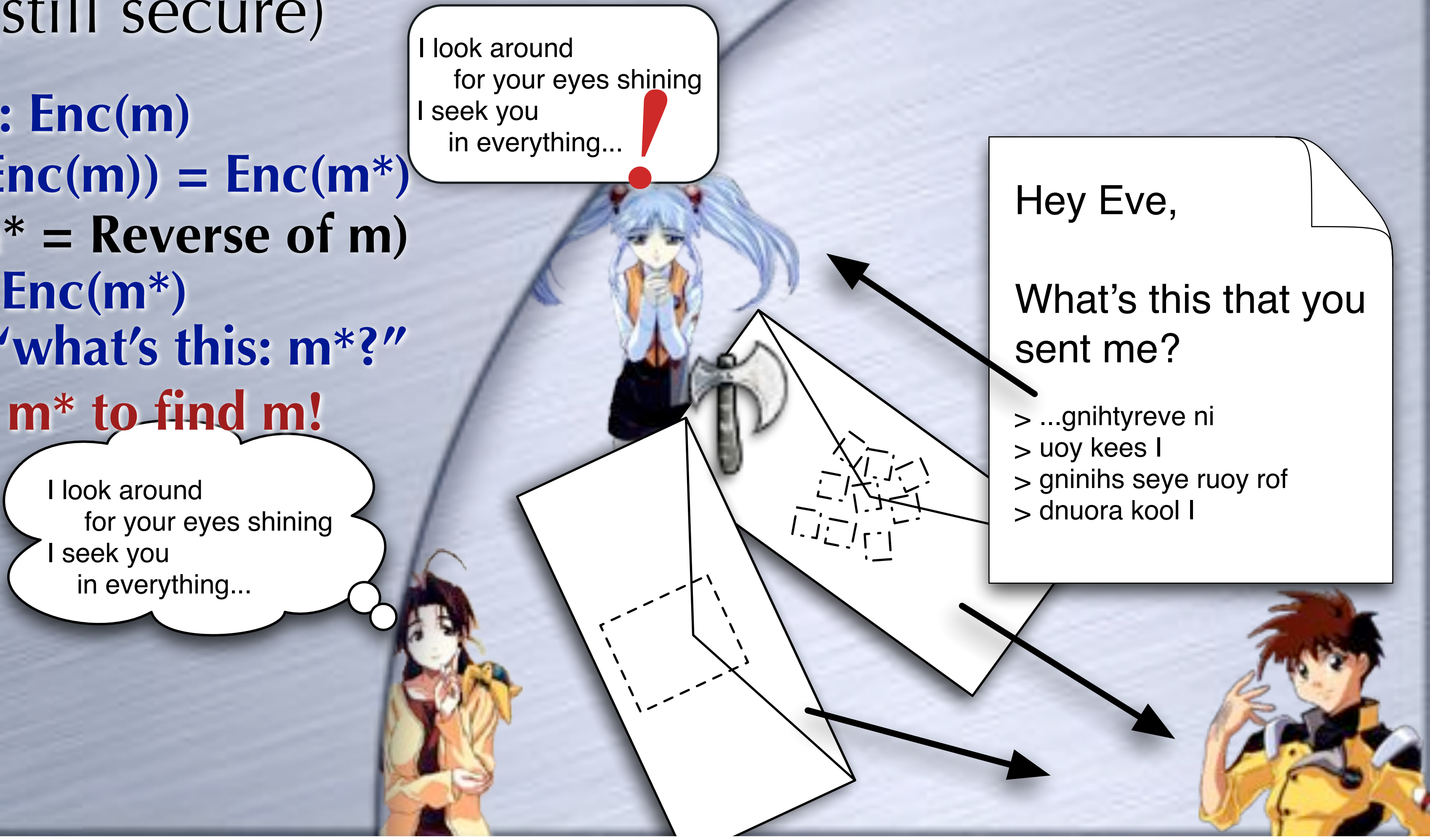
Eve: Reverse m* to find m!



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  - Then Eve can exploit malleability to learn something "related to" Alice's messages

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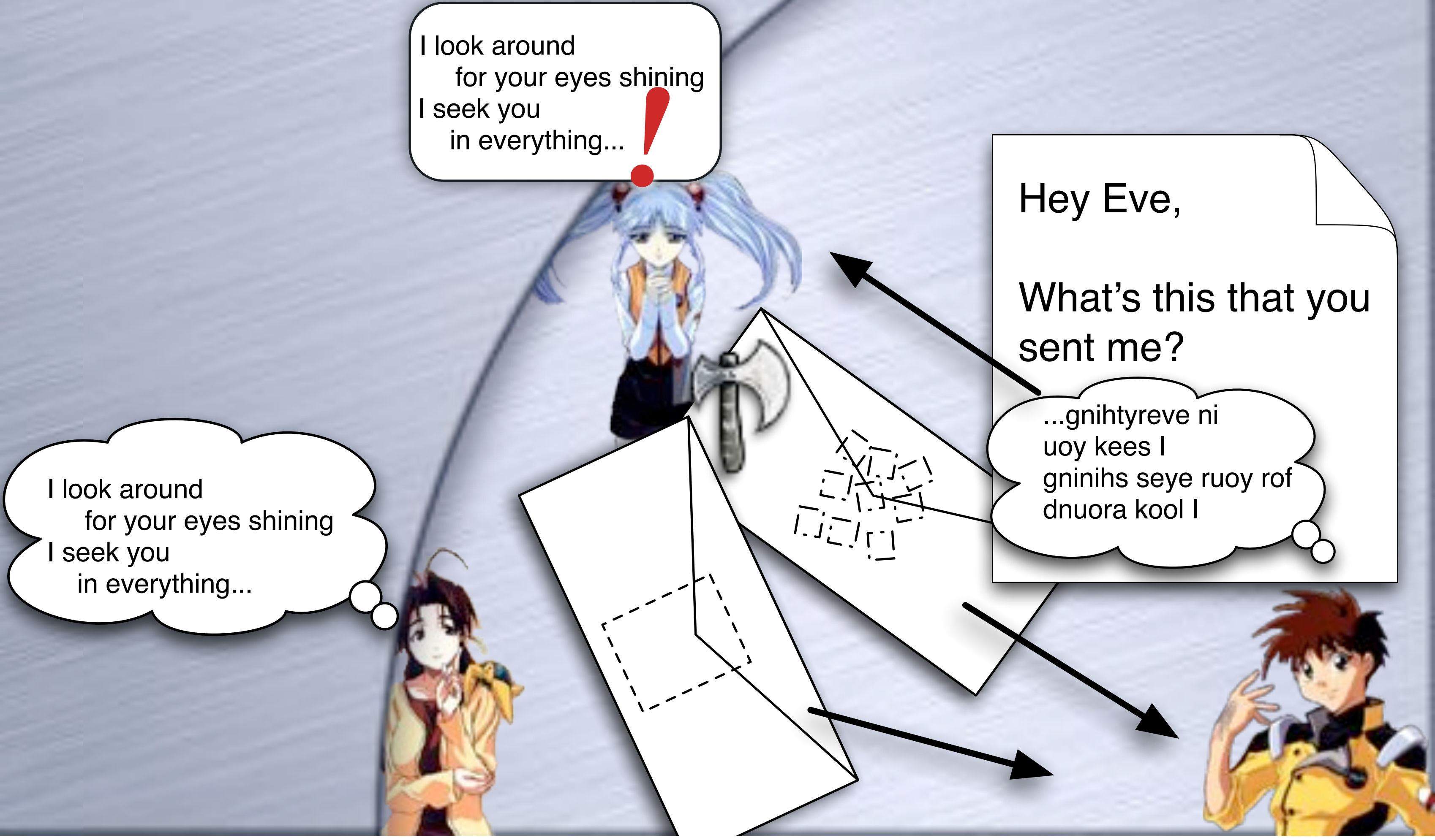

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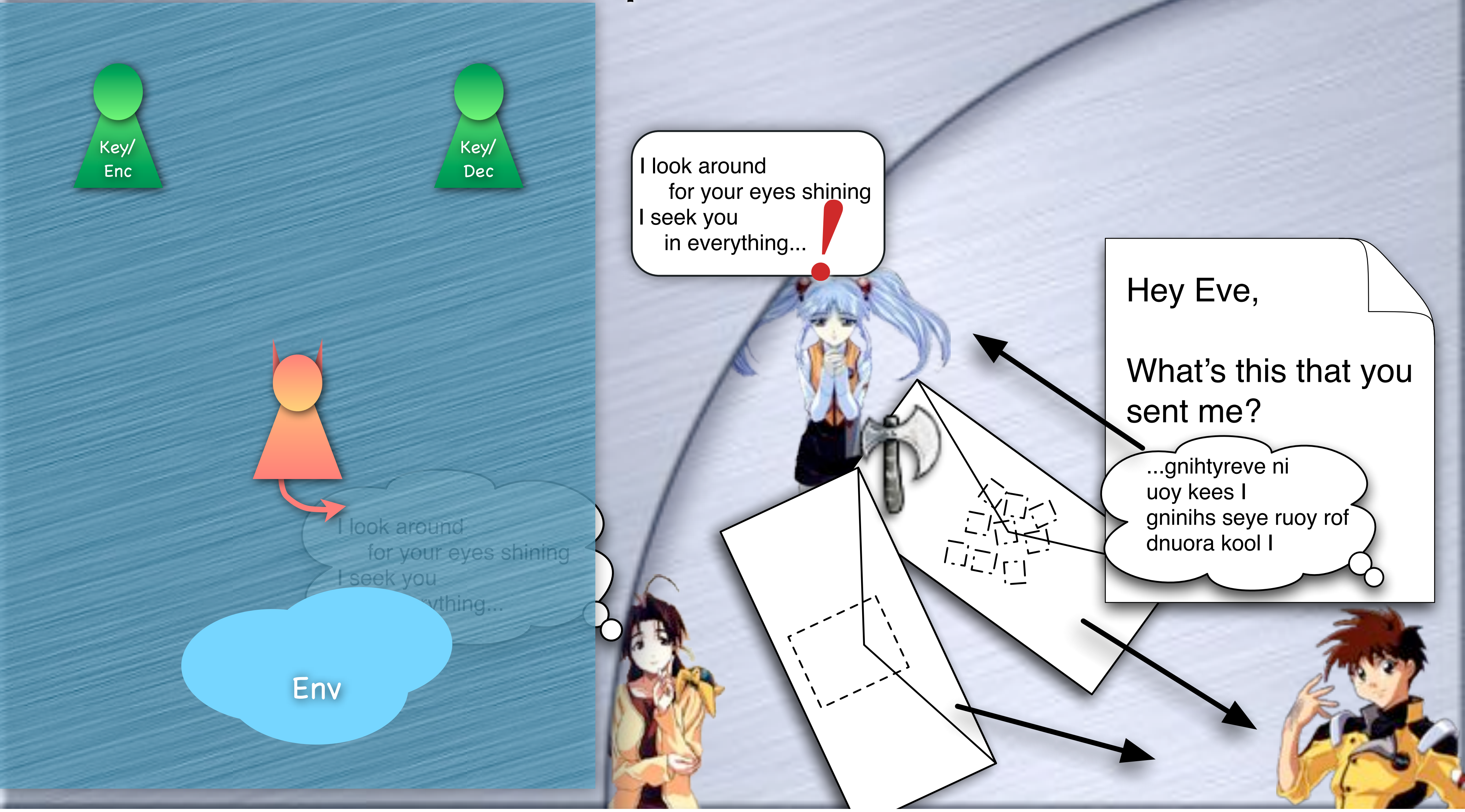

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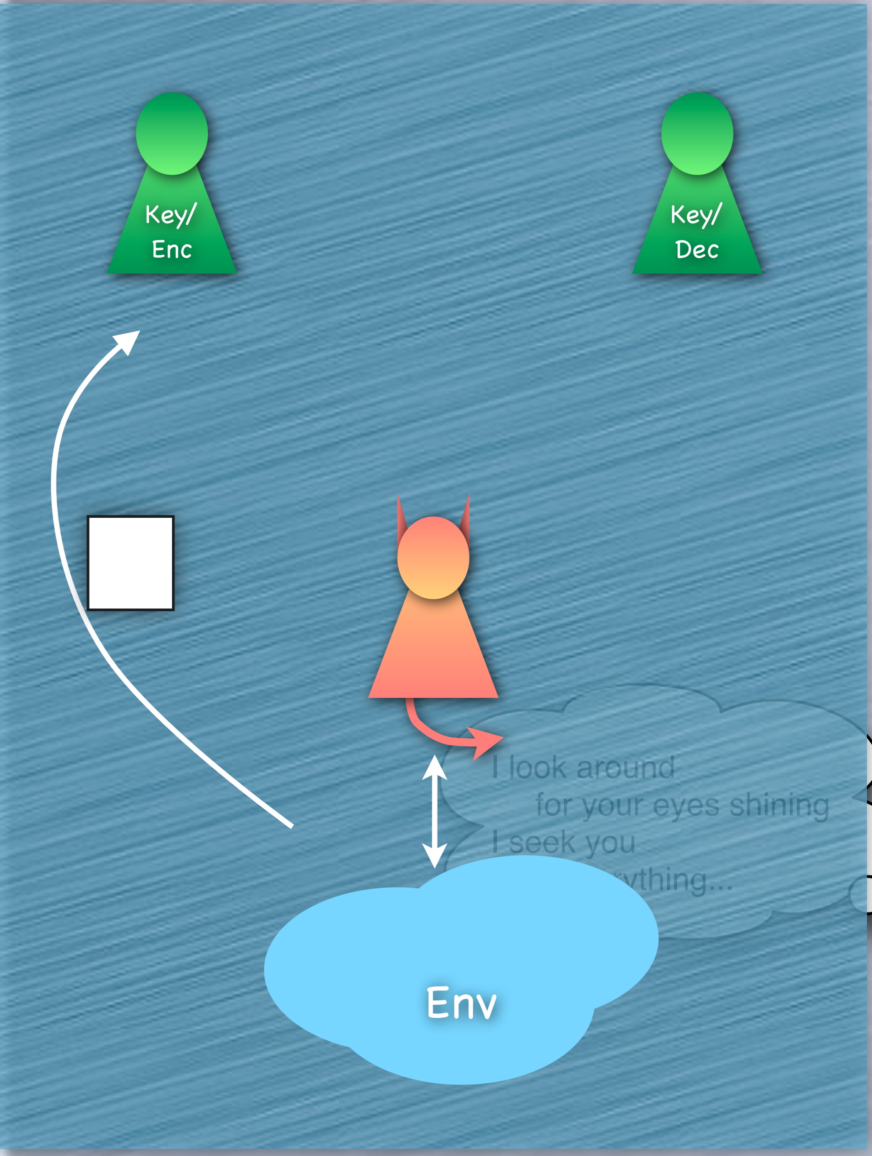

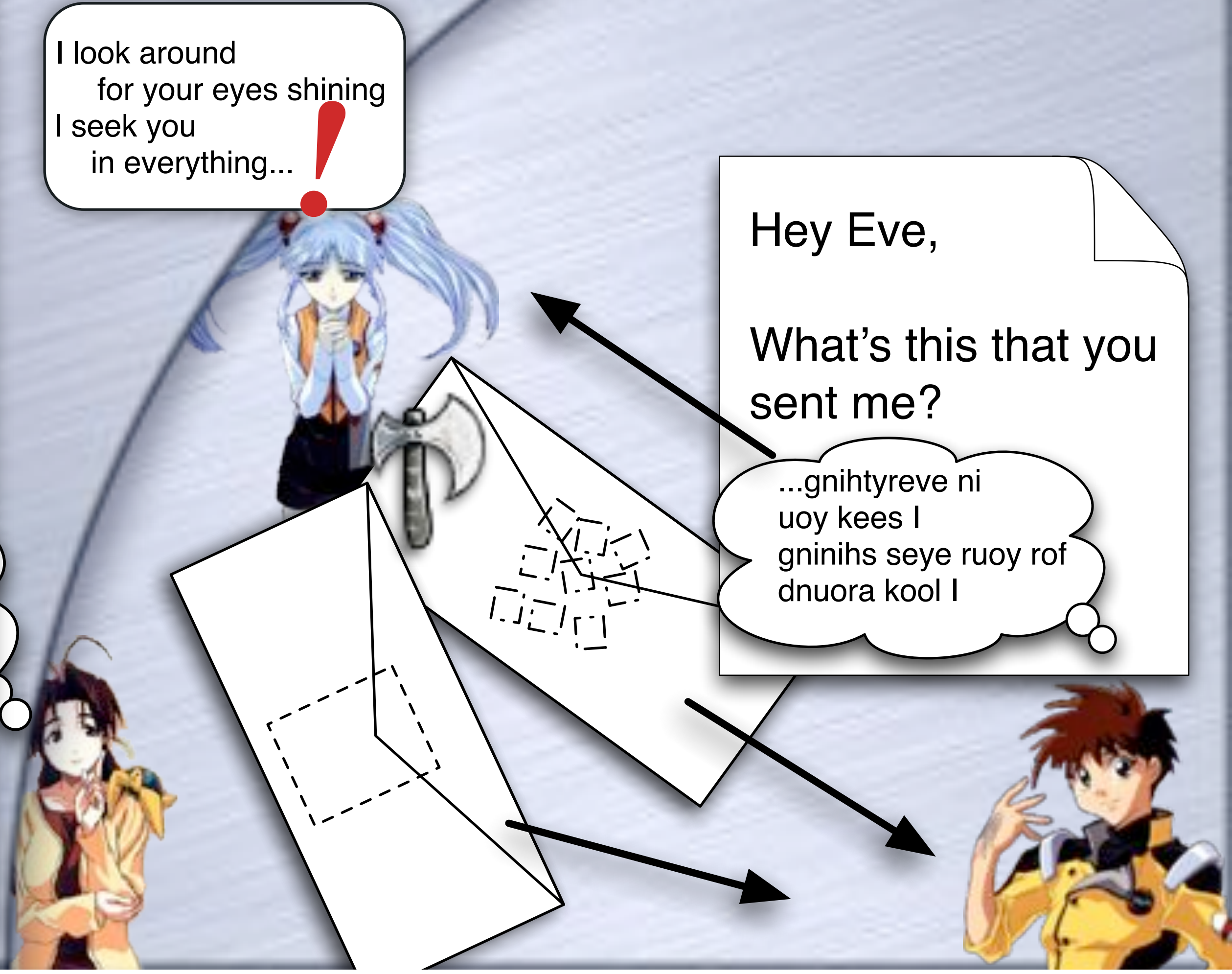

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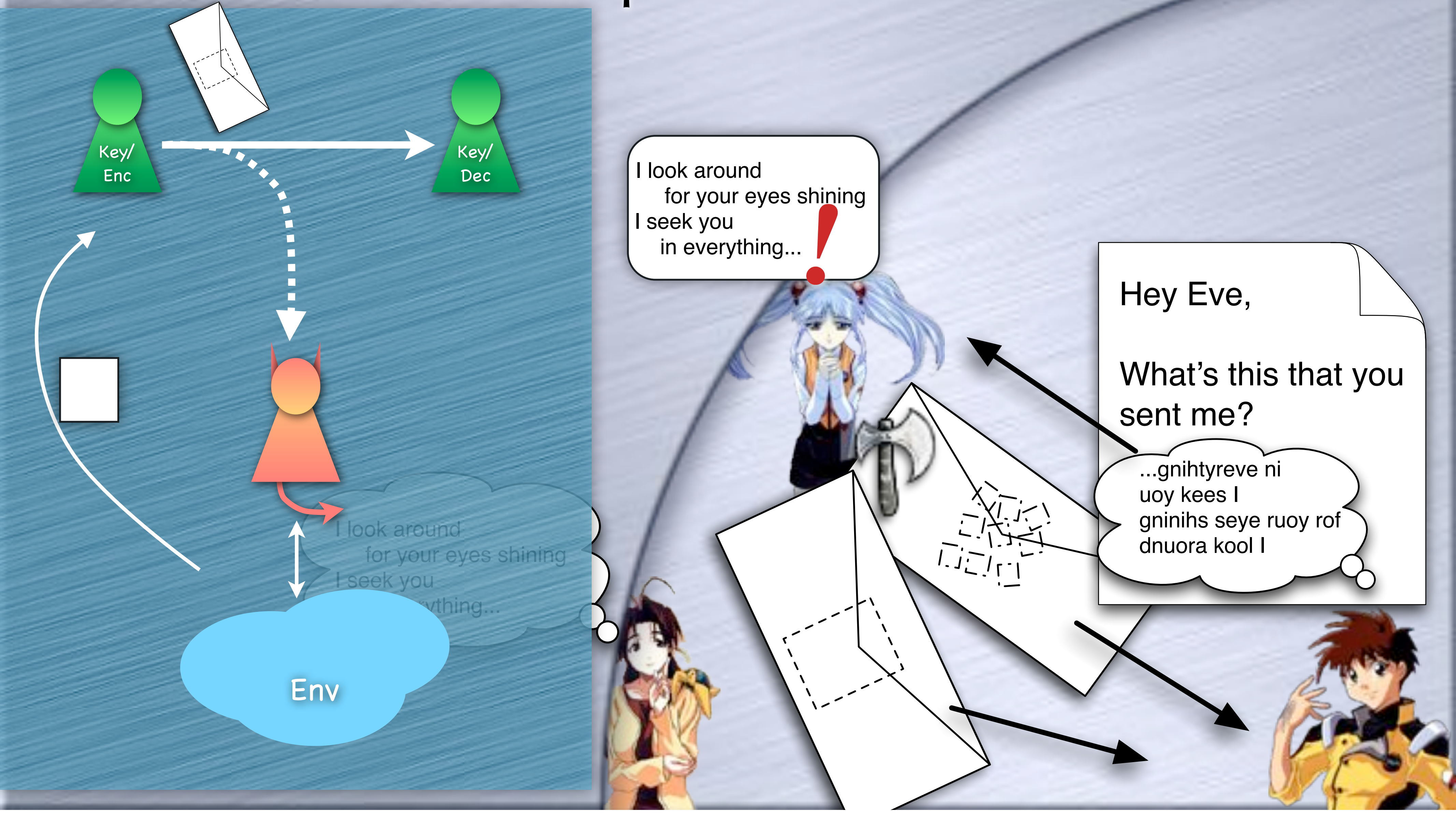
Key/ Enc
Key/ Dec
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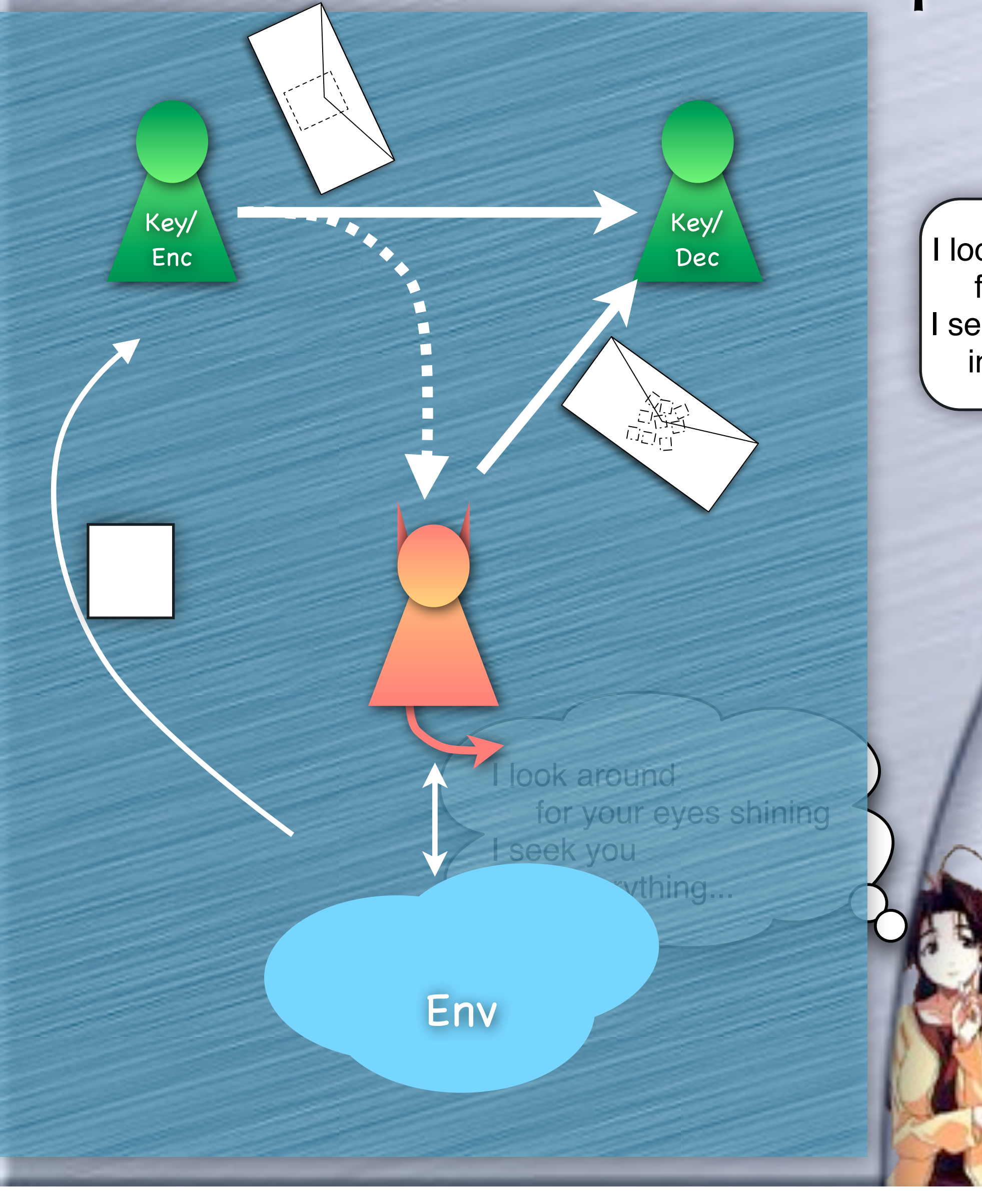

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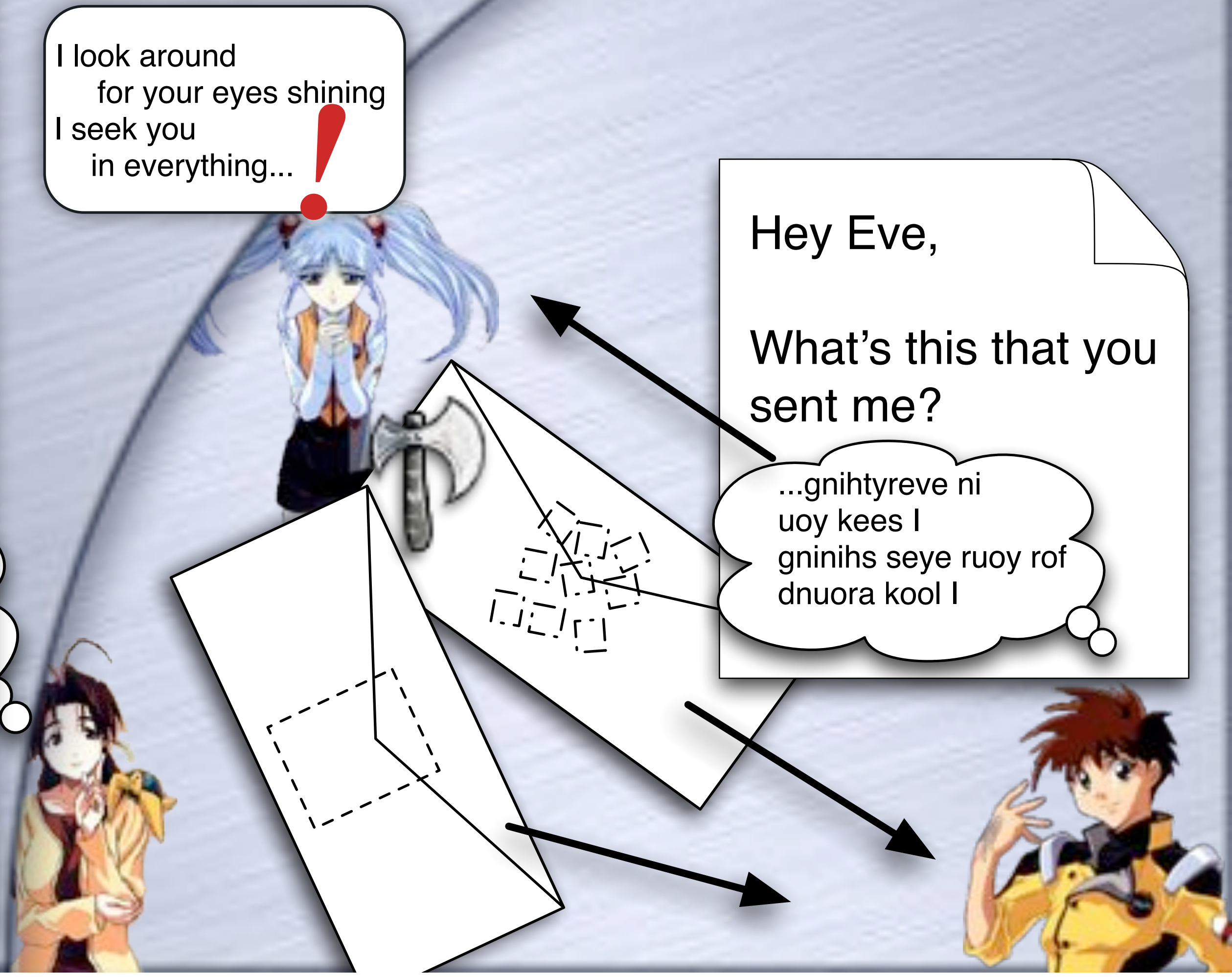

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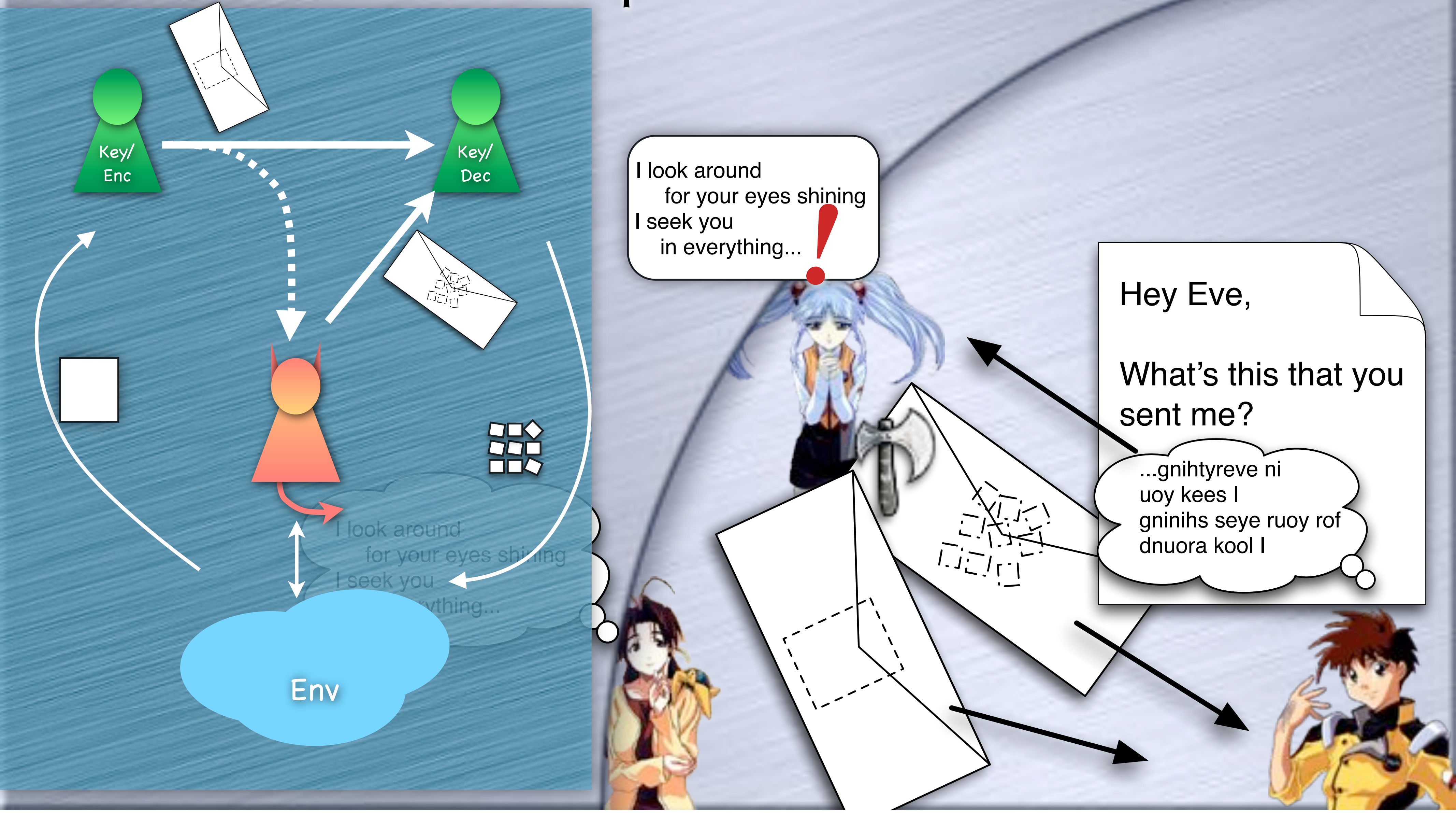
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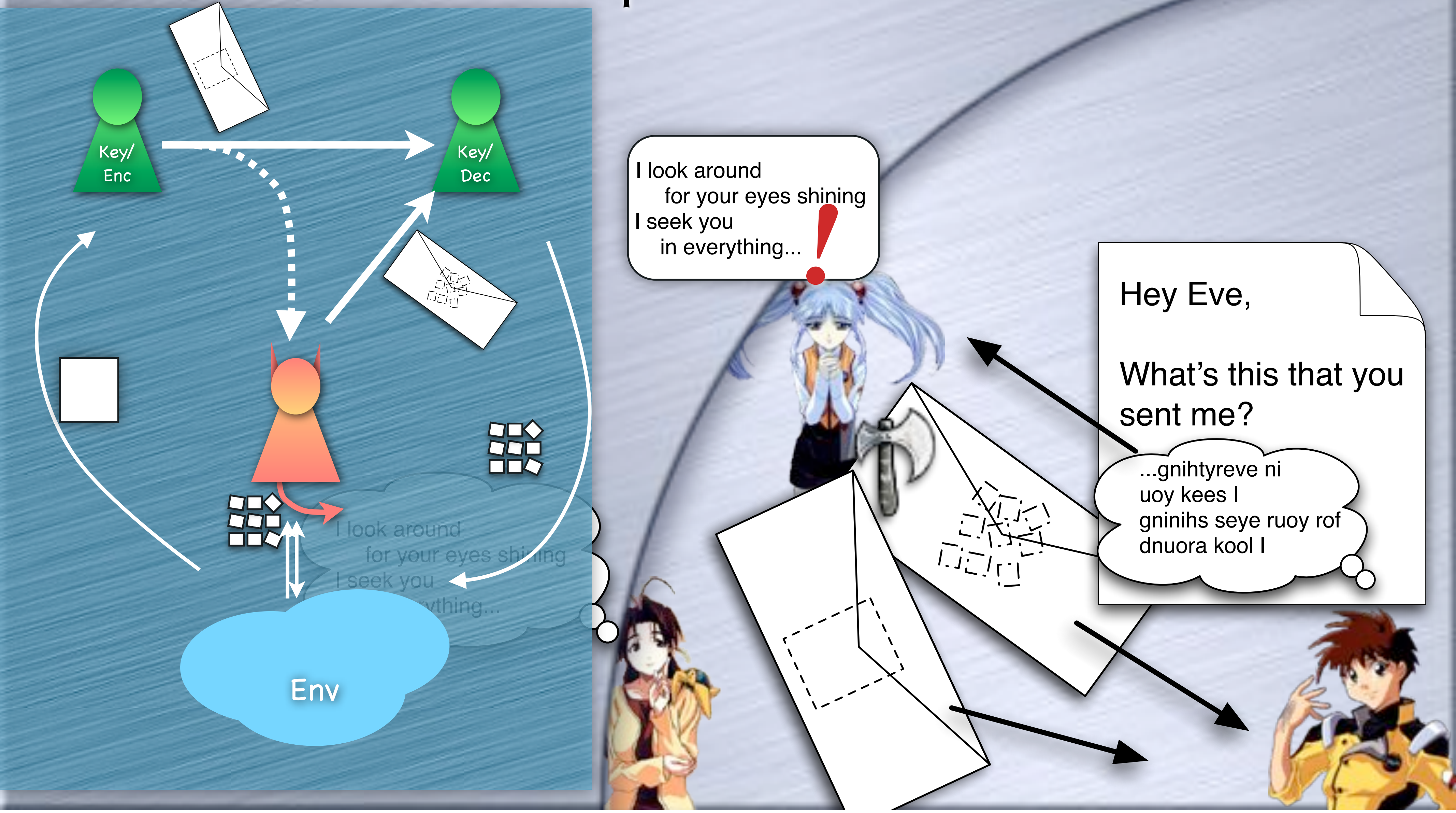
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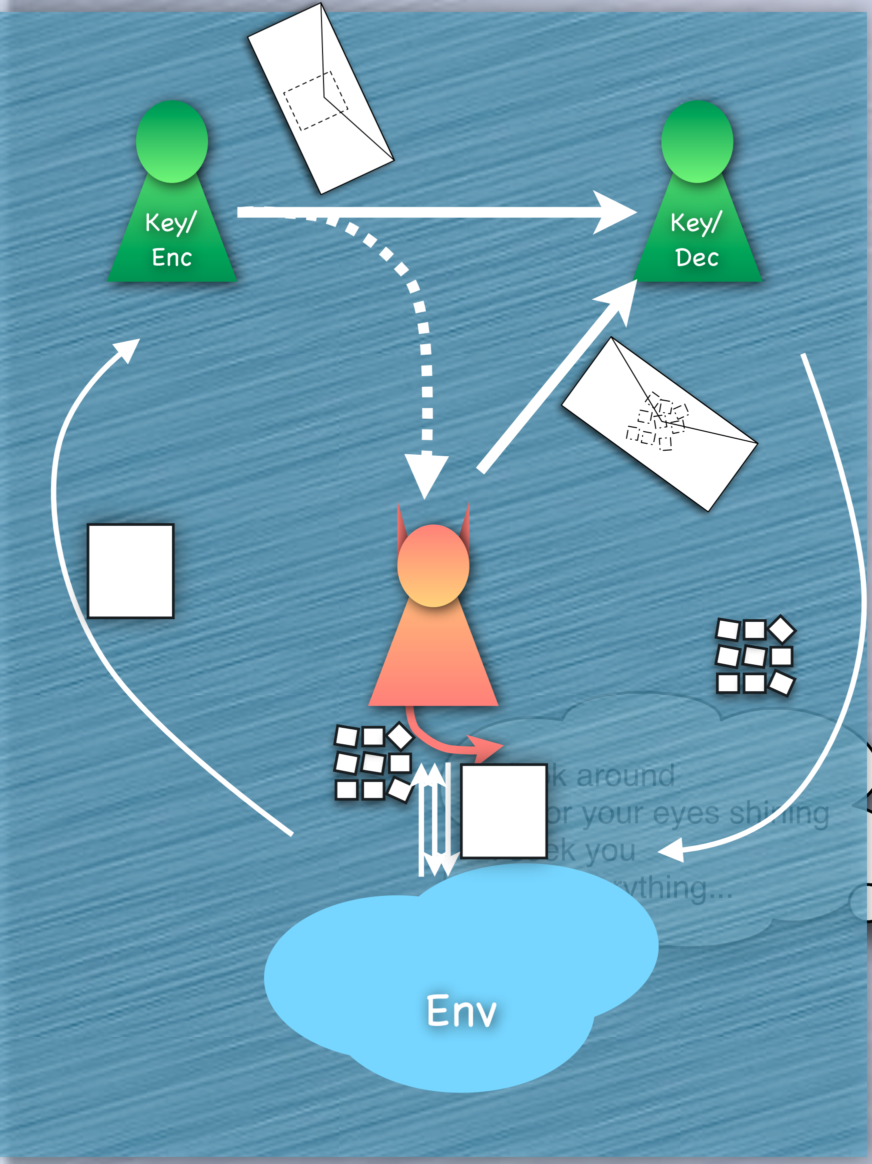

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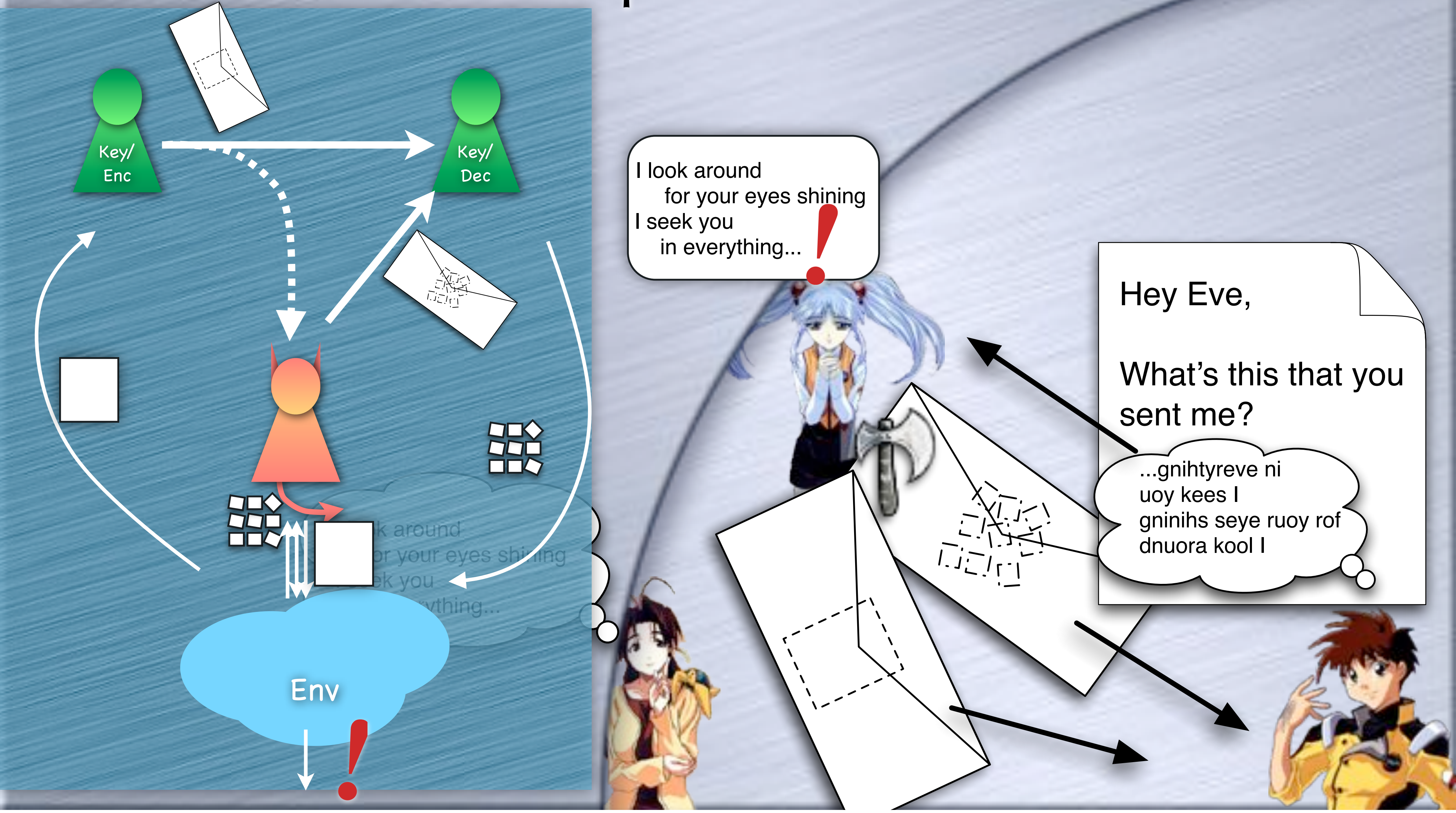
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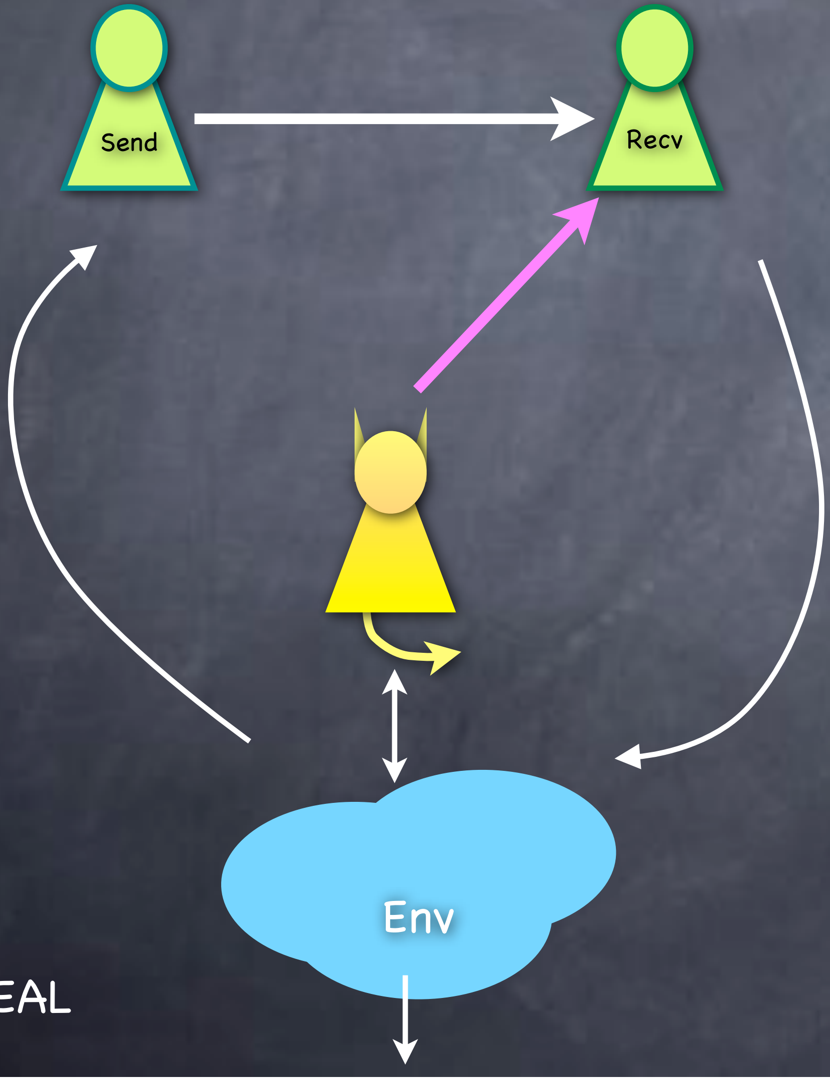


Secure (and correct) if:

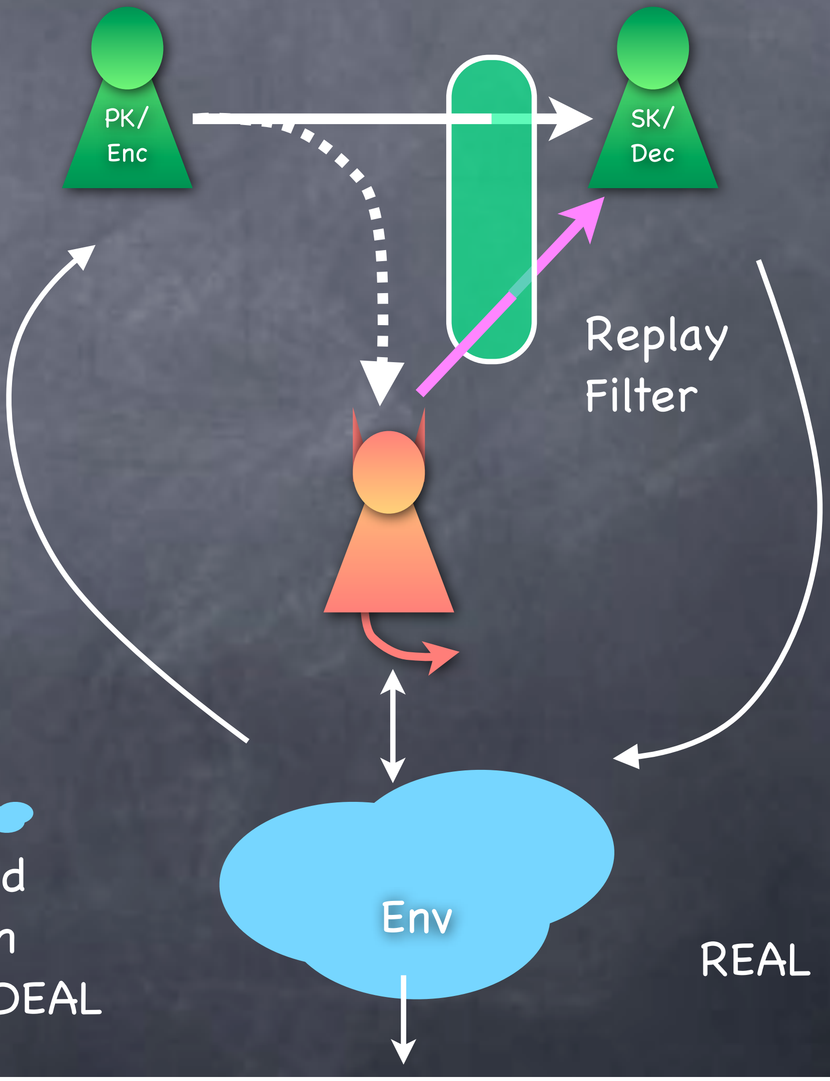
∀

∃ s.t.

∀

output of

is distributed identically in REAL and IDEAL



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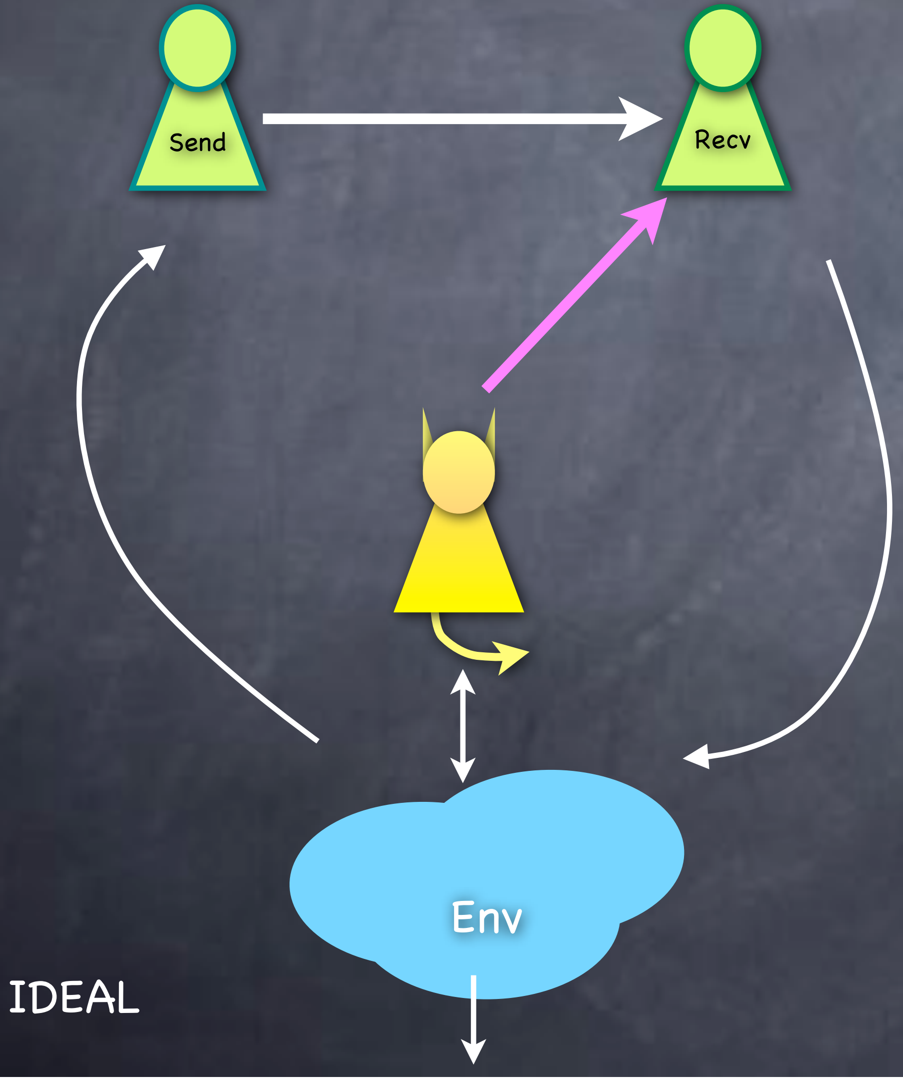

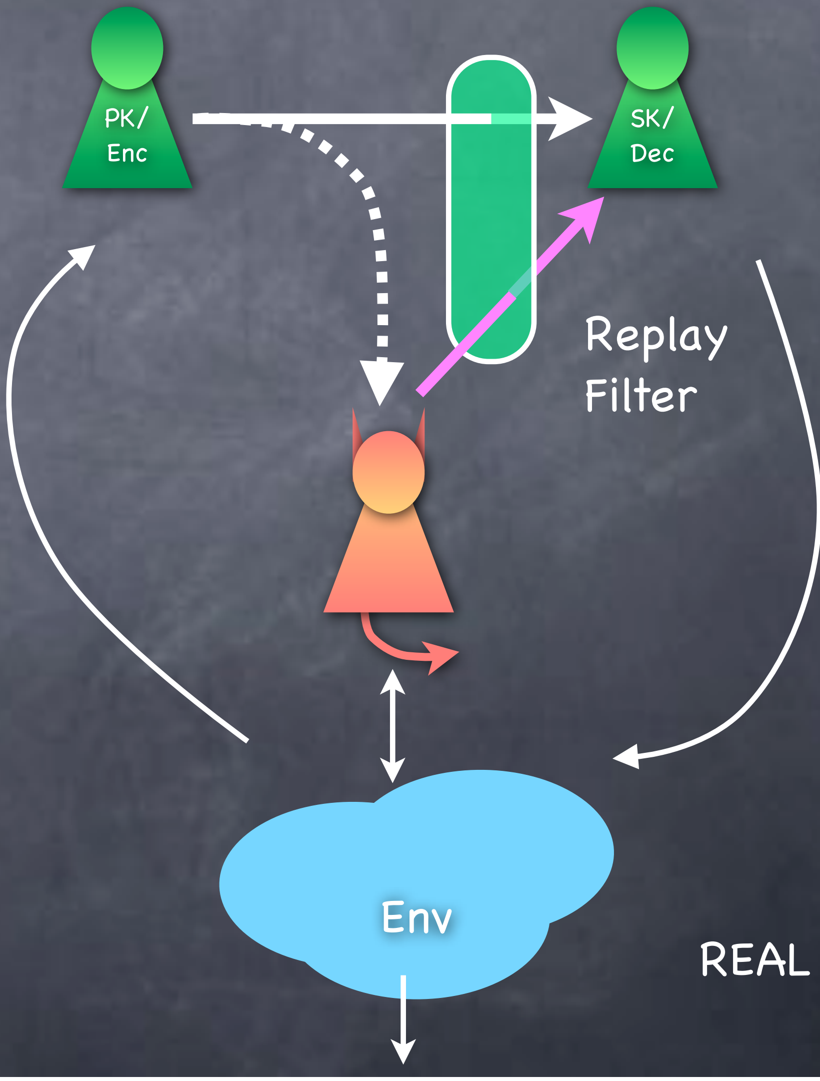

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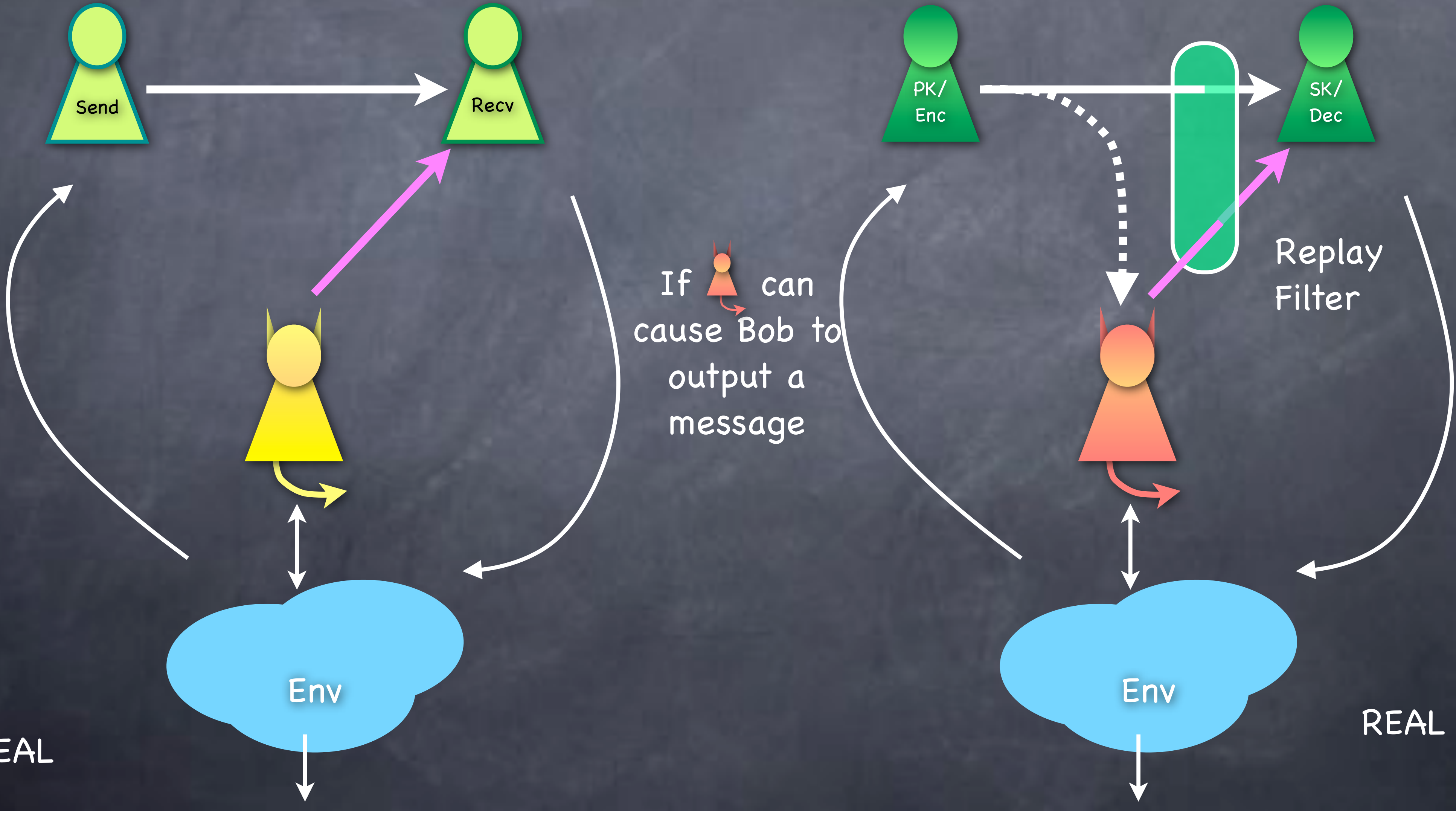

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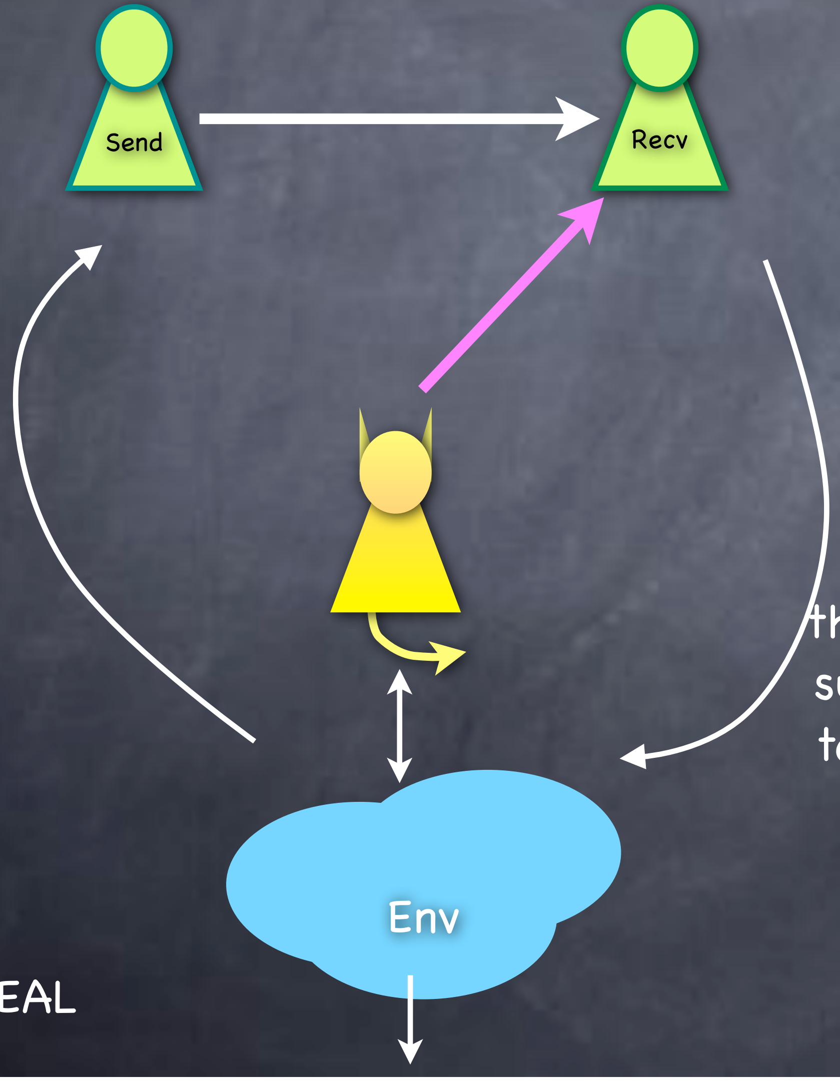

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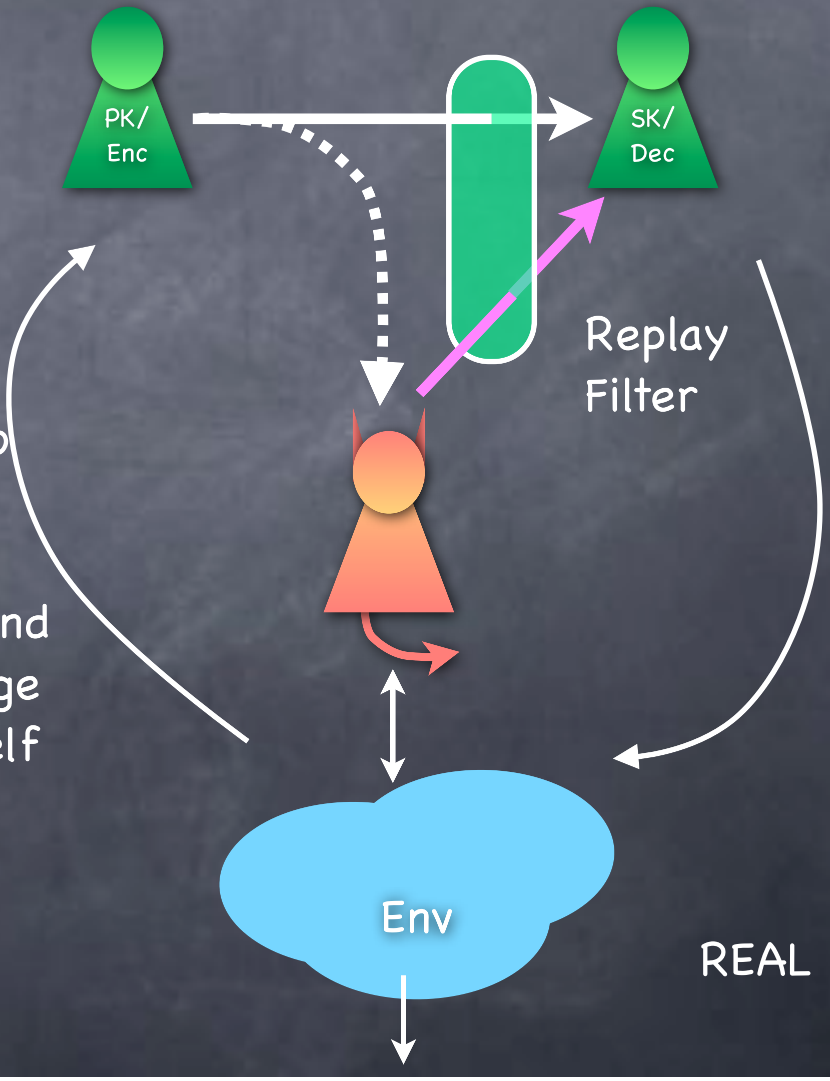

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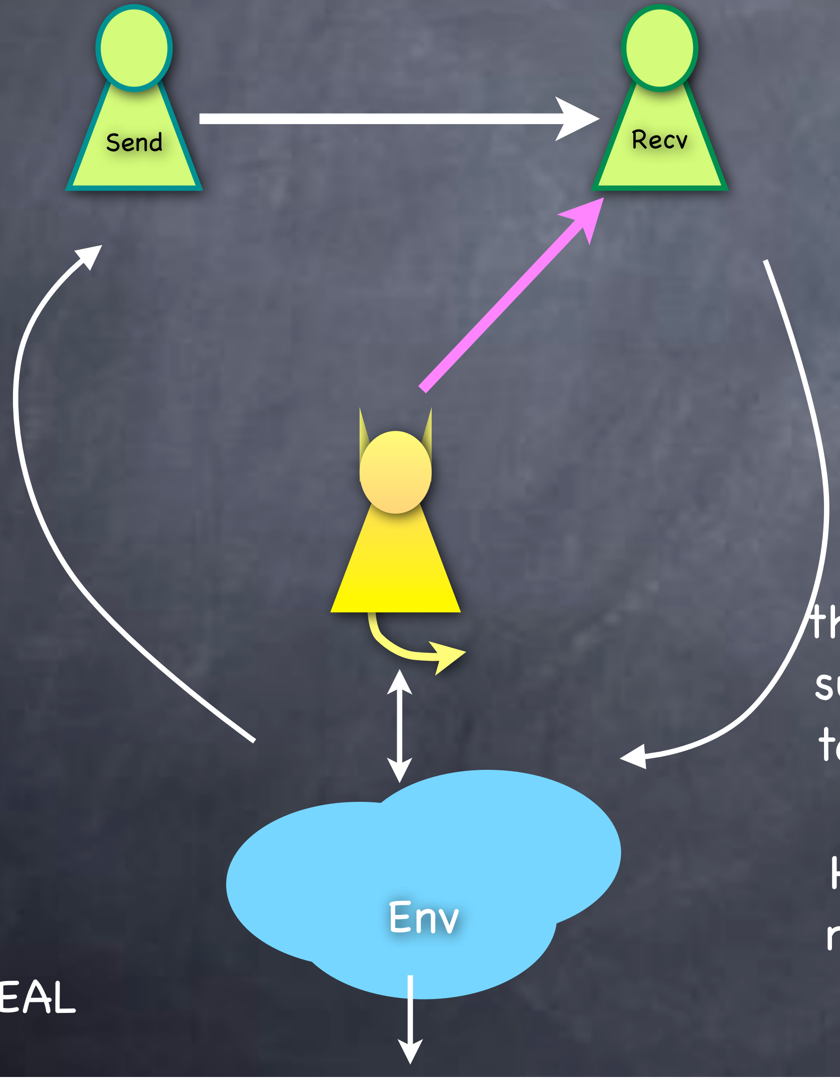

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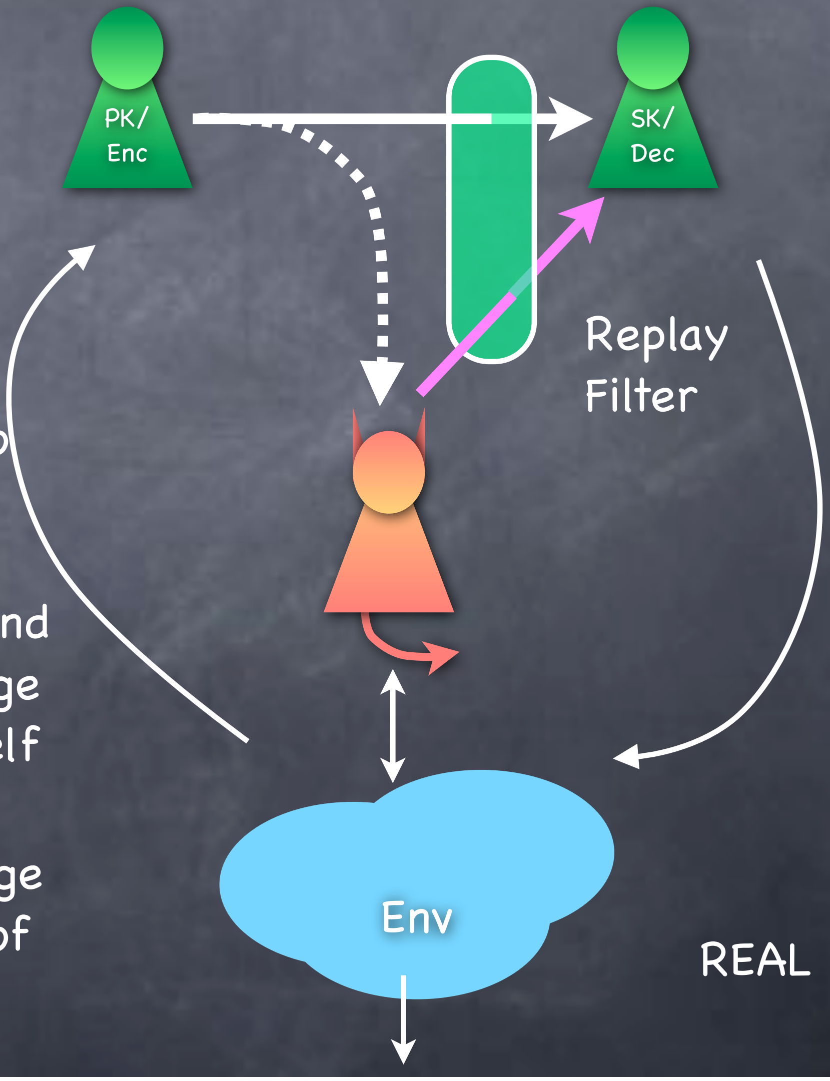

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- Significant efficiency gain using "Hybrid Encryption"

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    - $C = (g_1^x, g_2^x, MY^x)$ and $S = (WZ^{H(C)})^x$
    - g_1, g_2, Y, W, Z part of PK
      - $Y = g_1^{y1}\, g_2^{y2}$, $W = g_1^{w1}\, g_2^{w2}$, $Z = g_1^{z1}\, g_2^{z2}$. SK contains $(y_1,y_2,w_1,w_2,z_1,z_2)$

Multiple SKs can explain the same PK (unlike El Gamal)

  - Trapdoor: Using SK, and (g_1^x,g_2^x) can find Y^x, W^x, Z^x
    - If (g_1^{x1},g_2^{x2}), $x_1 \neq x_2$, then "Y^x, W^x, Z^x" vary with different SKs
  - Decryption: Check S (assuming $x_1=x_2$) and extract M

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- Formally using "hybrid argument" (0 advantage in last hybrid)

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  - Part of RSA Cryptography Standard (PKCS#1 Ver 2.1). Commonly used in SSL/TLS implementations

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  - Secure against attacks that treat H as a blackbox (and for which H is pseudorandom)

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    - Relatively low overhead on top of the (fast) SKE encryption

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  - Less security sufficient: KEM used to transfer a random key; DEM uses a new key every time.

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- CCA security based on a strong (non-standard) assumption involving Hash and the group: "Oracle Diffie-Hellman Assumption"

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  - Very weak security sufficient for encryptions used in KEM and DEM (but only with H, G modeled as random oracles)

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