Symmetric-Key Encryption: constructions

Lecture 4

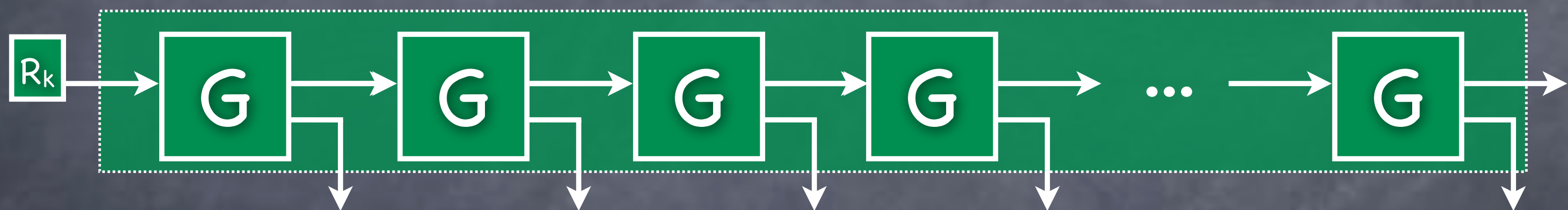
PRF, Block Cipher

RECALL

PRG from One-Way Permutations

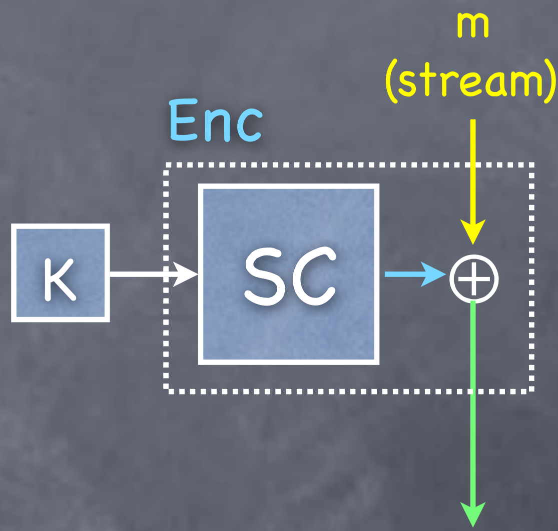
- One-bit stretch PRG, G_k: $\{0,1\}^k \rightarrow \{0,1\}^{k+1}$

- Increasing the stretch
  - Can use part of the PRG output as a new seed



  - If the intermediate seeds are never output, can keep stretching on demand (for any "polynomial length")
  - A stream cipher

One-time CPA-secure SKE with a Stream-Cipher



- One-time Encryption with a stream-cipher:
  - Generate a one-time pad from a short seed
  - Can share just the seed as the key
  - Mask message with the pseudorandom pad
- Security: indistinguishability from using a real random pad
- If SC can spit out bits on demand, the message can arrive bit by bit, and the length of the message doesn't have to be a priori fixed

Beyond One-Time?

- Need to make sure same part of the one-time pad is never reused
  - Sender and receiver will need to maintain state
    - Or Sender can send the index, but then receiver will need to run the stream-cipher to get to that index
    - A PRG with direct access to any part of the output stream?
- Pseudo Random Function (PRF)

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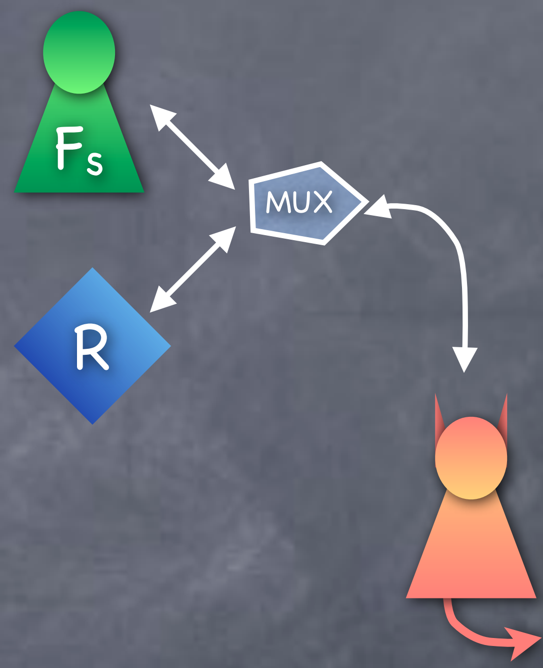

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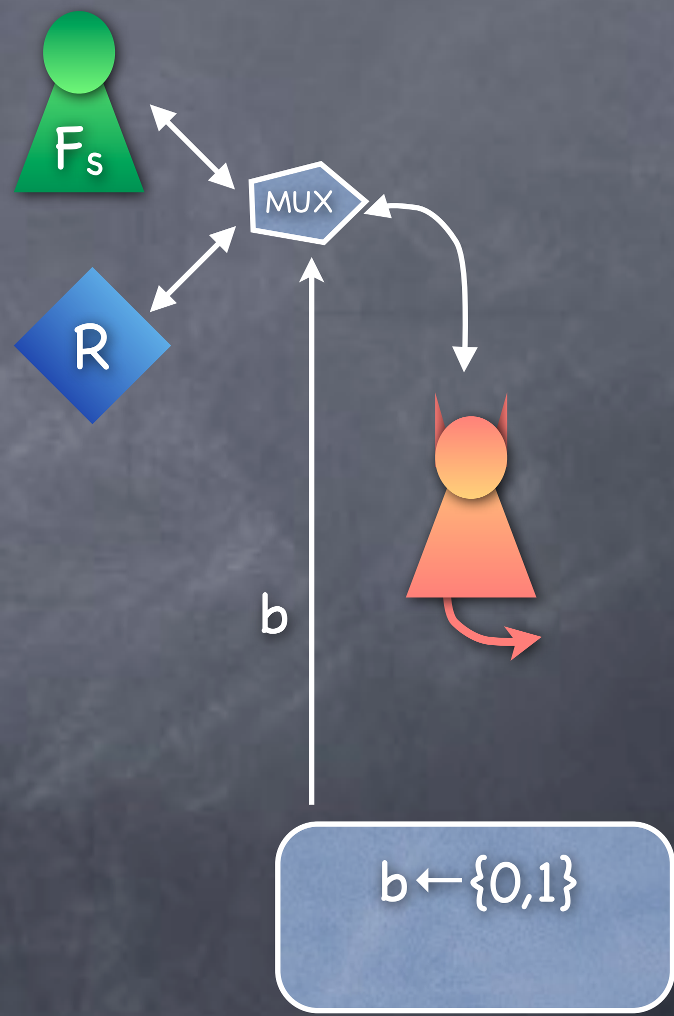

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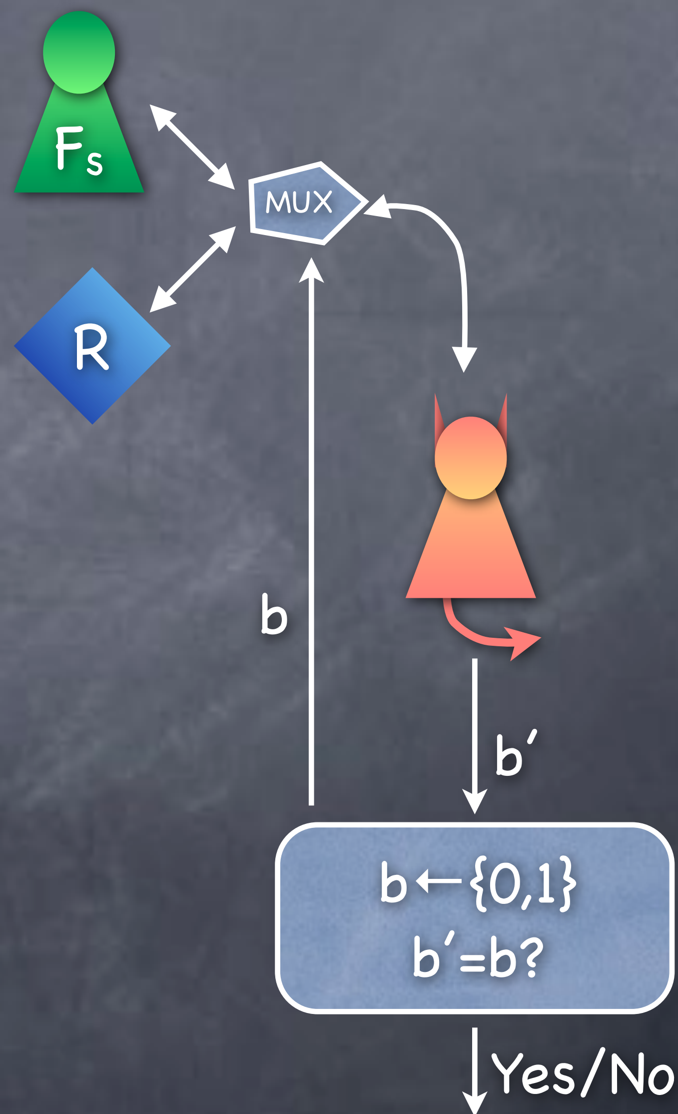

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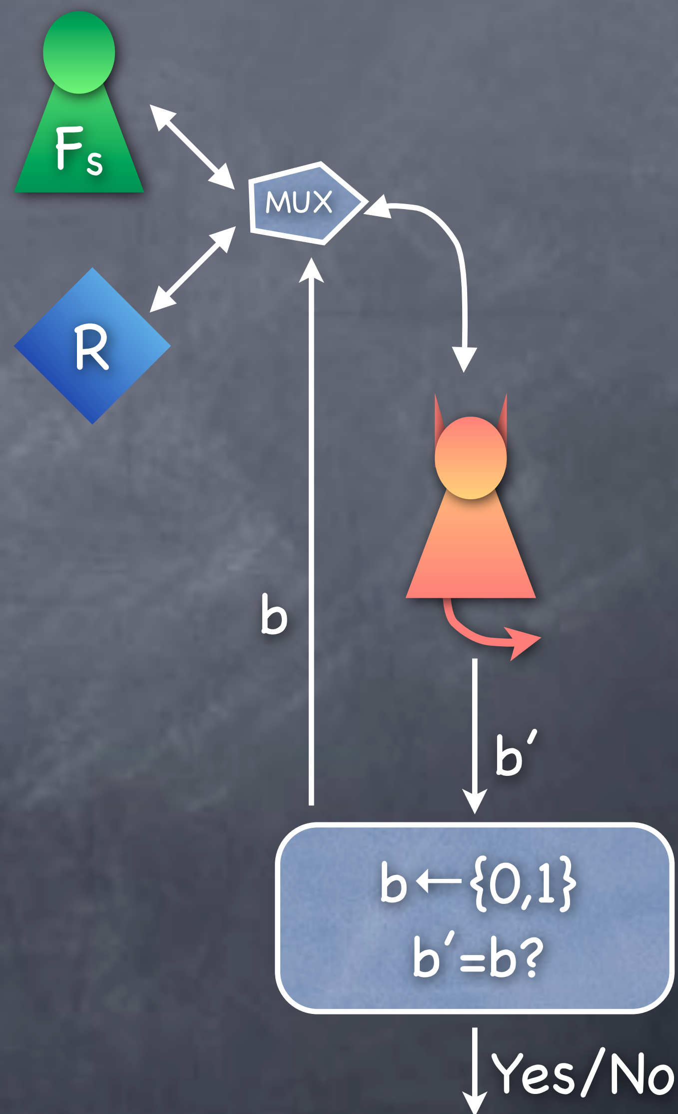

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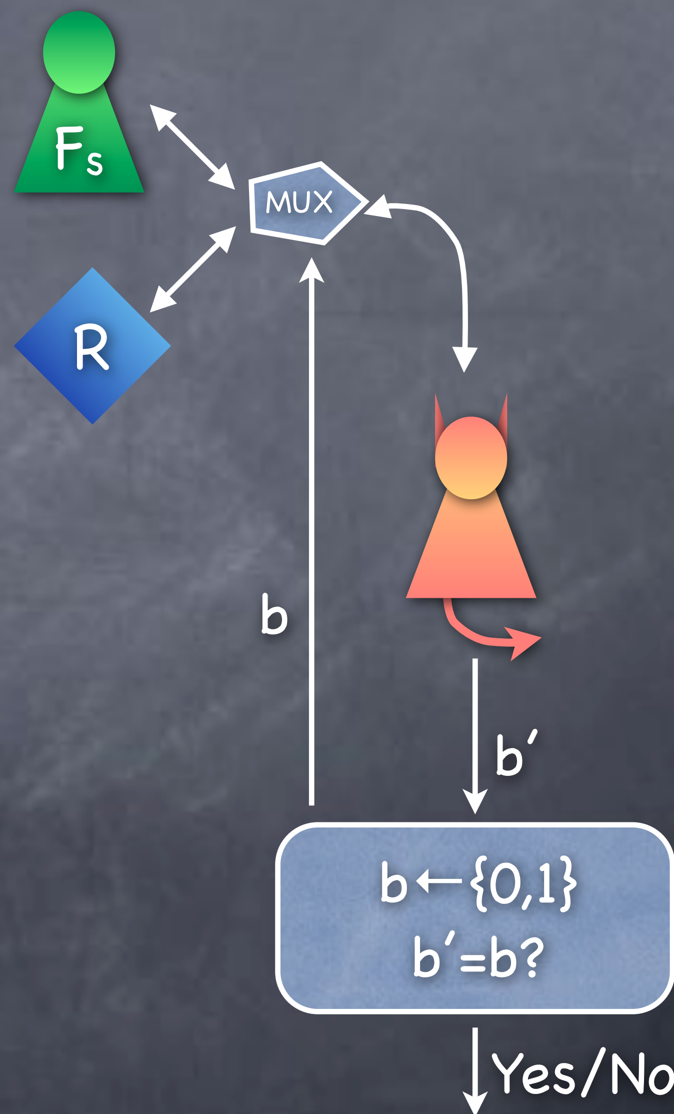

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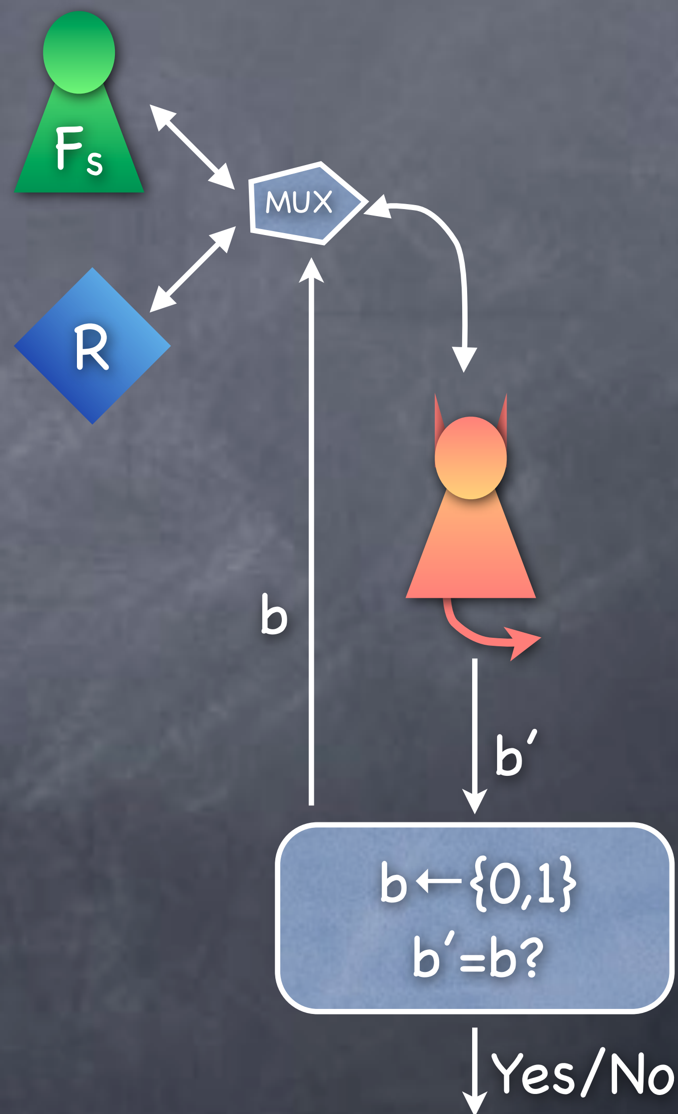

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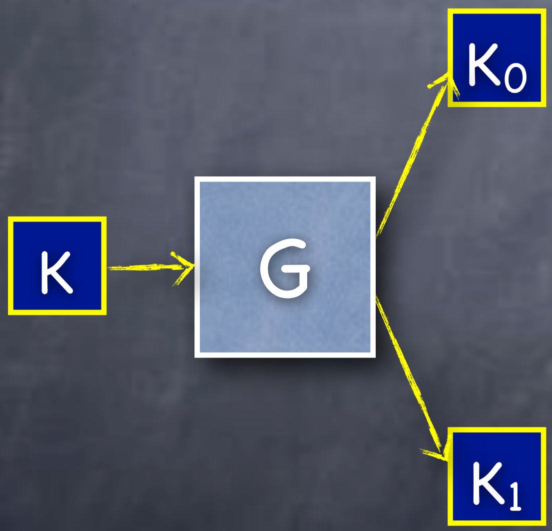

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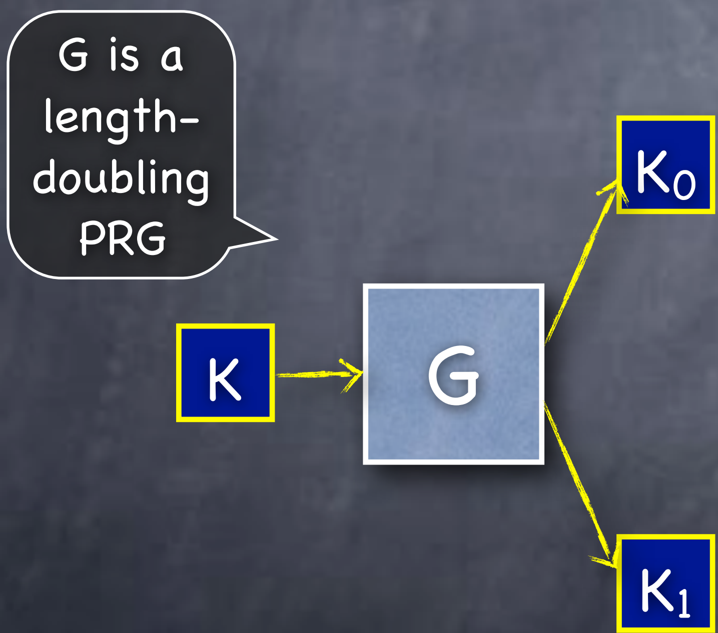

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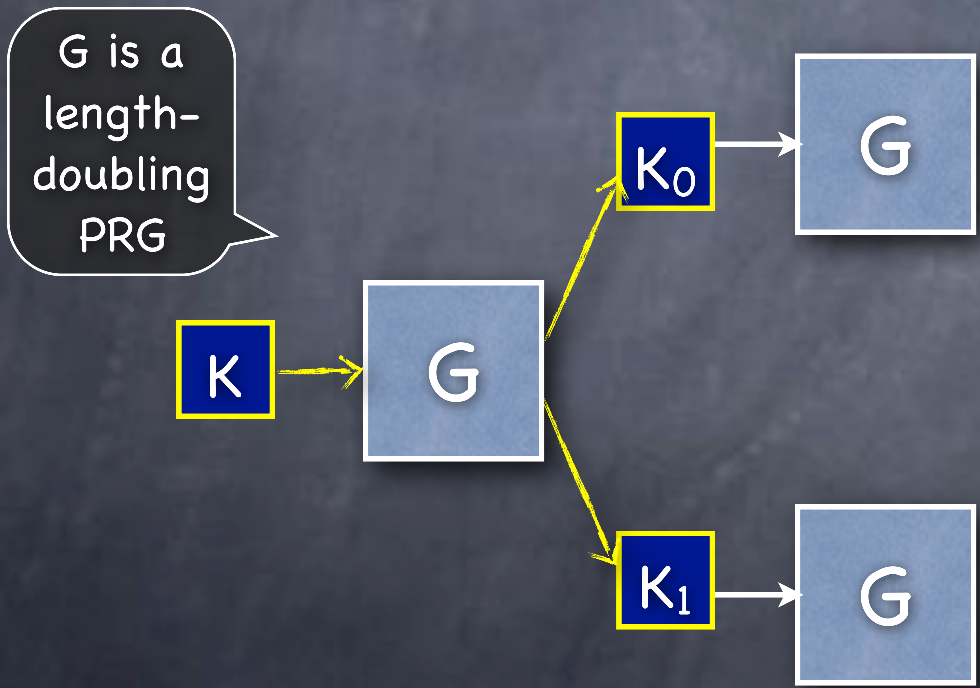

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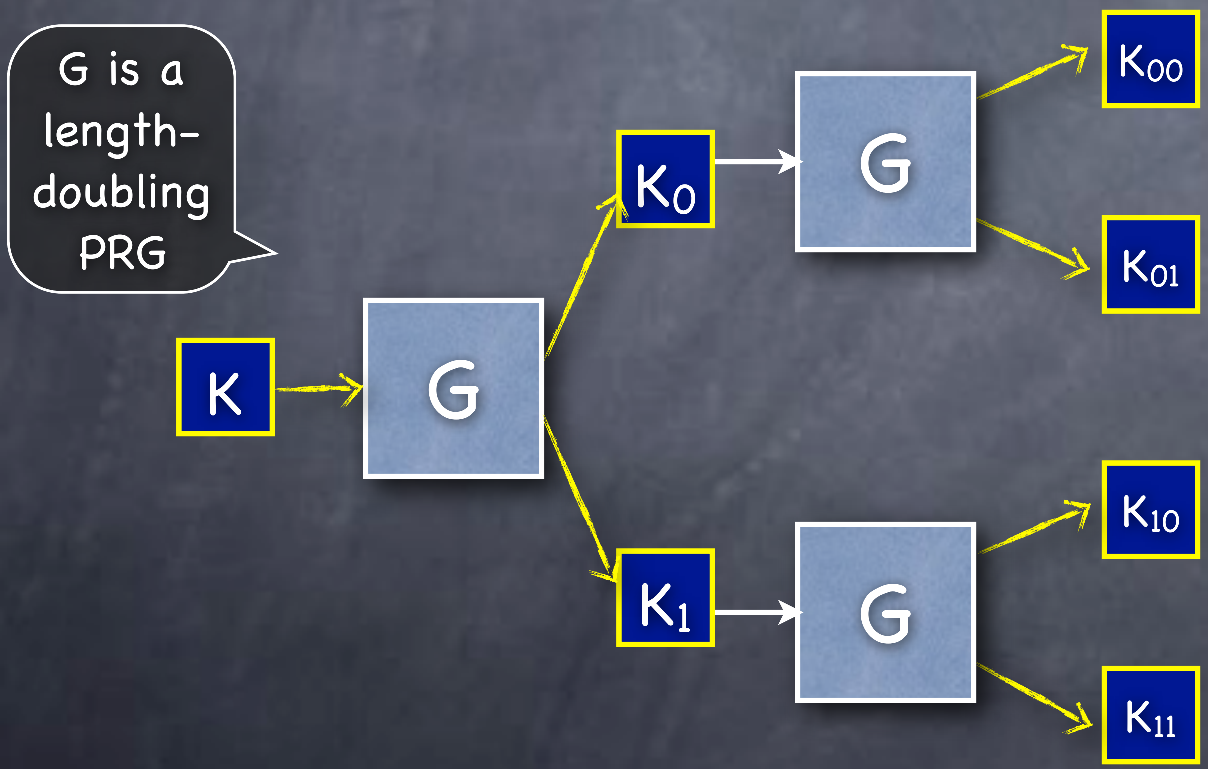

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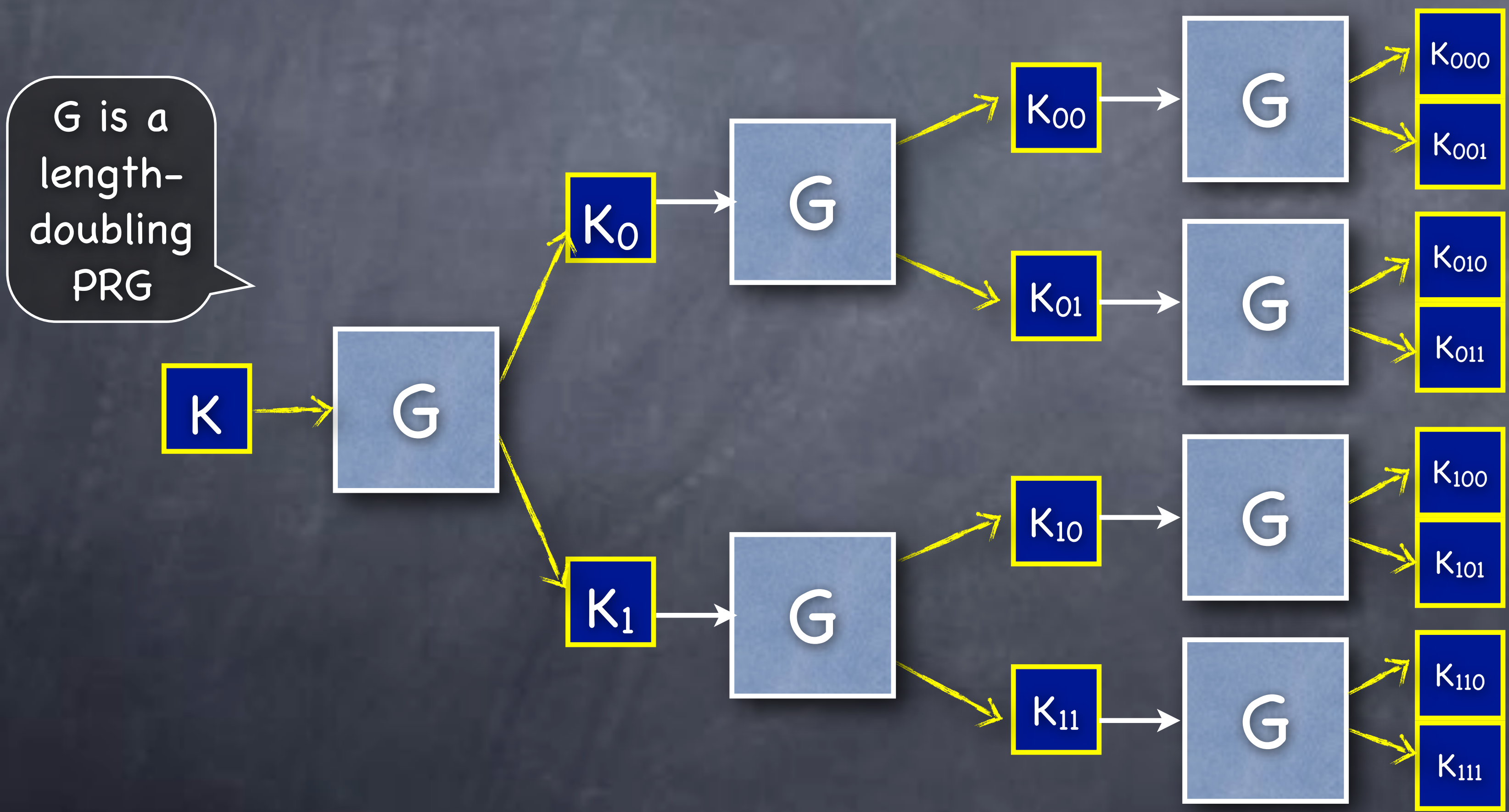

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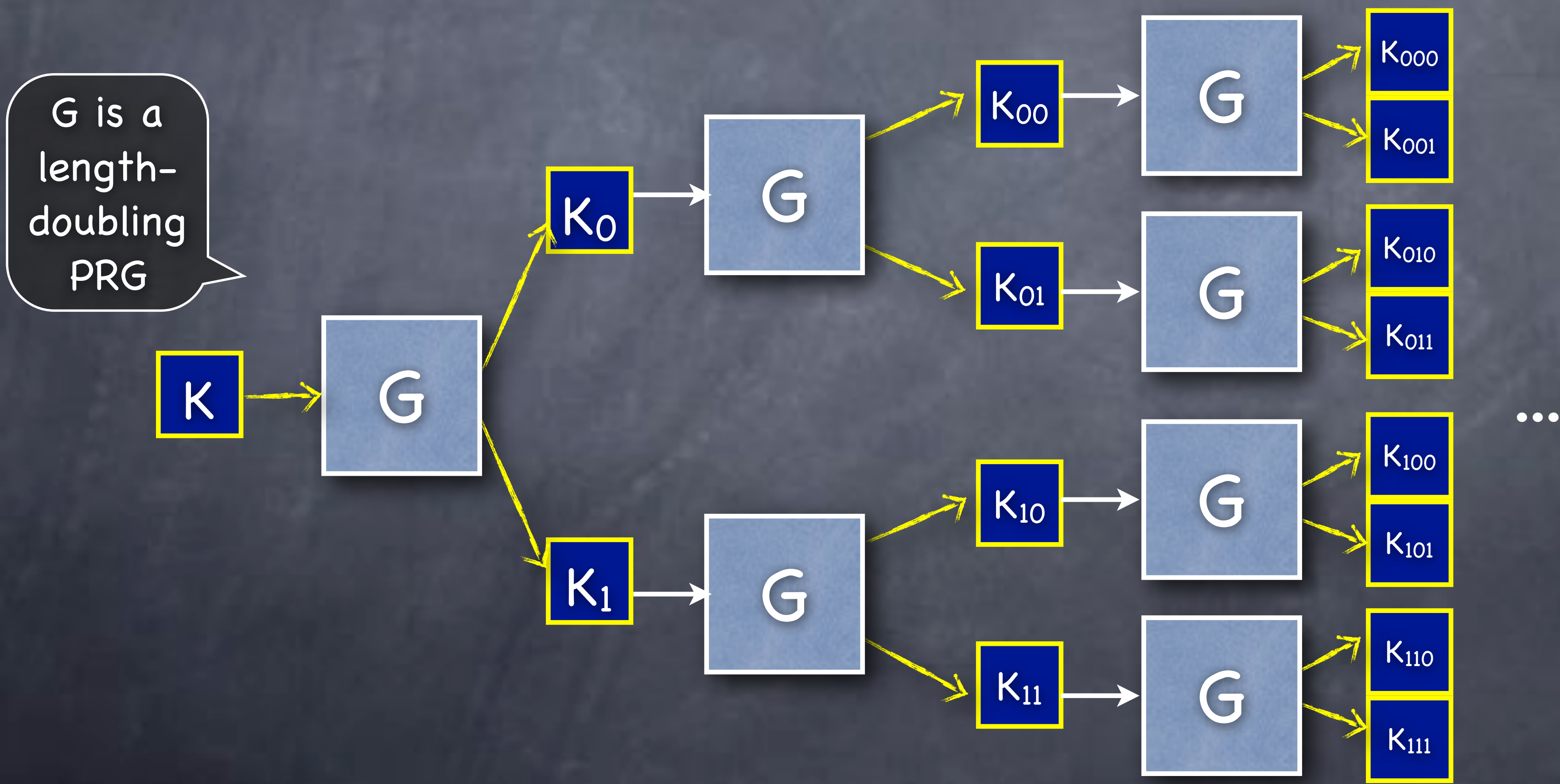

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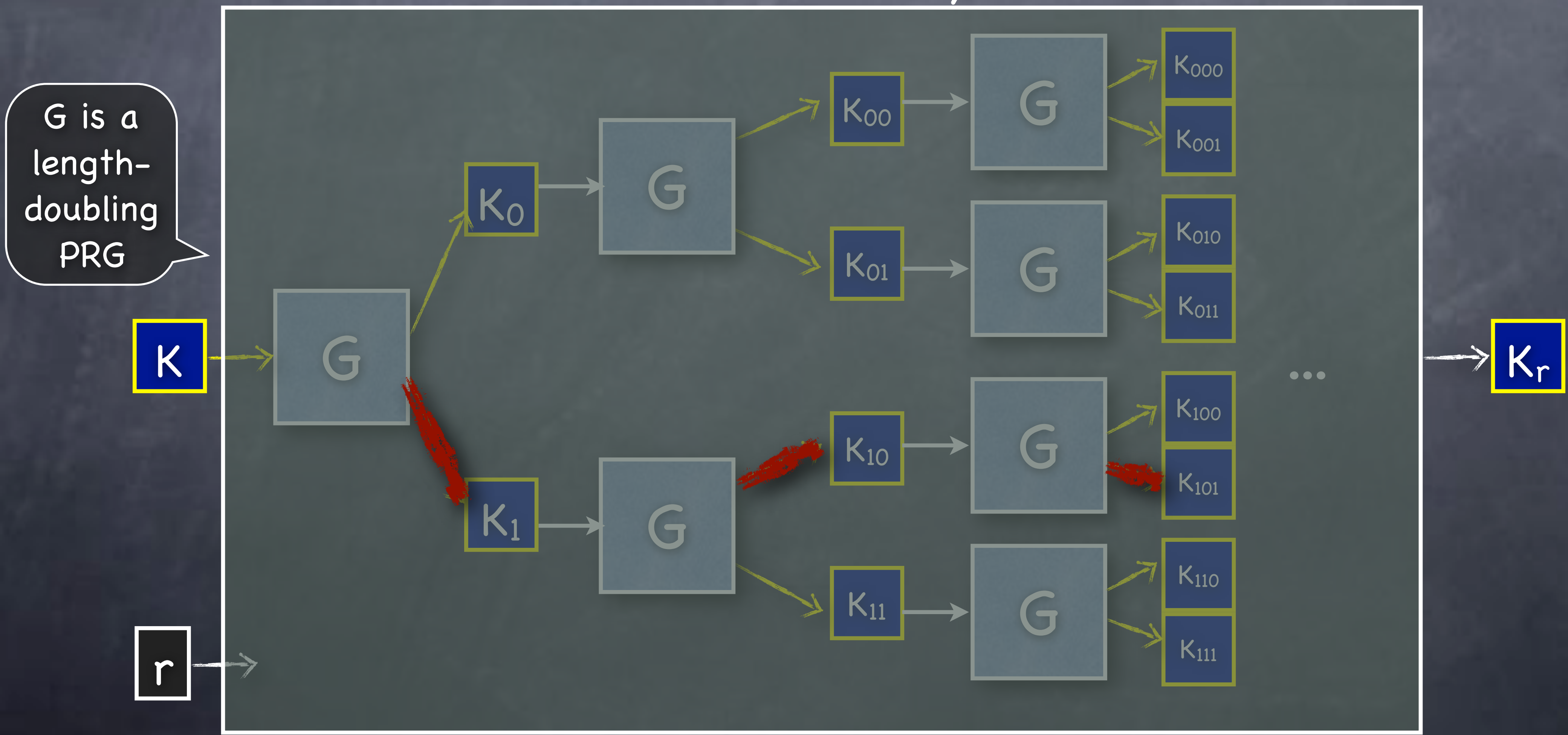

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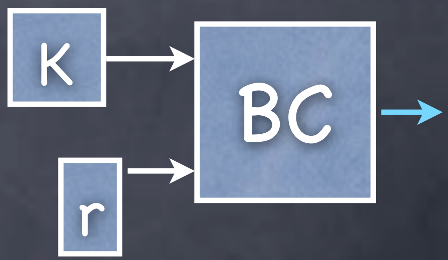

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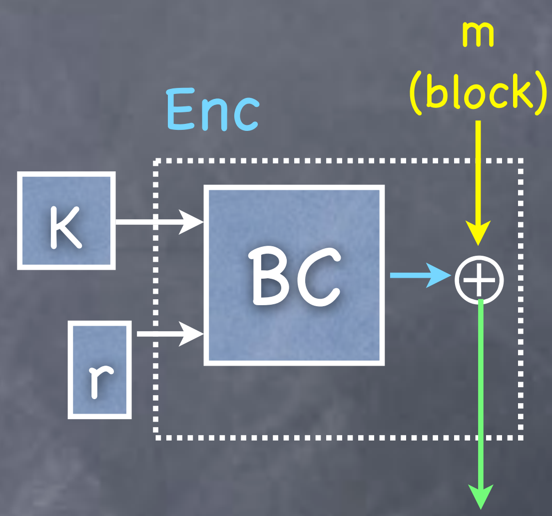

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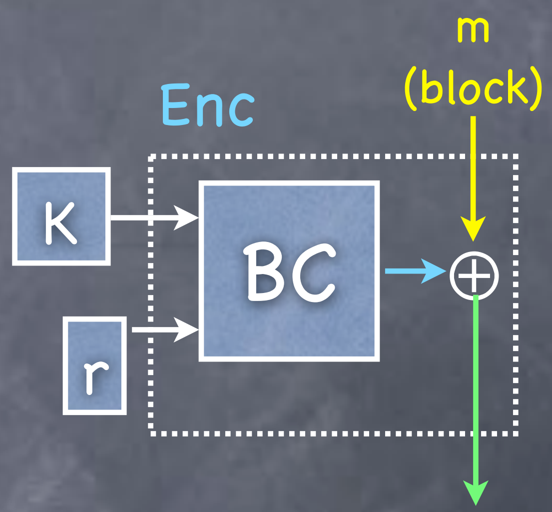

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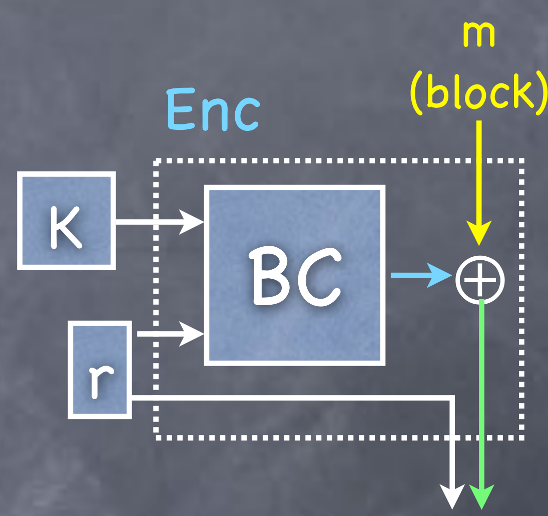

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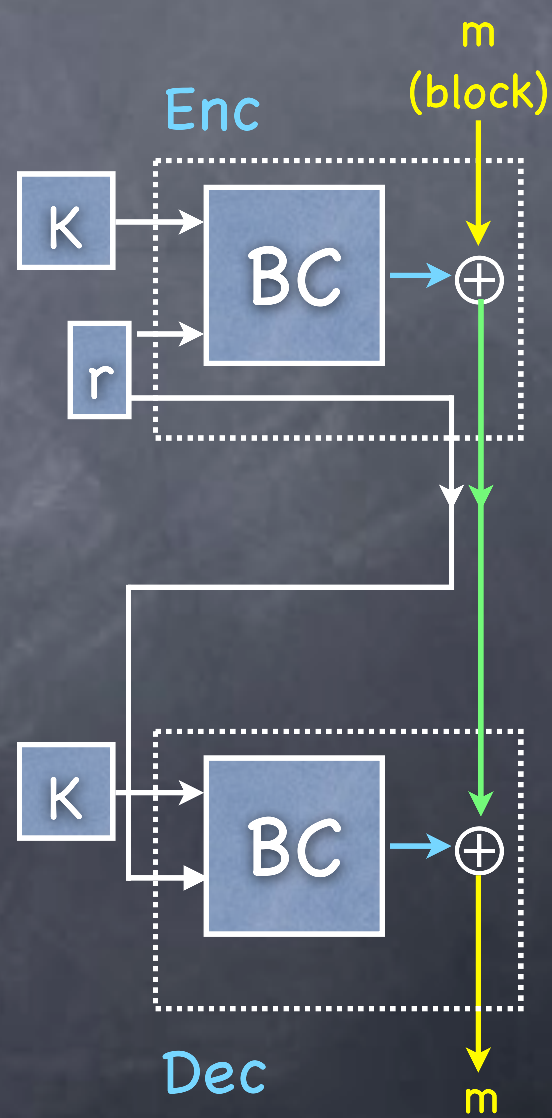

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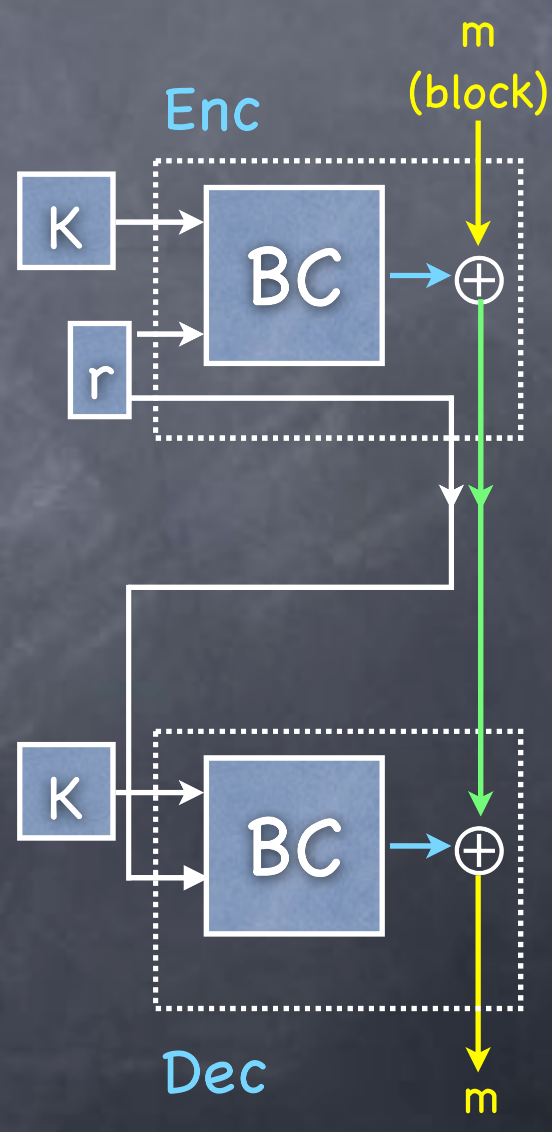

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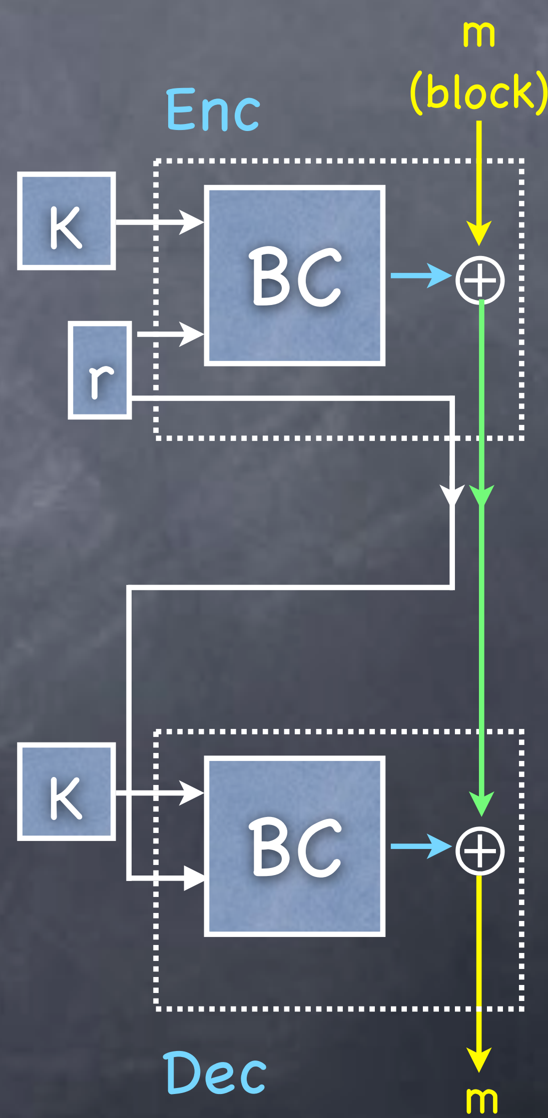

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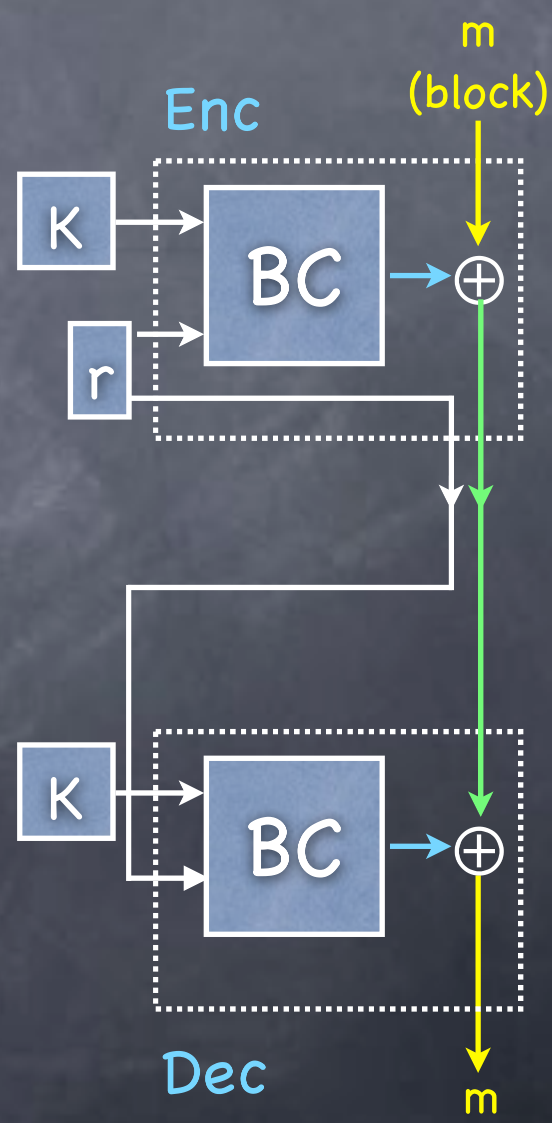

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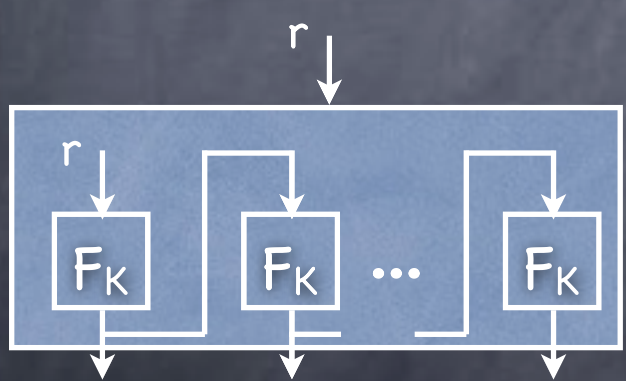

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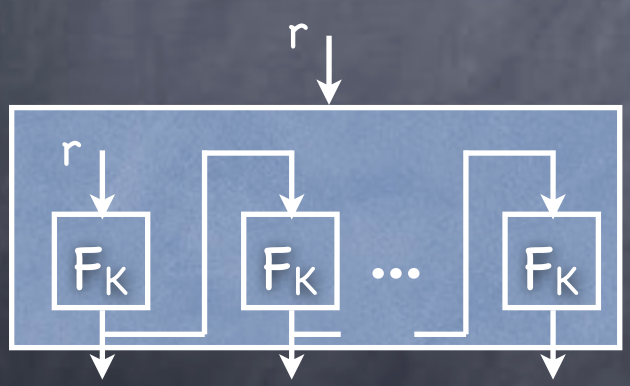

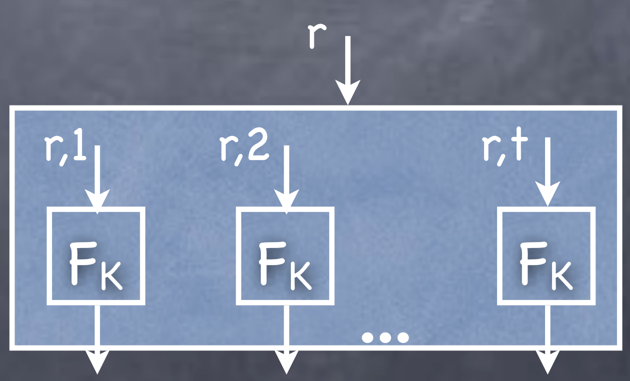

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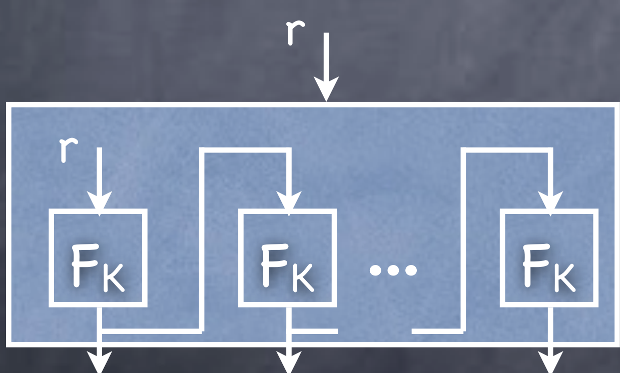

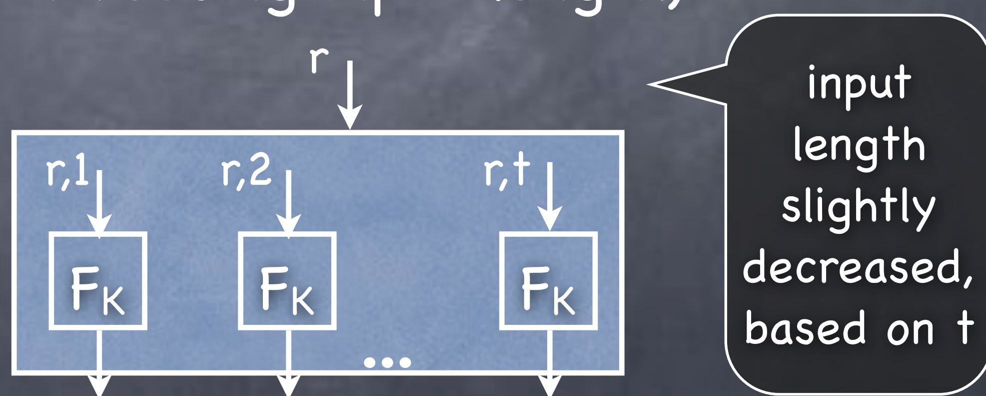

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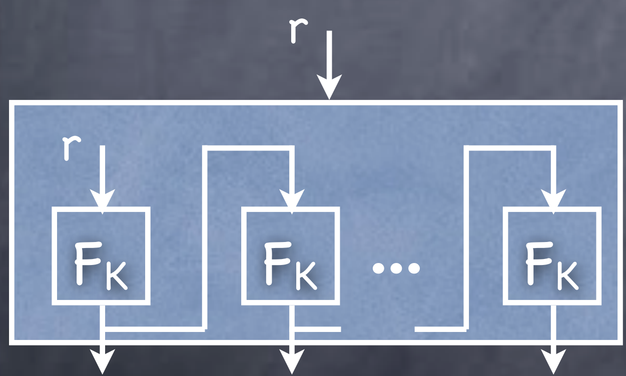

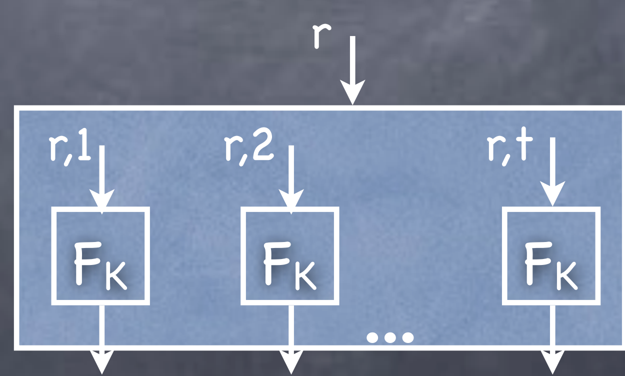

input length slightly decreased, based on t

- Output is indistinguishable from t random blocks (even if input to F_K known/chosen)

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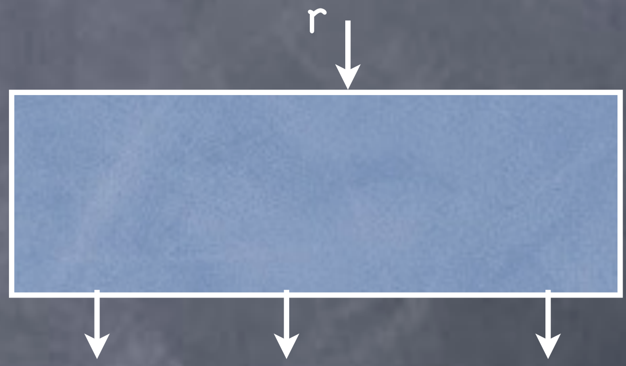

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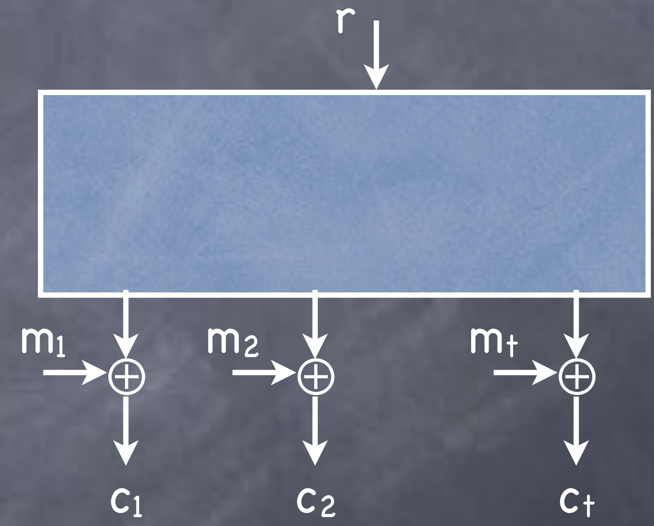

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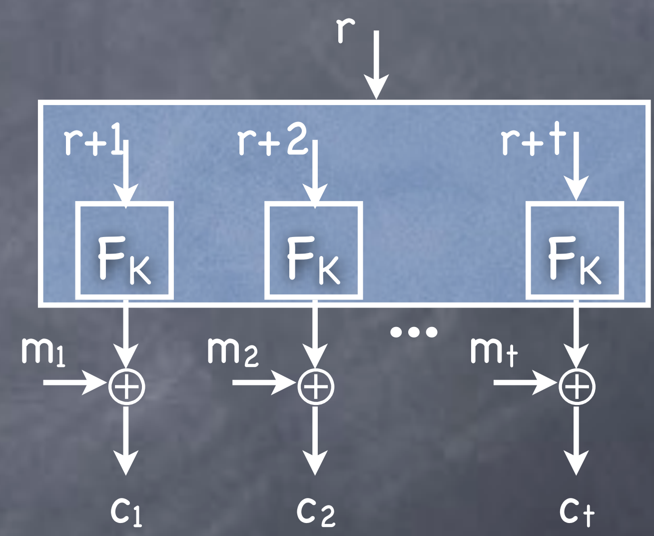

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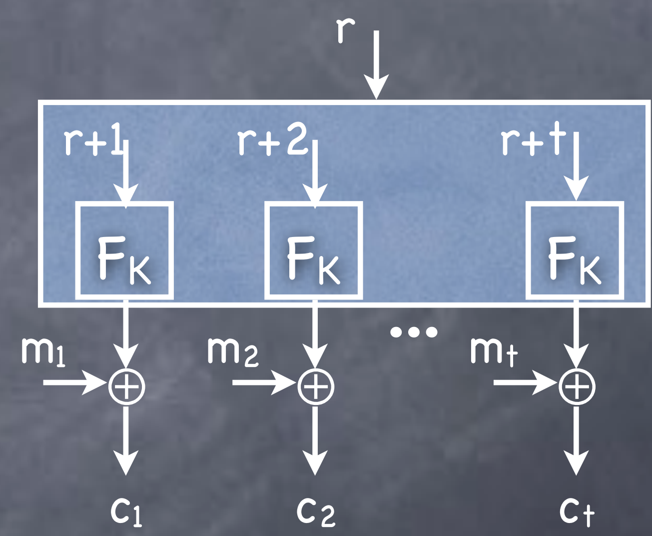

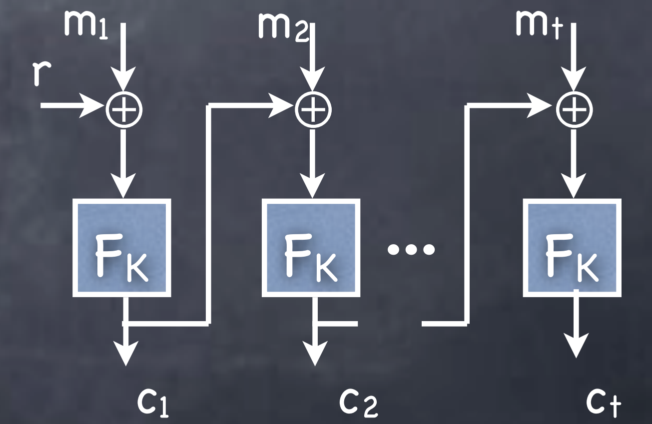

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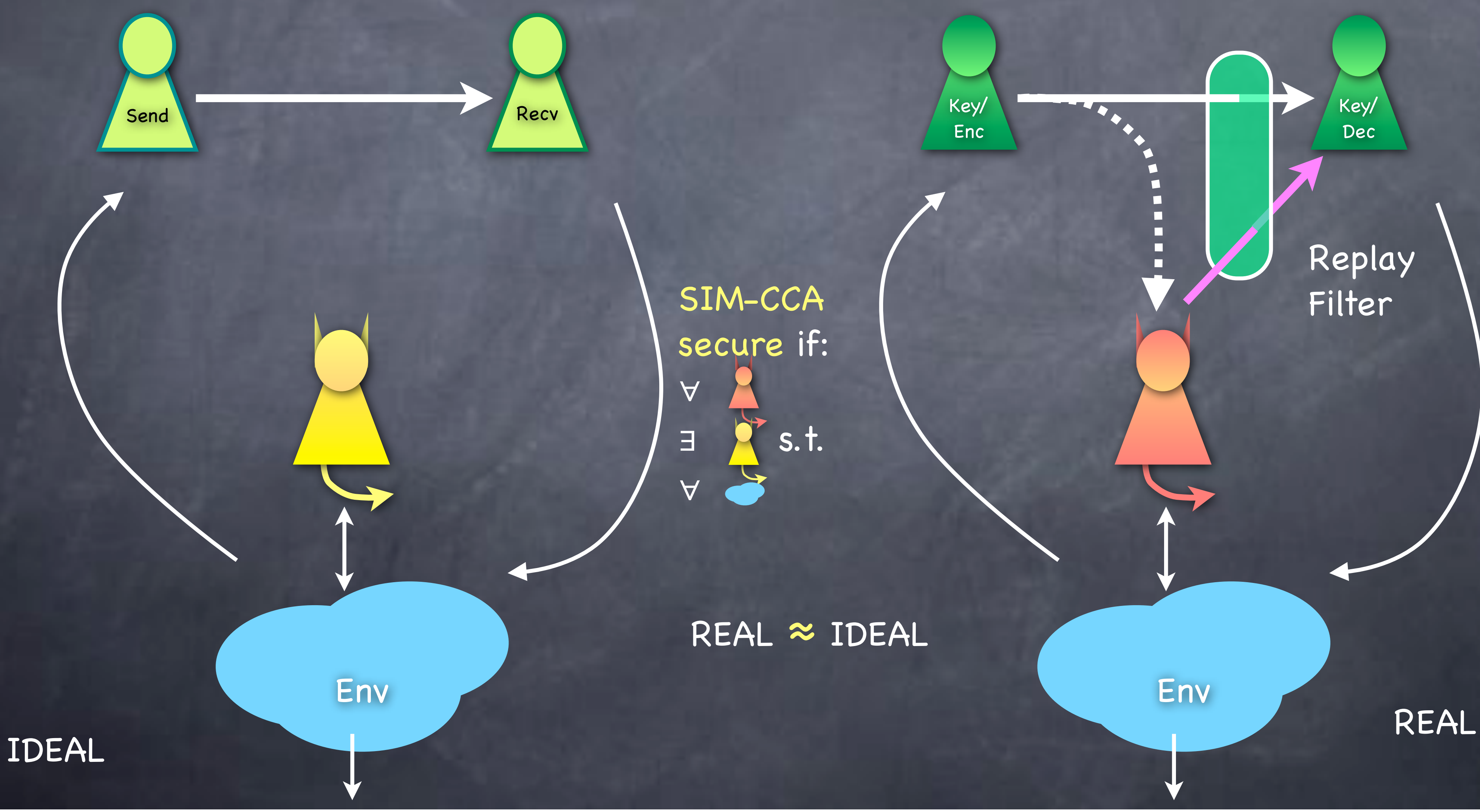
Active Adversary

- An active adversary can inject messages into the channel
  - Eve can send ciphertexts to Bob and get them decrypted
    - Chosen Ciphertext Attack (CCA)
  - If Bob decrypts all ciphertexts for Eve, no security possible
    - What can Bob do?

RECALL

Symmetric-Key Encryption

SIM-CCA Security



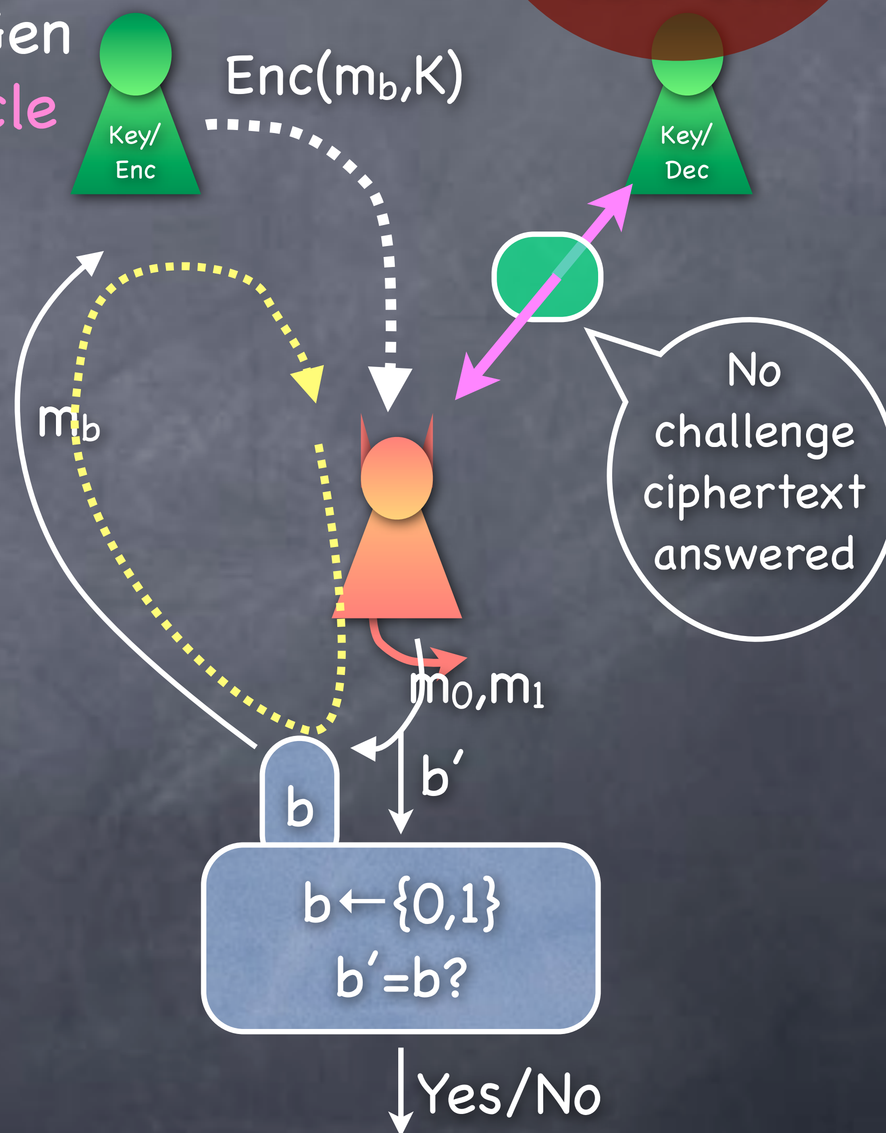
RECALL

Symmetric-Key Encryption

IND-CCA Security

IND-CCA + ~correctness equivalent to SIM-CCA

- Experiment picks b←{0,1} and K←KeyGen
  Adv gets (guarded) access to Dec_K oracle
  - For as long as Adversary wants
    - Adv sends two messages m_0, m_1 to the experiment
    - Expt returns $Enc(m_b,K)$ to the adversary
- Adversary returns a guess b′
- Experiments outputs 1 iff b′=b
- IND-CCA secure if for all feasible adversaries $Pr[b'=b] \approx 1/2$



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  - MAC: Message Authentication Code

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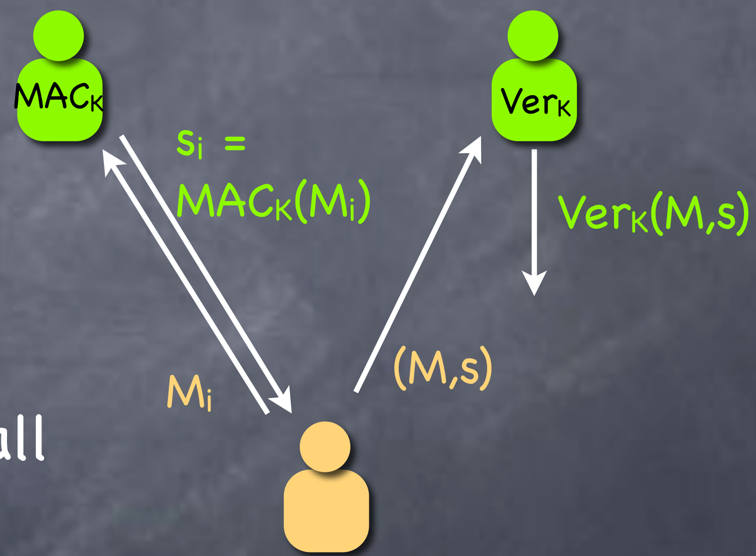
- A triple (KeyGen, MAC, Verify)
- Correctness: For all K from KeyGen, and all messages M, $\text{Verify}_K(M,\text{MAC}_K(M))=1$
- Security: probability that an adversary can produce (M,s) s.t. $\text{Verify}_K(M,s)=1$ is negligible unless Alice produced an output $s=\text{MAC}_K(M)$



Advantage
$= \Pr[\ \text{Ver}_K(M,s)=1 \text{ and } (M,s) \notin \{(M_i,s_i)\}\ ]$

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- SKE in practice uses Block-Cipher standards (next time)
- In principle, constructions (less efficient) based on any One-Way Permutation or even One-Way Function (hence more secure)