Defining Encryption

Lecture 2

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Secrecy when Computationally Bounded



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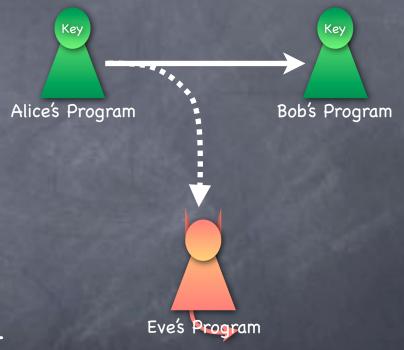




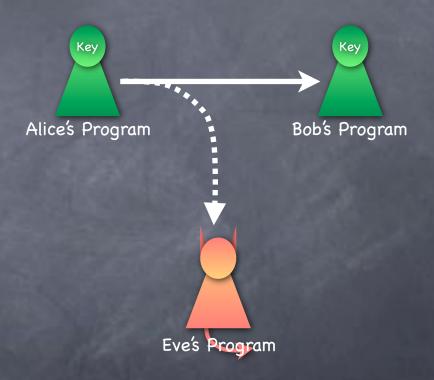
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- Alice, Bob and Eve. Alice and Bob share a key (a bit string)
- Alice wants Bob to learn a message, "without Eve learning it"
- Alice can send out a bit string on the channel. Bob and Eve both get it

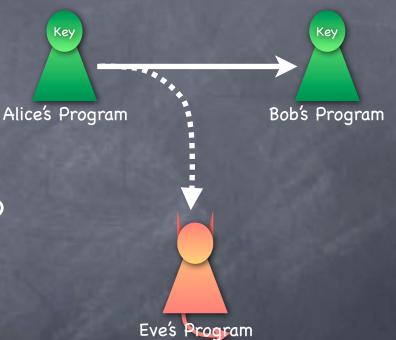


Encryption: Syntax



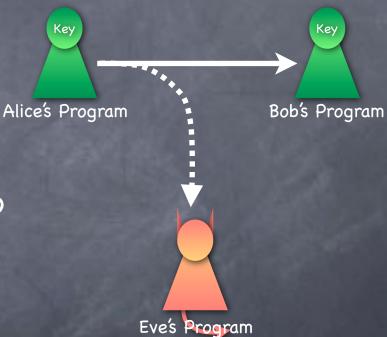
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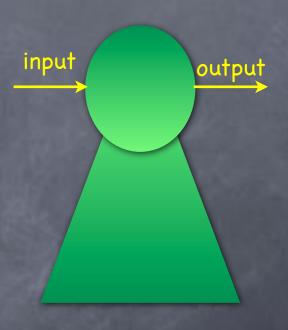
- Three algorithms
 - Key Generation: What Alice and Bob do a priori, for creating the shared secret key
 - Encryption: What Alice does with the message and the key to obtain a "ciphertext"
 - Decryption: What Bob does with the ciphertext and the key to get the message out of it



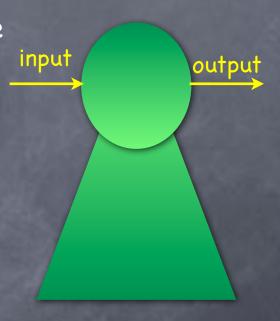
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- All of these are (probabilistic) computations

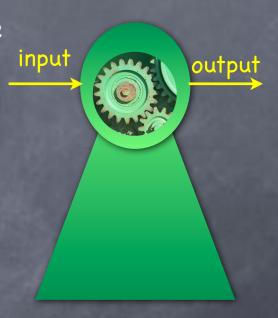




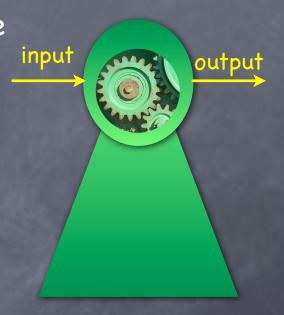
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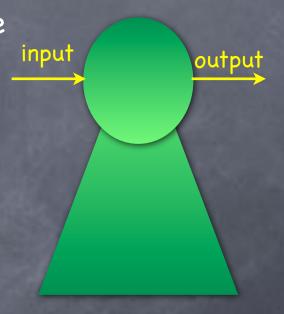
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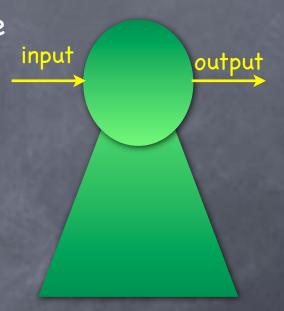
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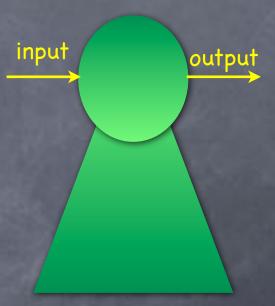
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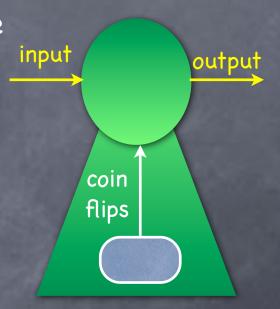
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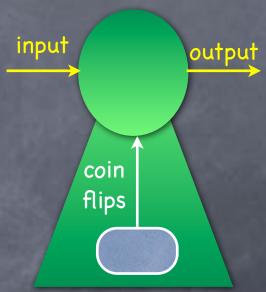
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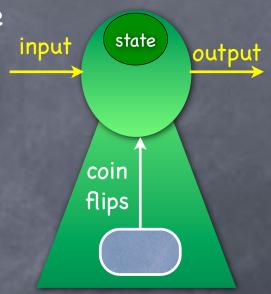


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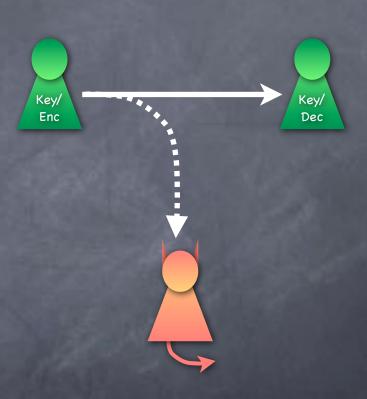


Ideal coin flips: If n coins flipped, each outcome has probability 2⁻ⁿ

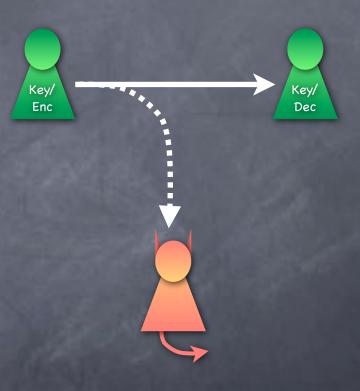
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 - Can be probabilistic
 - Sometimes stateful



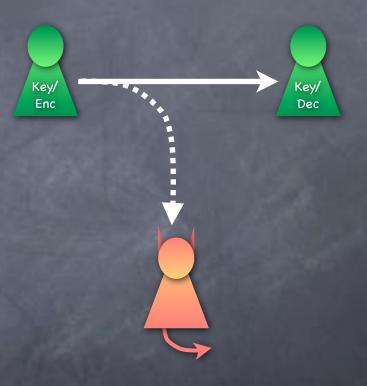
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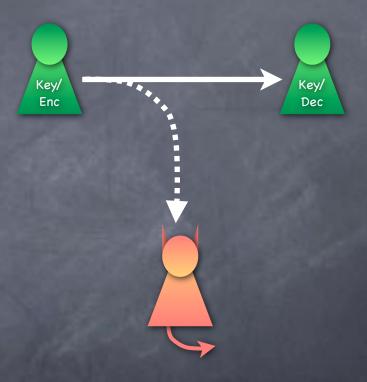
Where does the message come from?



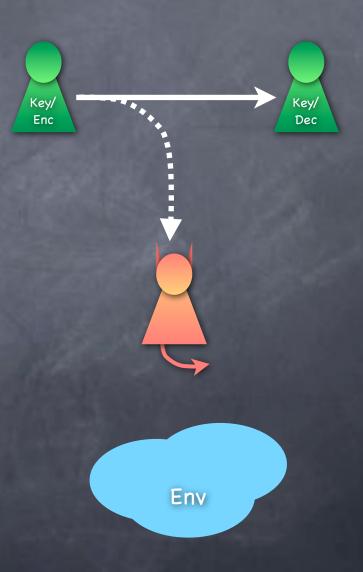
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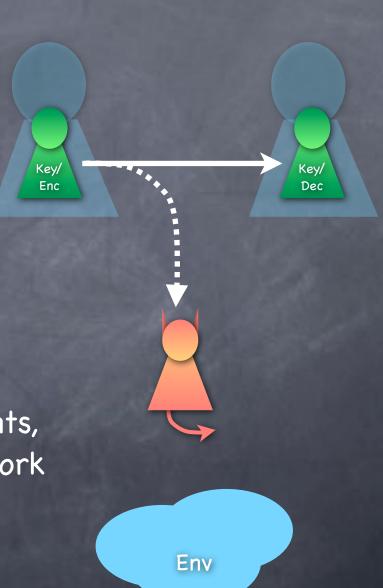
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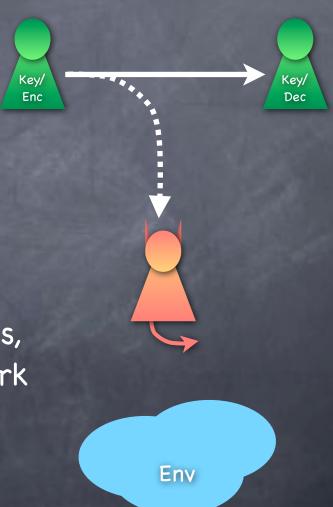
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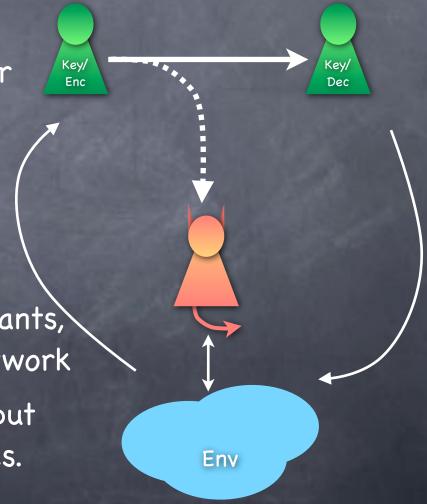
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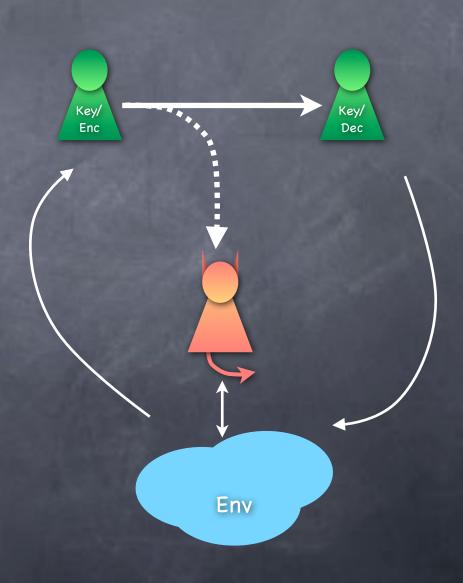


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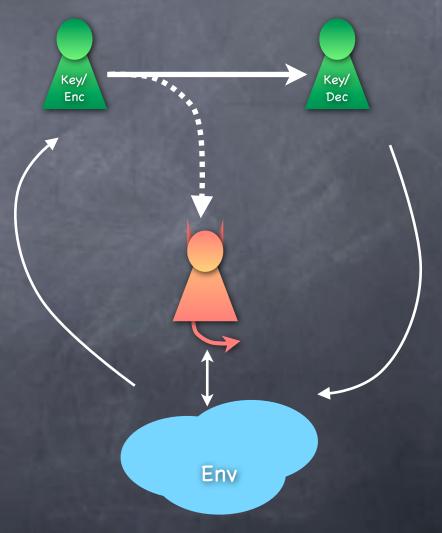


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 - Includes the operating systems and other programs run by the participants, as well as other parties, if in a network
 - Abstract entity from which the input comes and to which the output goes. Arbitrarily influenced by Eve

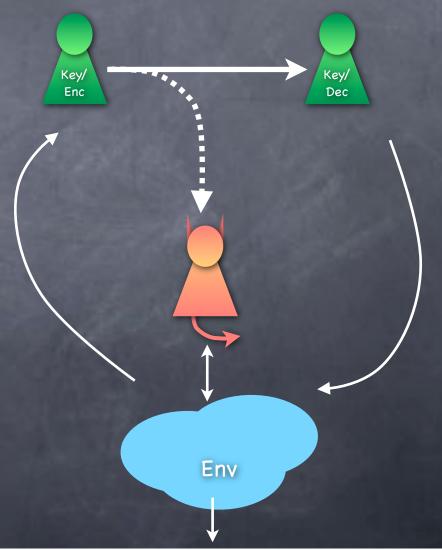




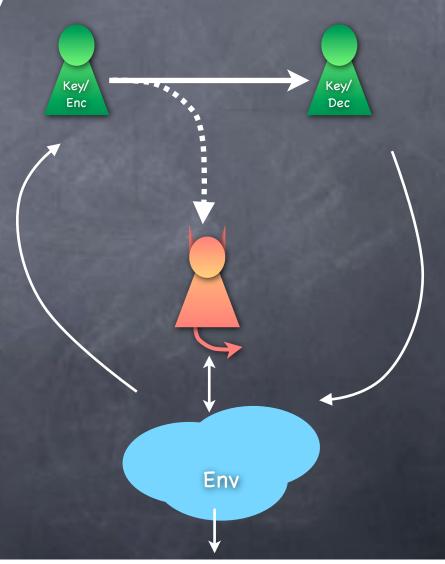
Eve shouldn't be able to produce any "bad effects" in any environment



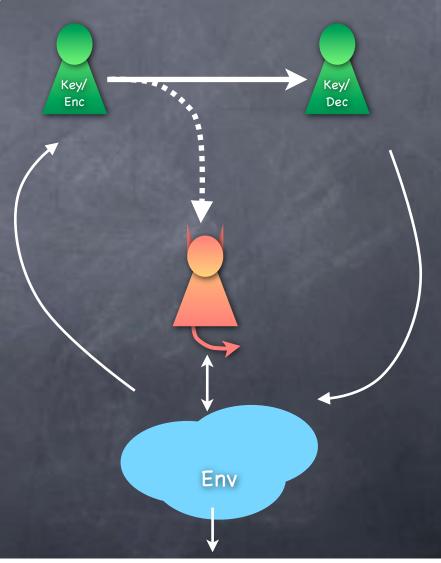
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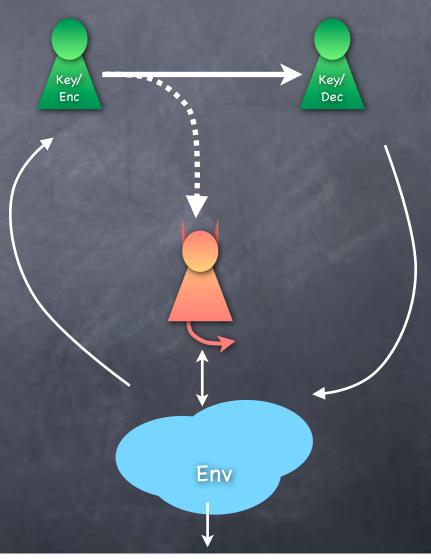


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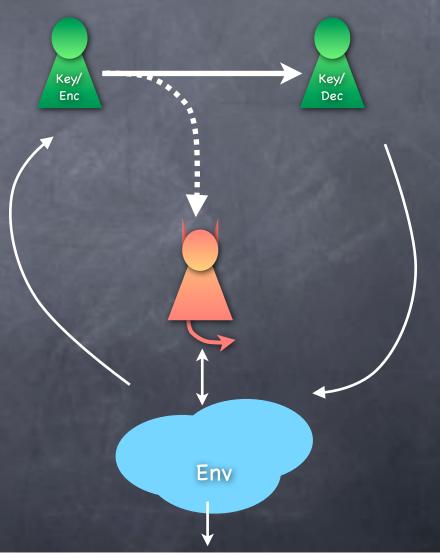


- Eve shouldn't be able to produce any "bad effects" in any environment
- Effects in the environment: modeled as a bit in the environment (called the output bit)
- What is bad?
 - Anything that Eve couldn't have caused if an "ideal channel" was used

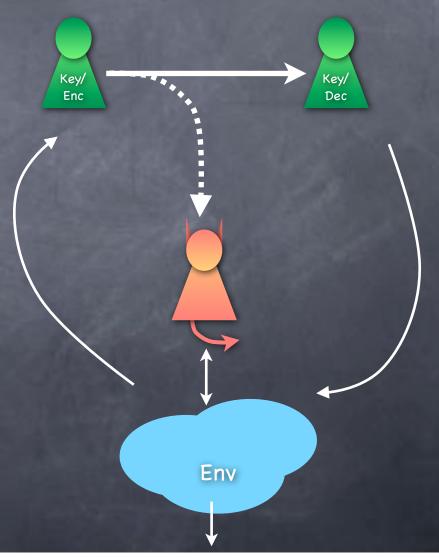




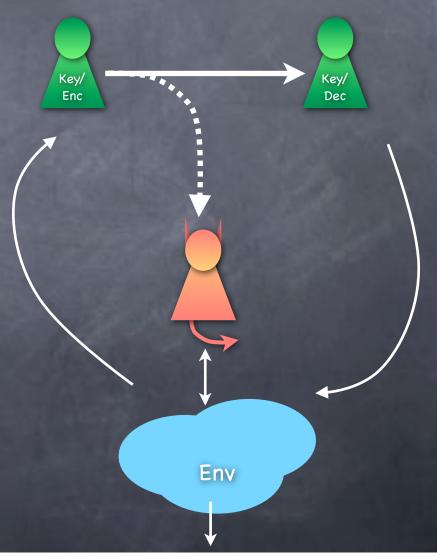
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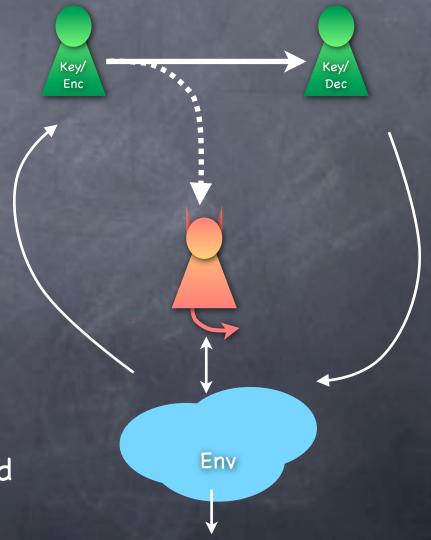
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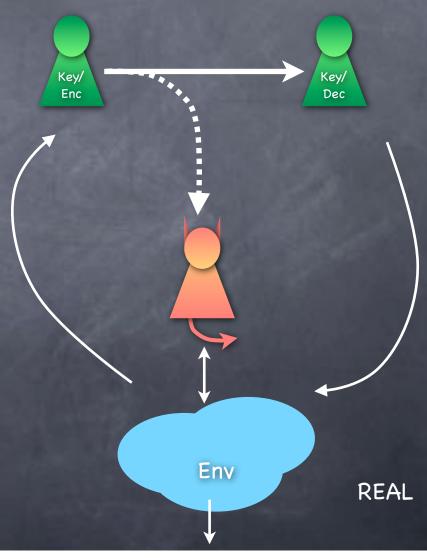


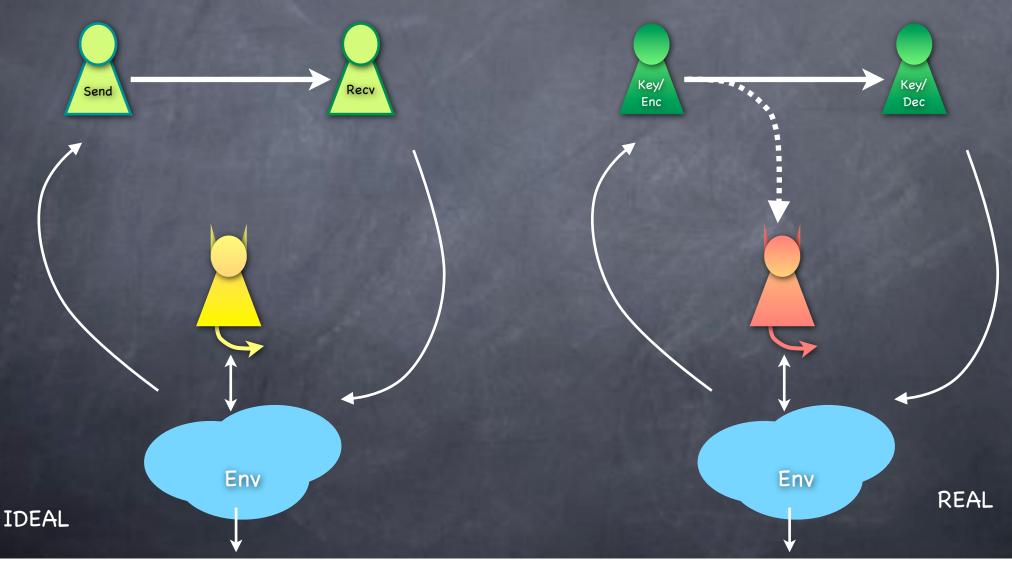
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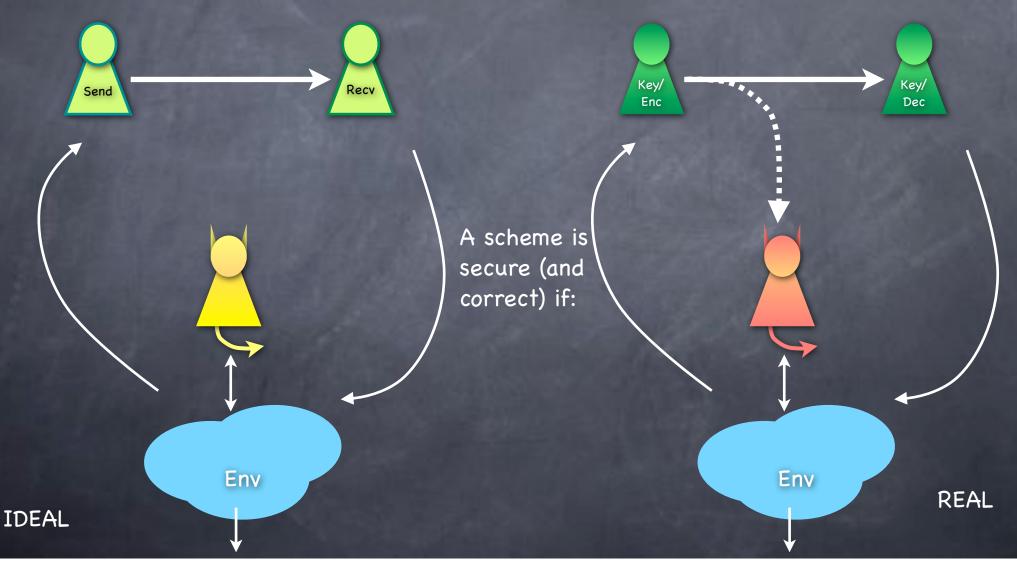


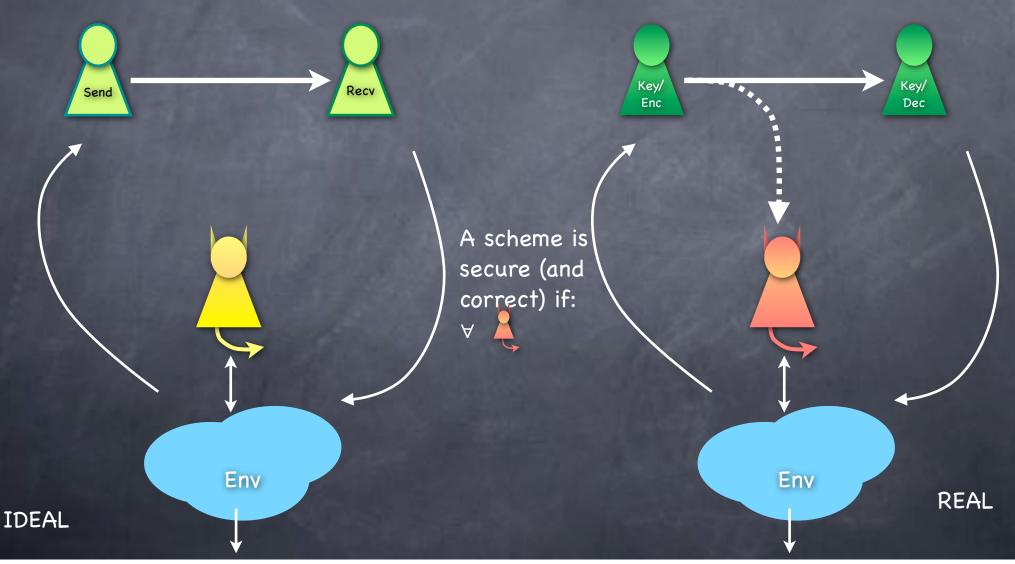
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 - Encryption is secure if whatever an Eve can do in the REAL world, an Eve' can do in the IDEAL world

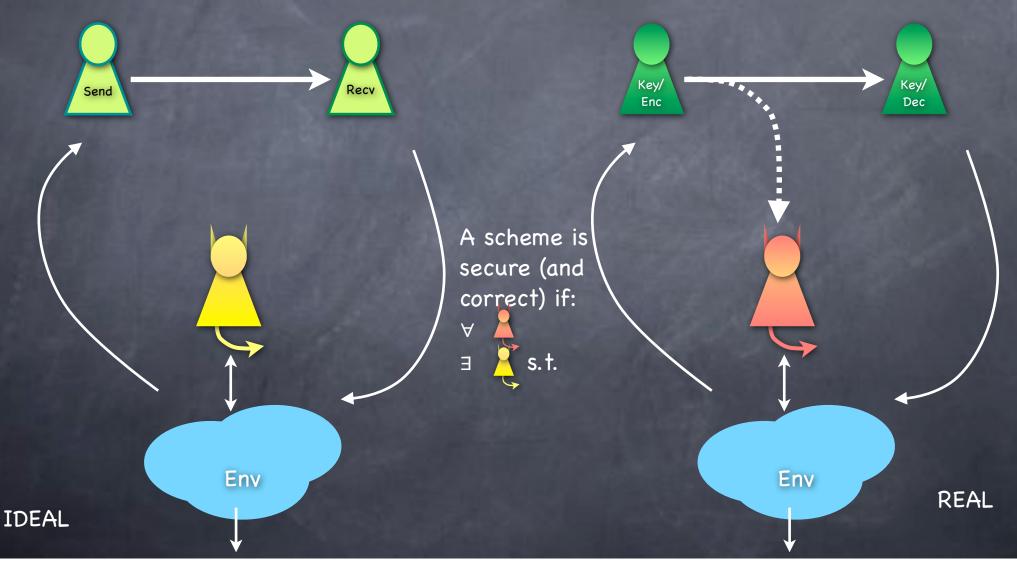


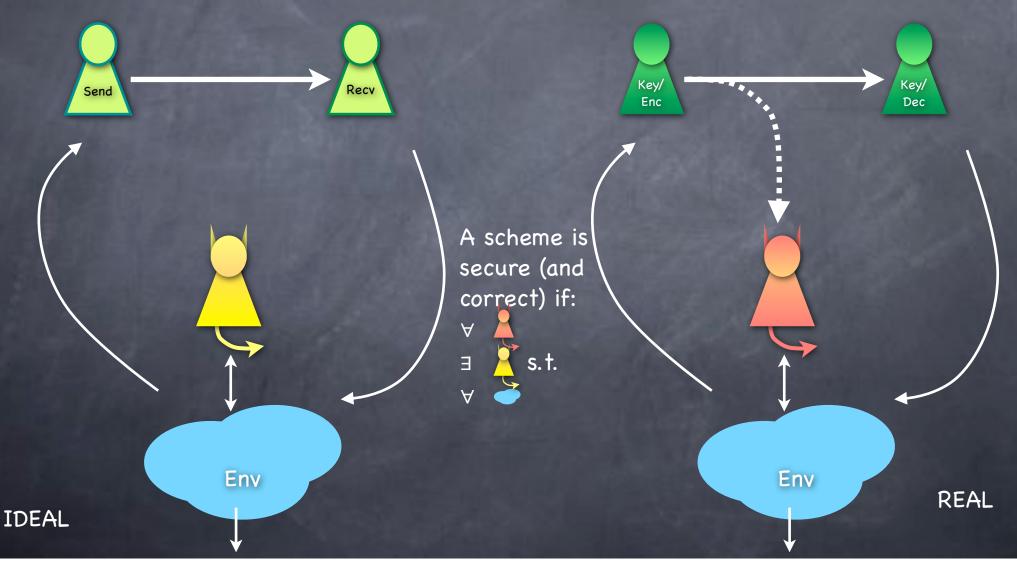


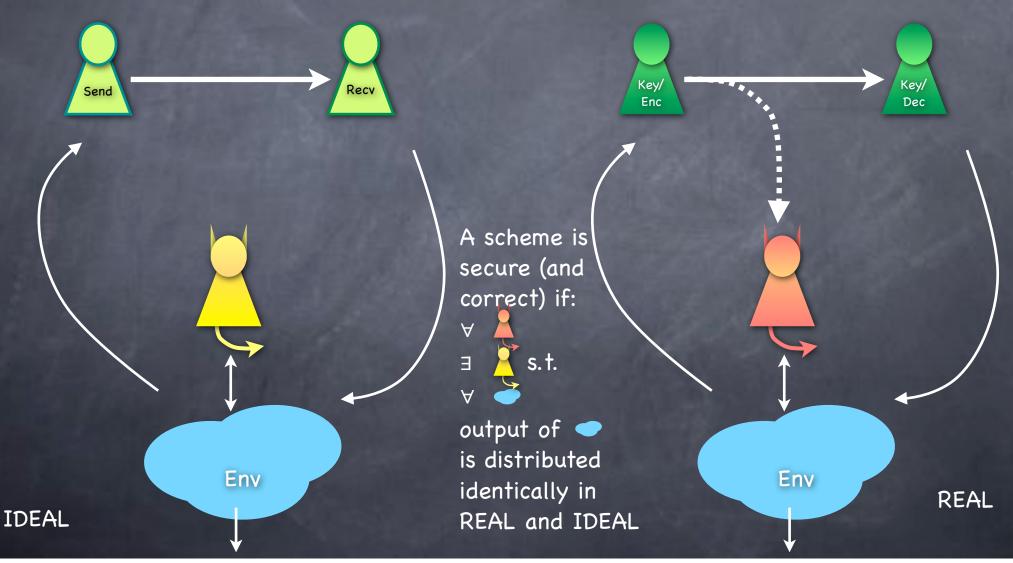












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- We will see three definitions of encryption
 - Security of "one-time encryption"
 - Security of (muti-message) encryption
 - Security against "active attacks"
- Will also see alternate (but essentially equivalent) security definitions

Onetime Encryption The Syntax

- Shared-key (Private-key) Encryption
 - Key Generation: Randomized
 - $_{\odot}$ K \leftarrow % , uniformly randomly drawn from the key-space (or according to a key-distribution)
 - Encryption: Deterministic
 - \bullet Enc: $\mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
 - Decryption: Deterministic
 - \bullet Dec: $C \times \mathcal{H} \rightarrow \mathcal{M}$

Onetime Encryption Perfect Secrecy

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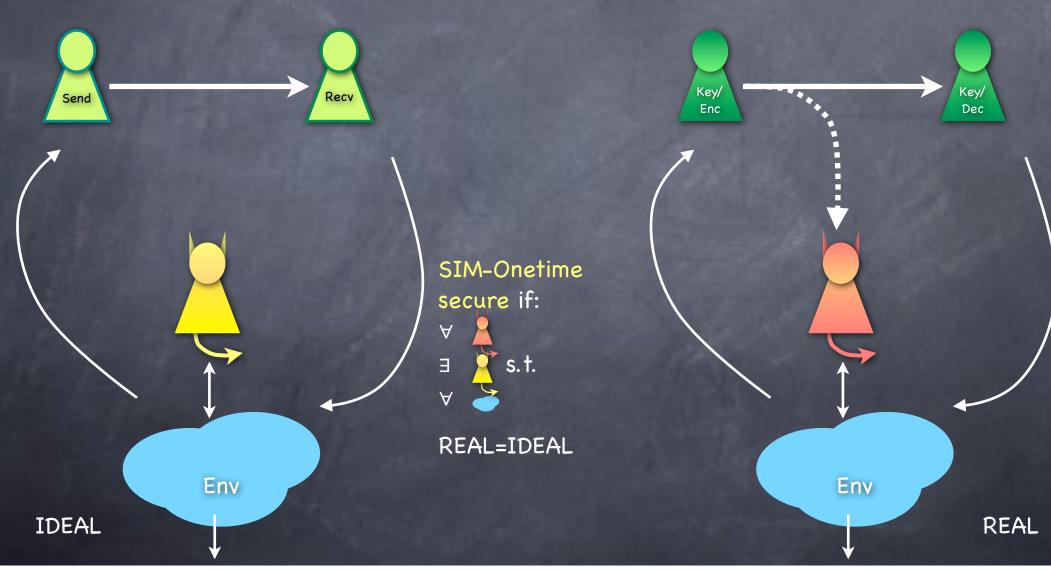
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	and $Enc(m,K) = m+K$,	Dec(c,K) = c-K

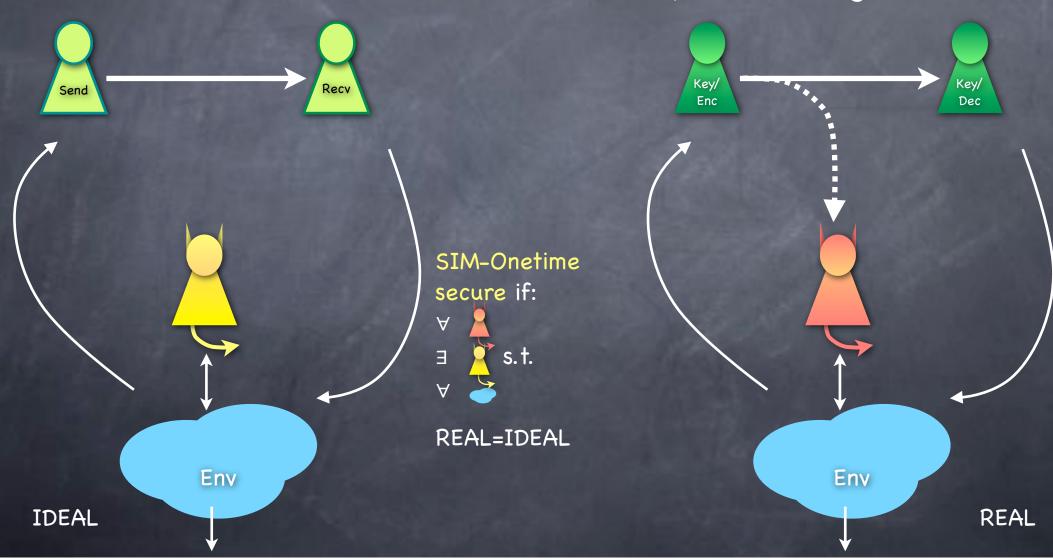
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SIM-Onetime Security



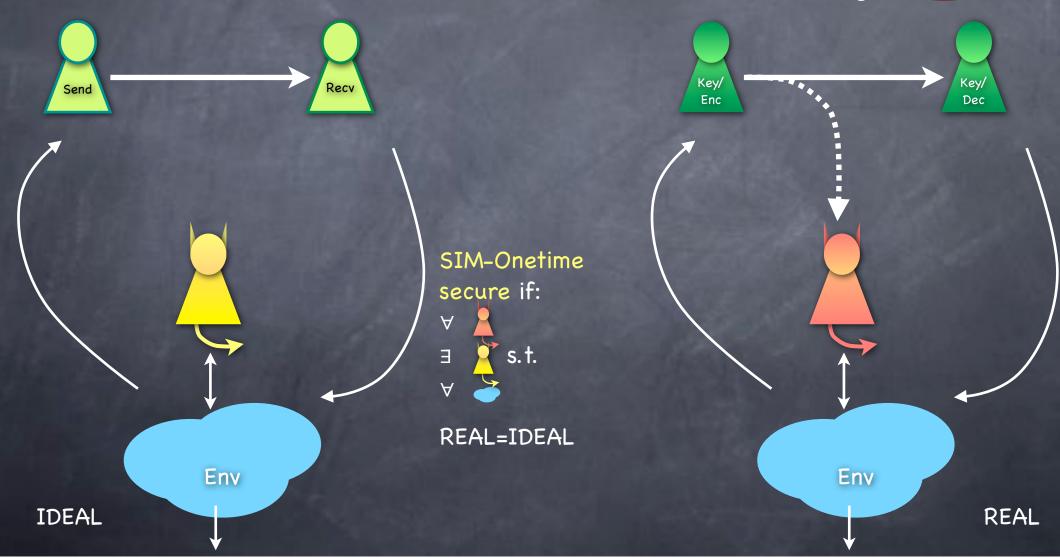
Onetime Encryption SIM-Onetime Security

Class of environments which send only one message

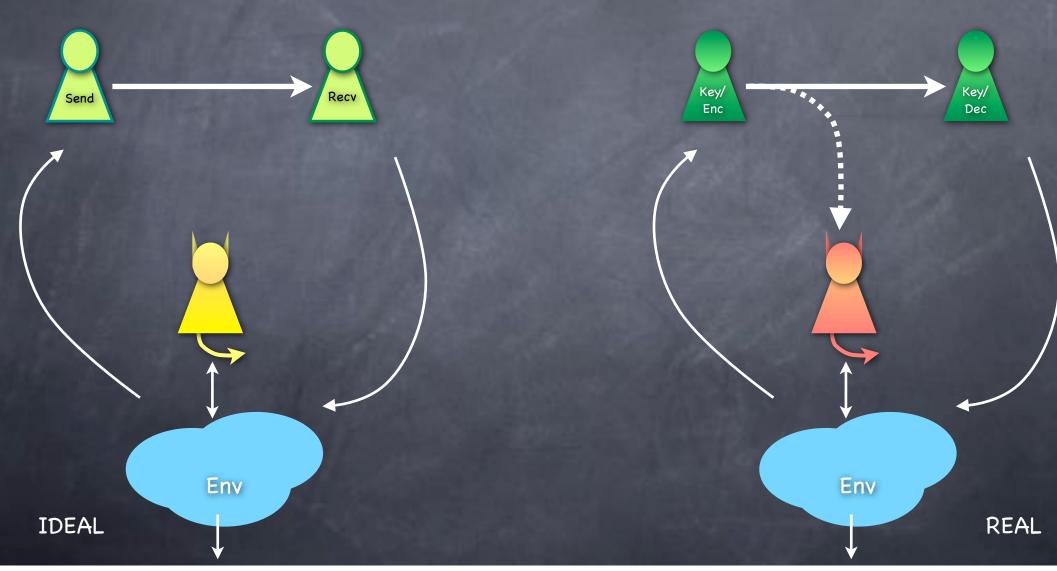


Onetime Encryption Equivalent to perfect secrecy + correctness

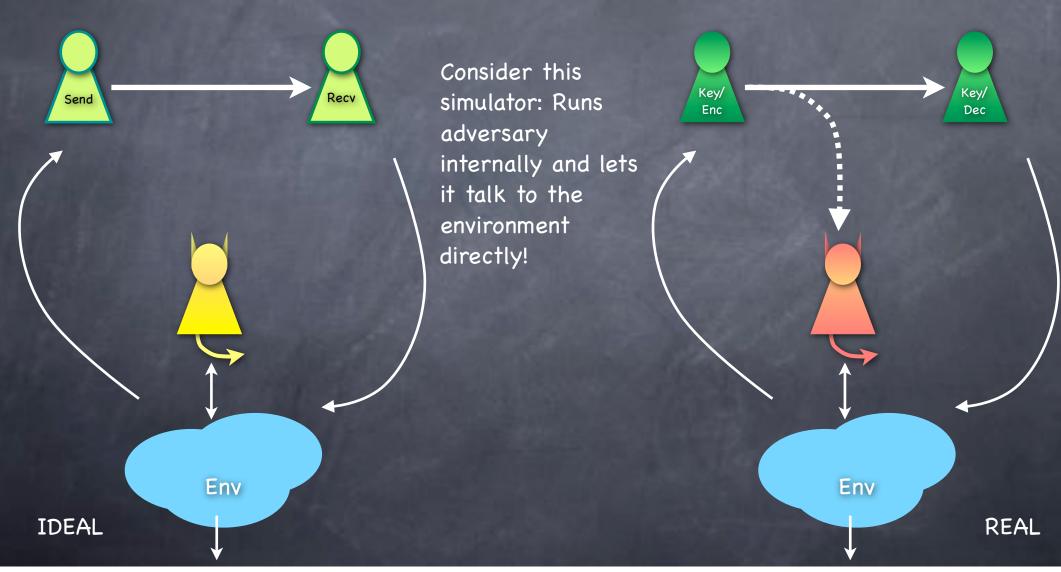
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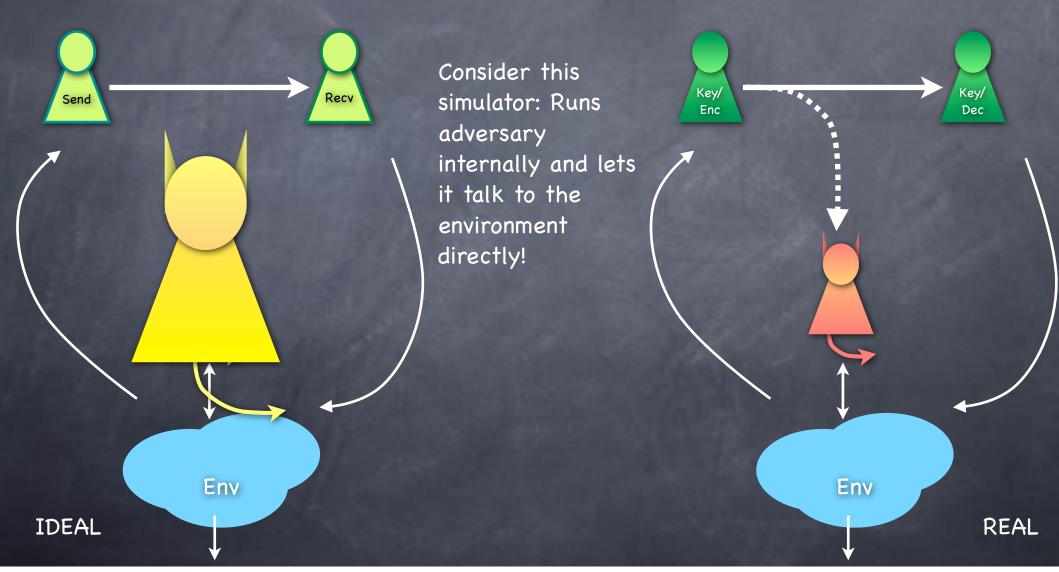
Perfect Secrecy + Correctness ⇒ SIM-Onetime Security



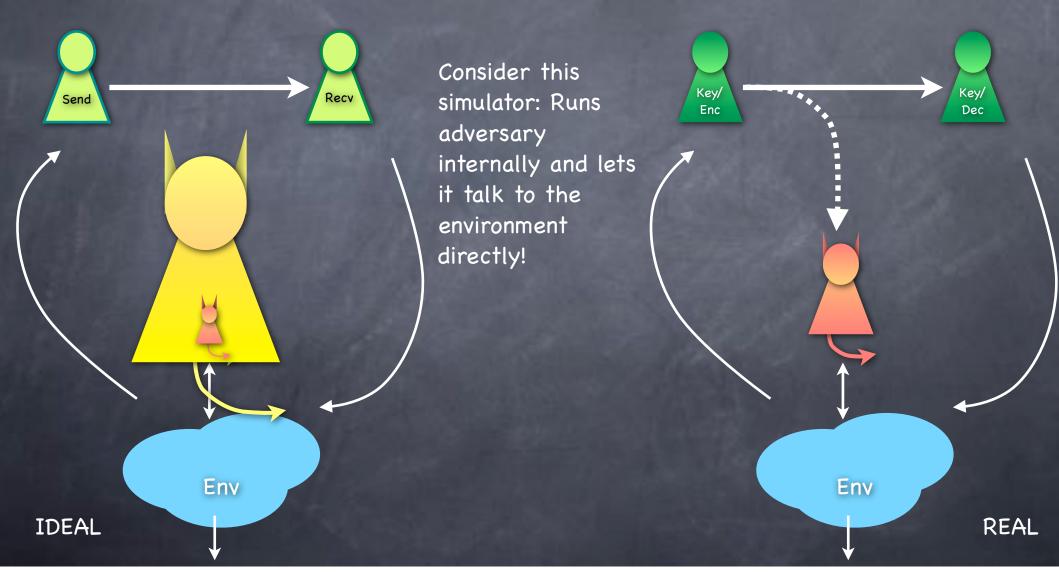
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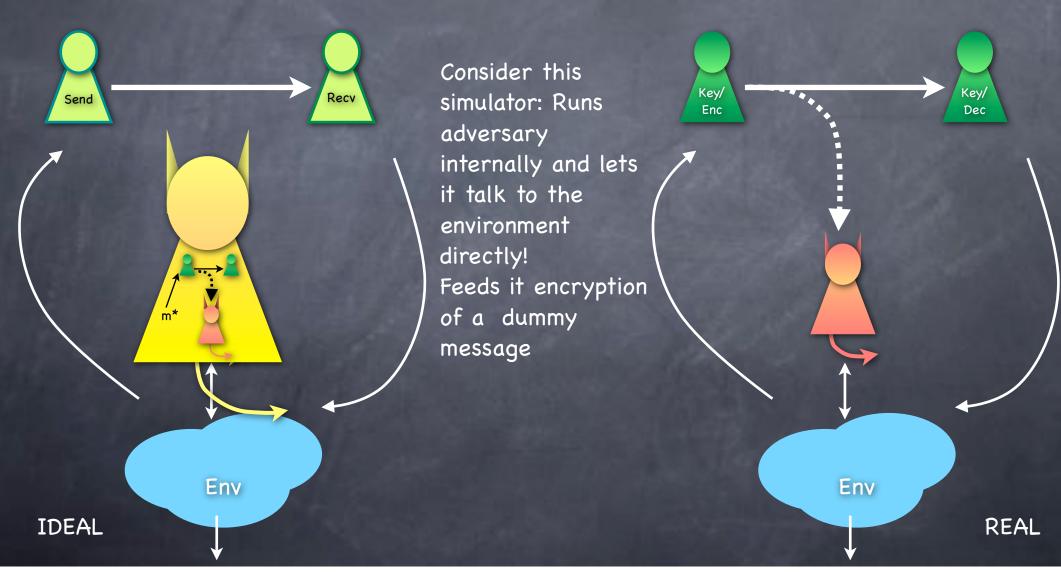
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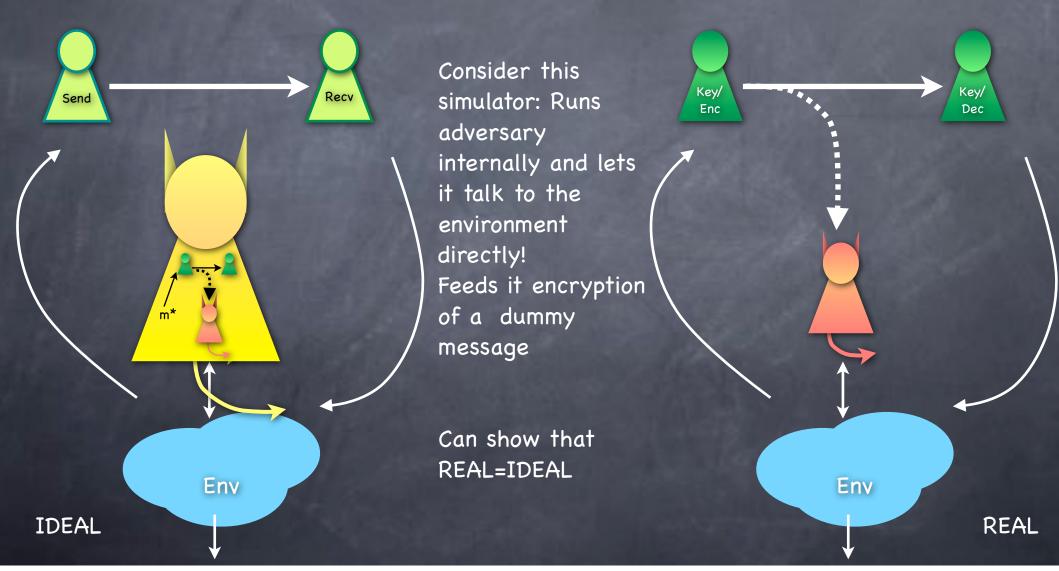
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 - Also, Eve' allowed to learn when a message is sent

IND-Onetime Security

IND-Onetime Experiment

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Onetime Encryption IND-Onetime Security

- IND-Onetime Experiment
 - Experiment picks a random bit b. It also runs KeyGen to get a key K



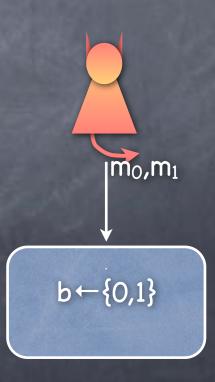
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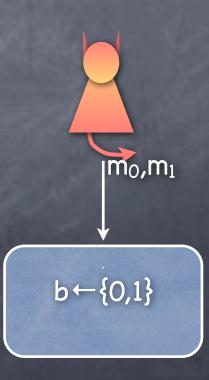
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 - Experiment replies with Enc(m_b,K)

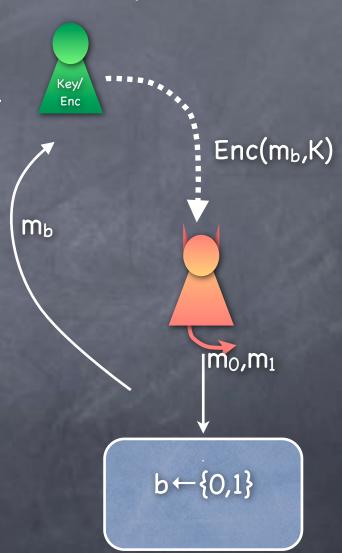




IND-Onetime Security

IND-Onetime Experiment

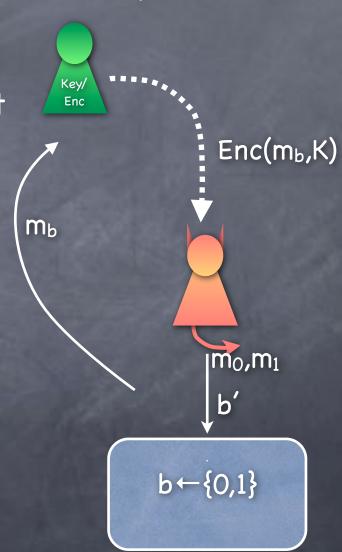
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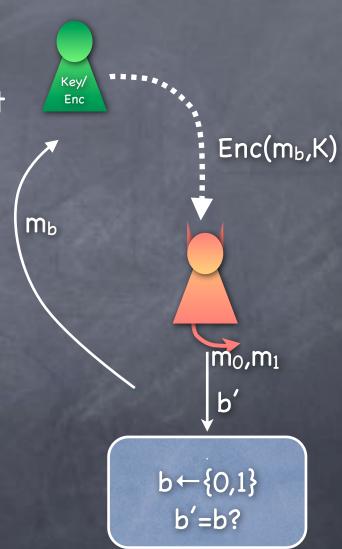
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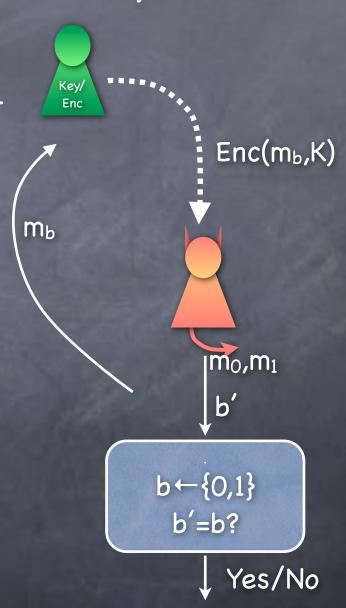
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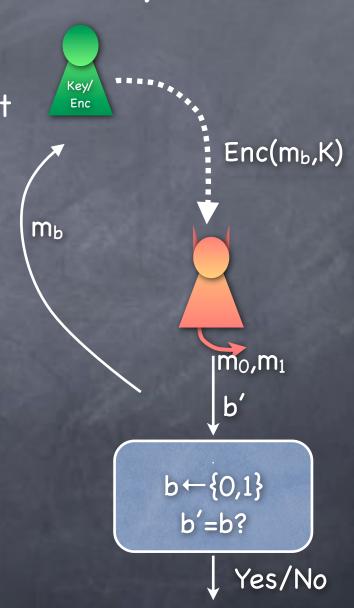
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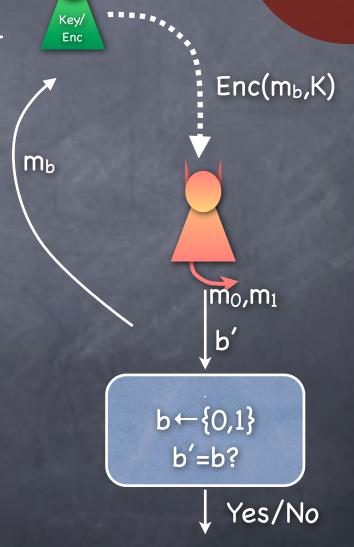
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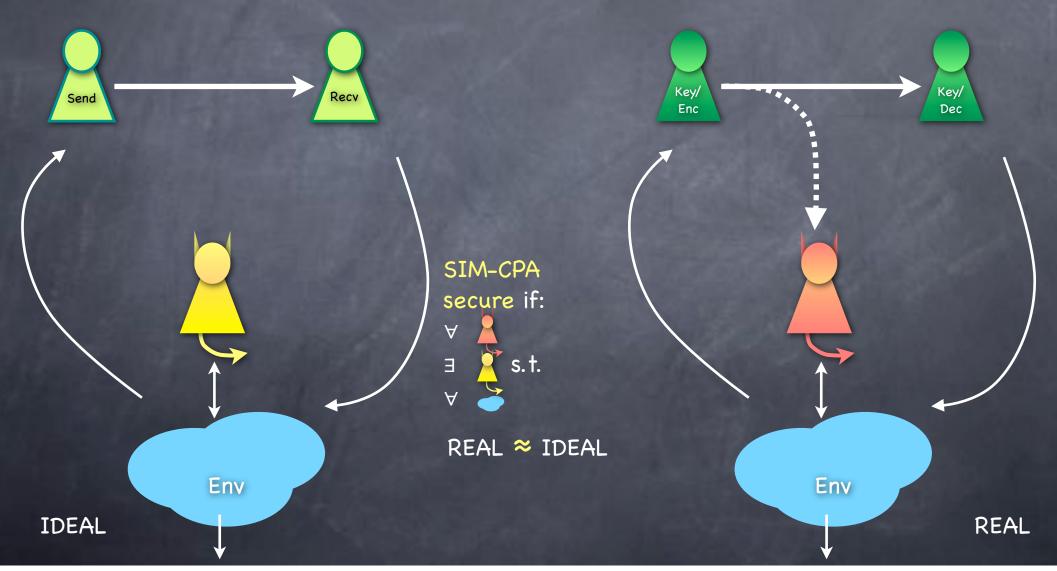
Equivalent to perfect secrecy



The Syntax

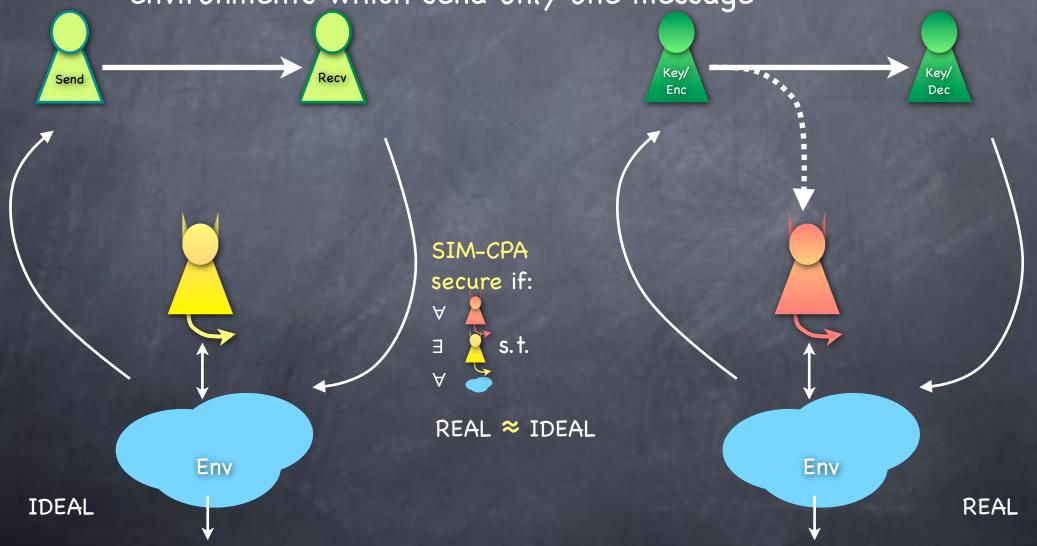
- Shared-key (Private-key) Encryption
 - Key Generation: Randomized
 - $_{\odot}$ K \leftarrow %, uniformly randomly drawn from the key-space (or according to a key-distribution)
 - Encryption: Randomized
 - The Enc: $\mathcal{M} \times \mathcal{K} \times \mathcal{R} \to \mathcal{C}$. During encryption a fresh random string will be chosen uniformly at random from \mathcal{R}
 - Decryption: Deterministic
 - \bullet Dec: $C \times \mathcal{H} \rightarrow \mathcal{M}$

Symmetric-Key Encryption SIM-CPA Security



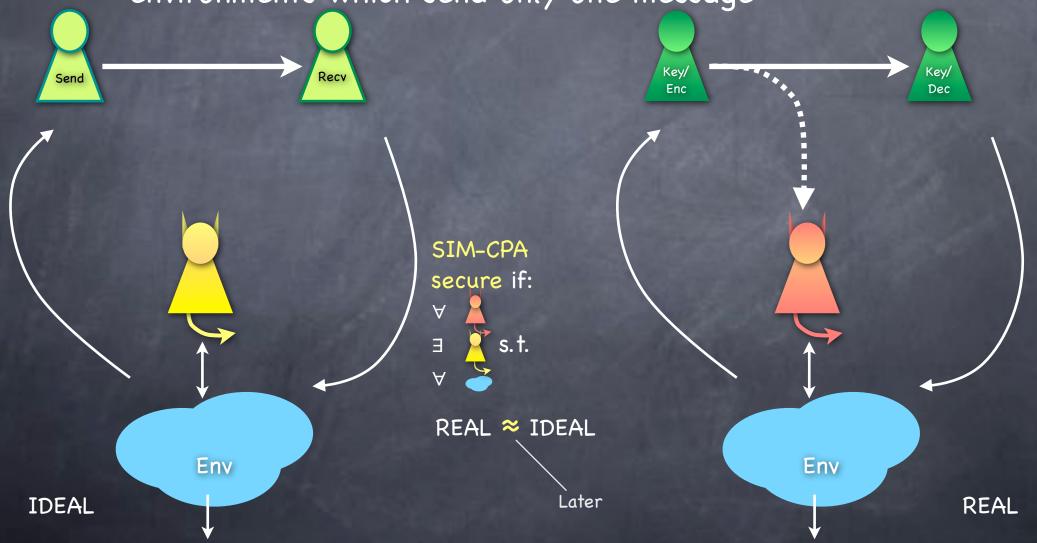
Symmetric-Key Encryption SIM-CPA Security

Same as SIM-onetime security, but not restricted to environments which send only one message



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IND-CPA Security





IND-CPA Security

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b←{0,1}

IND-CPA Security

- Experiment picks a random bit b. It also runs KeyGen to get a key K
 - For as long as Adversary wants



Key/ Enc

IND-CPA Security

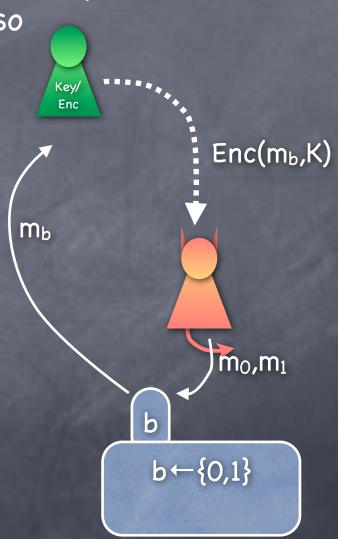
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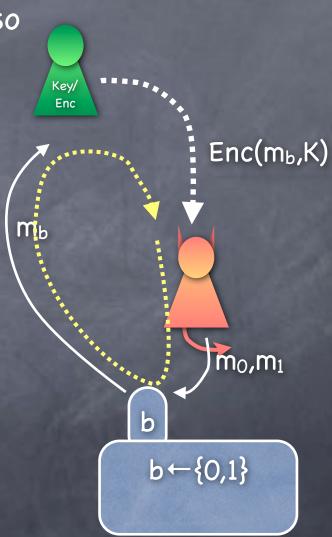
IND-CPA Security

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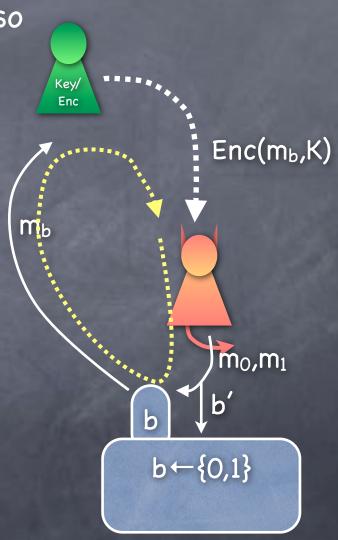
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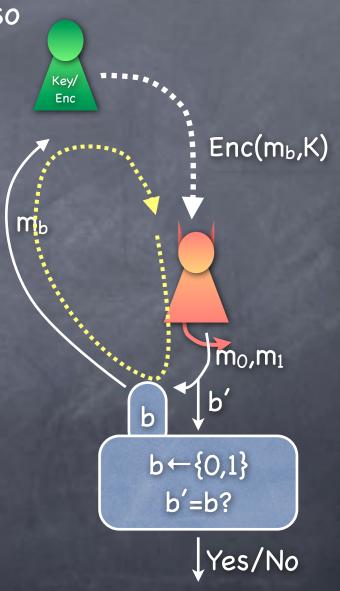
IND-CPA Security

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 - Expt returns Enc(m_b,K) to the adversary
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IND-CPA Security

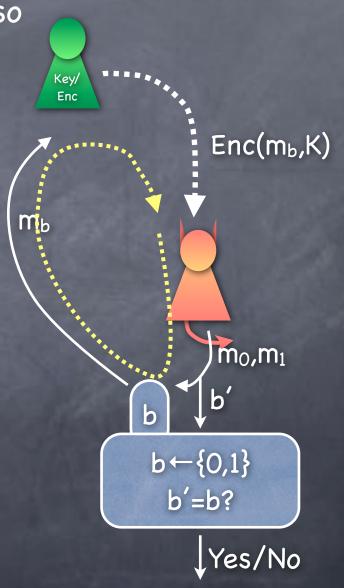
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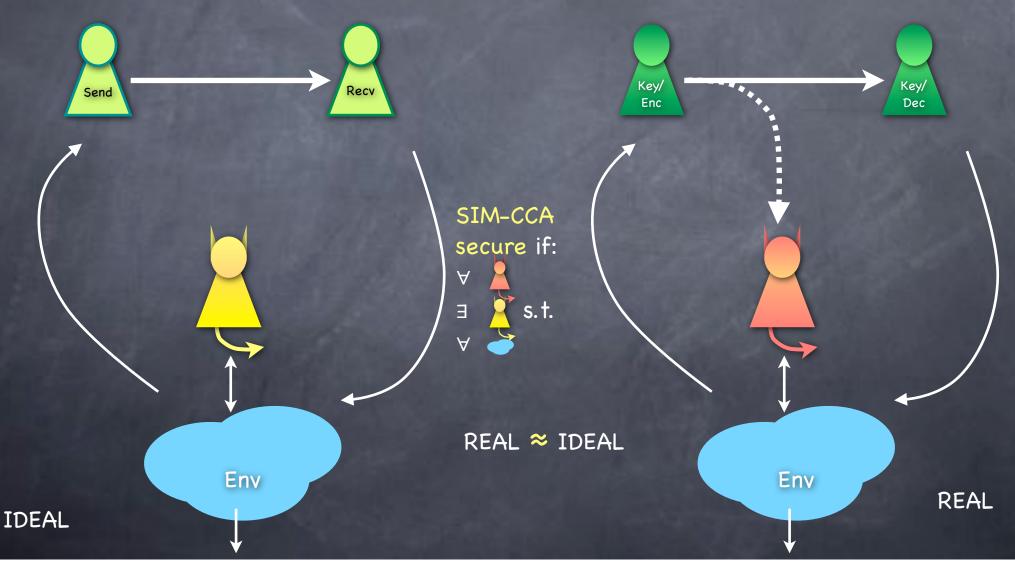
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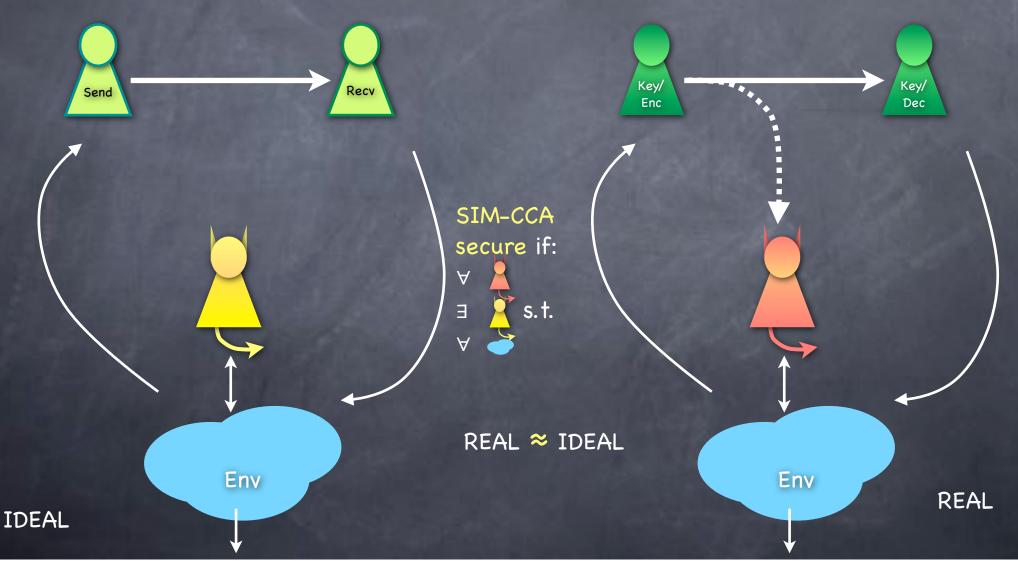
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IND-CPA + ~correctness equivalent to SIM-CPA Key/ Enc Enc(mb,K) Mb m_0, m_1 b ← {0,1} b'=b?

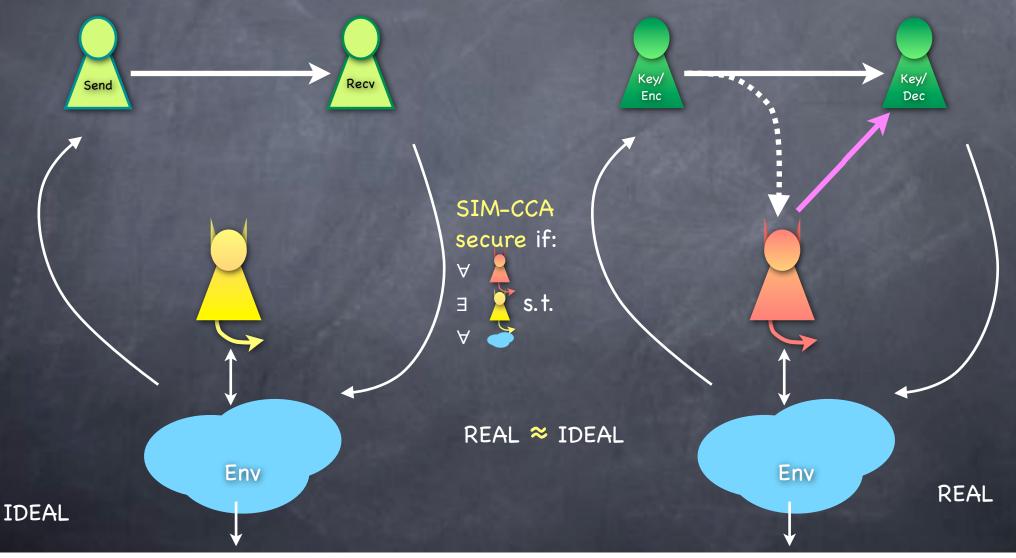
Yes/No



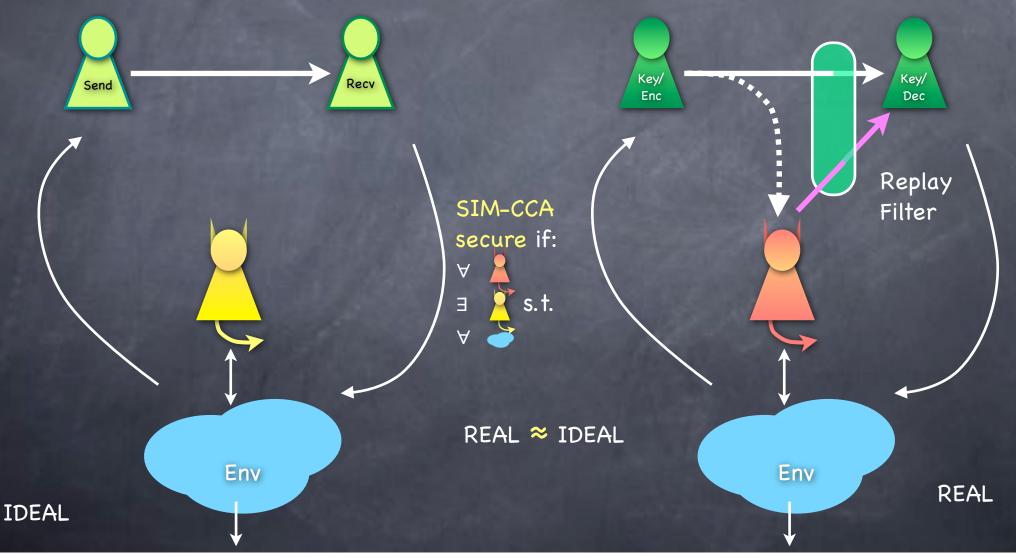
An active adversary can inject messages into the channel



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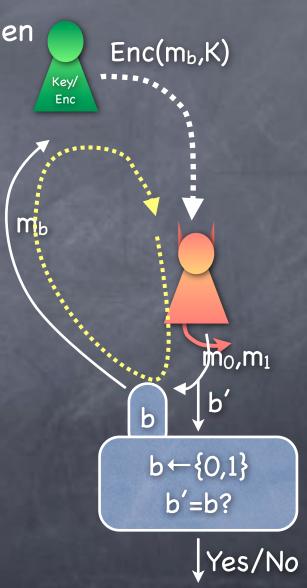
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IND-CCA Security

Experiment picks b←{0,1} and K←KeyGen

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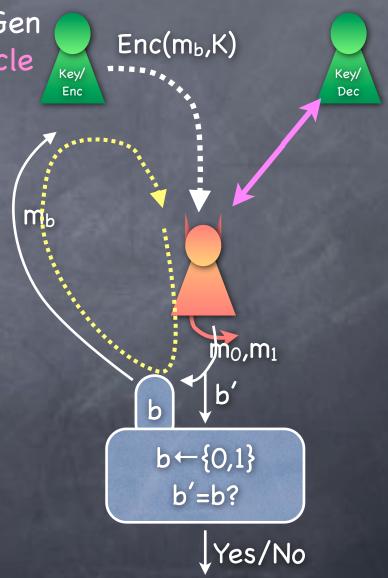


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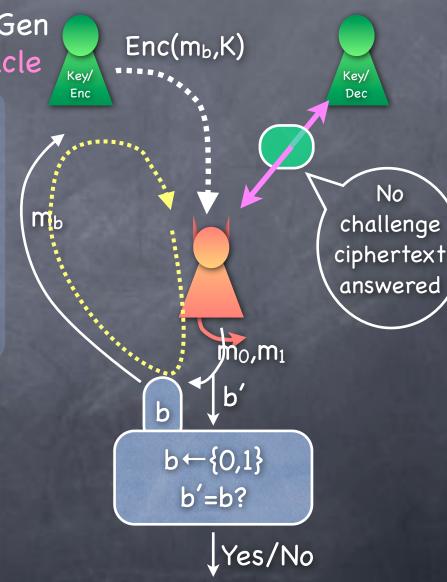


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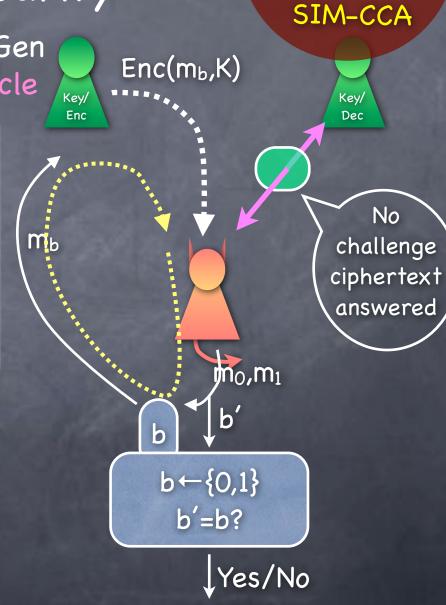


Symmetric-Key Encryp's IND-CCA Security

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equivalent to

"Technical" vs. "Convincing"

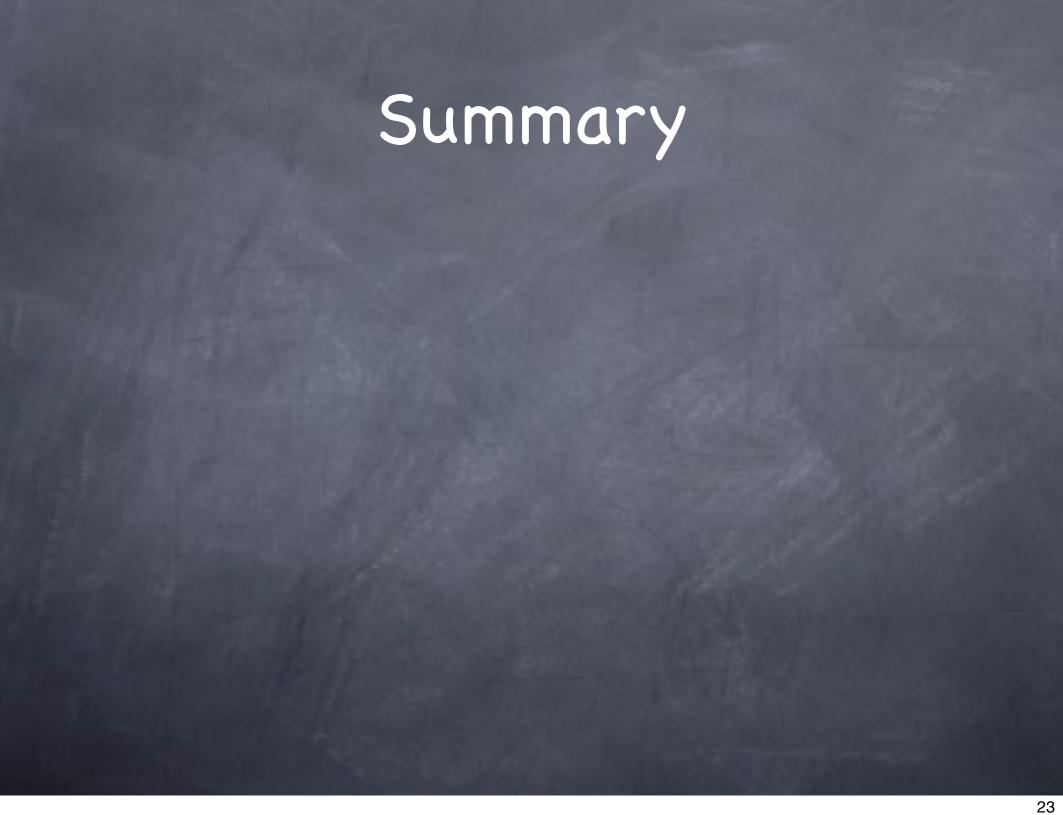
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- IND- definitions tend to be technical: more low-level details, but may not make the big picture clear. Could have "weaknesses"
- SIM- definitions give the big picture, but may not give details of what is involved in satisfying it. Could be "too strong"
- Best of both worlds when they are equivalent: use IND- definition while say proving security of a construction; use SIM- definition when low-level details are not important



Security definitions:

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 - But what is ≈ ?
 - And, how to build symmetric-key encryption schemes