Applied Cryptography

Lecture 1

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Our first encounter with secrecy:
Secret-Sharing

Secrecy

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  - Access to learning and/or influencing information
- One of the aspects of access control is secrecy

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- Other ideas?

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- Multiple bits can be shared independently: as, $m_1m_2 = a_1a_2 \oplus b_1b_2$
- Note: one share can be chosen before knowing the message [why?]

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  - i.e., for all possible values of the message, view is distributed the same way

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    - Leakage resilience ...

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    - our previous example: (2,2) secret-sharing

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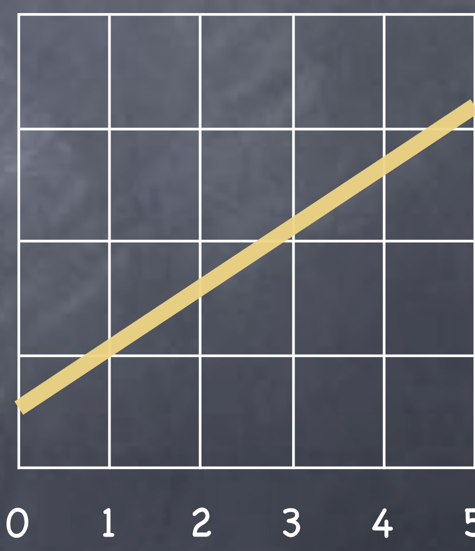

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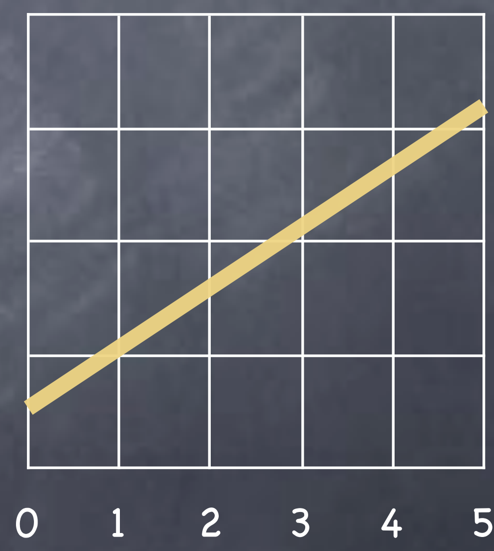

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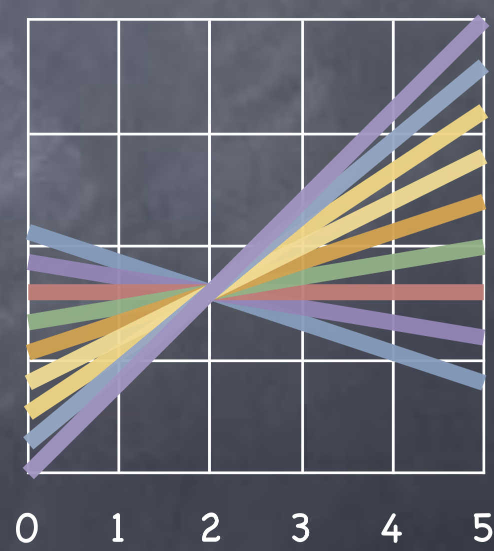

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    - But can reconstruct the line from two points!

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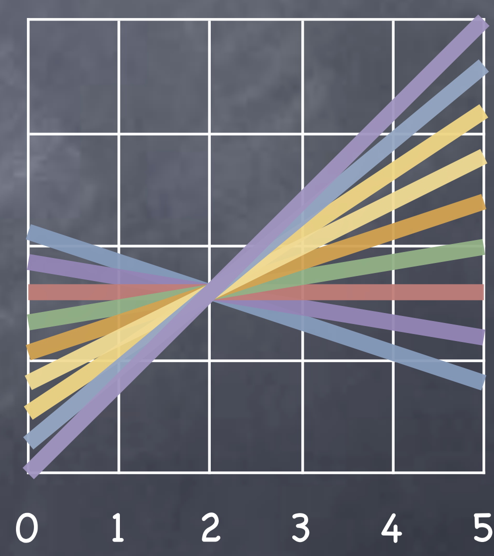

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      - Random polynomial with f(0)=M: $c_0 + c_1X + c_2X^2 + \ldots + c_{t-1}X^{t-1}$ by picking c_0=M and $c_1,\ldots,c_{t-1}$ at random.

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    - Reconstruct($s_1,\ldots,s_t$): Lagrange interpolation to find $M=c_0$
      - Need t points to reconstruct the polynomial. Given t-1 points, there is exactly one polynomial passing through (0,M′) for each M′

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- Shamir's secret-sharing solves threshold secret-sharing. How about the others?

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  - More efficient schemes known for large classes of access structures

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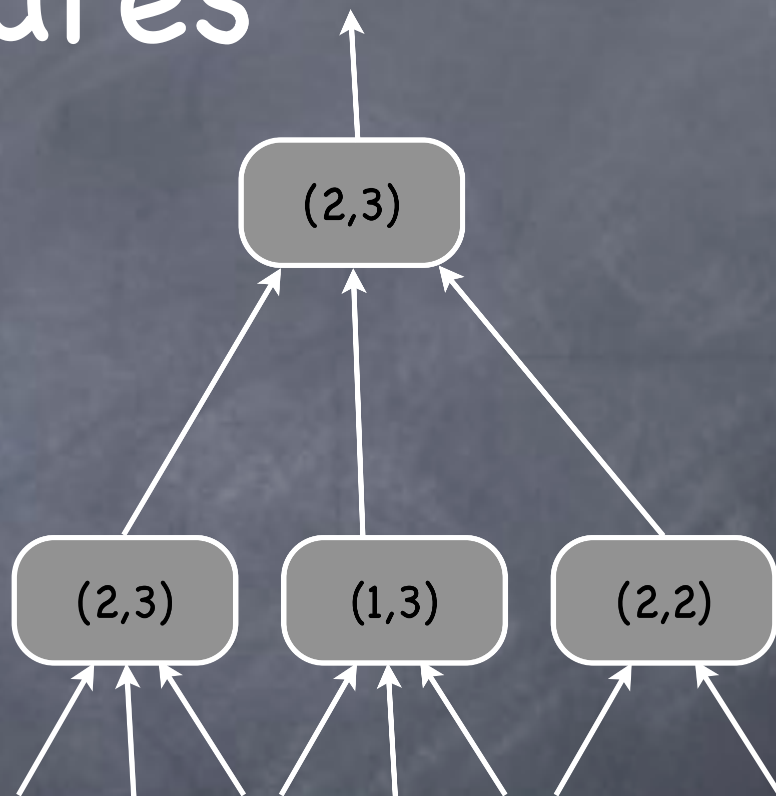

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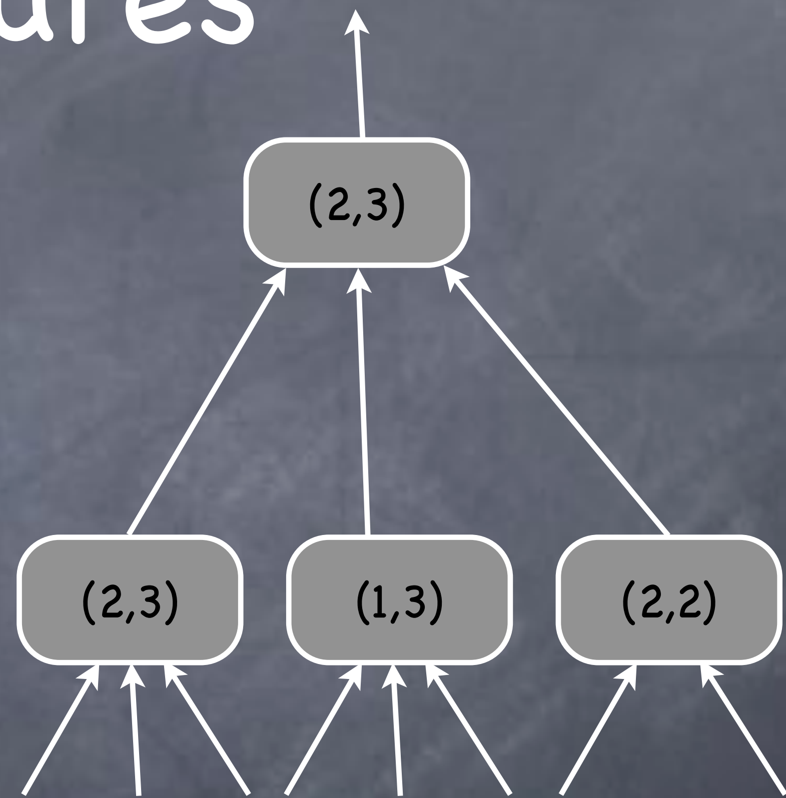

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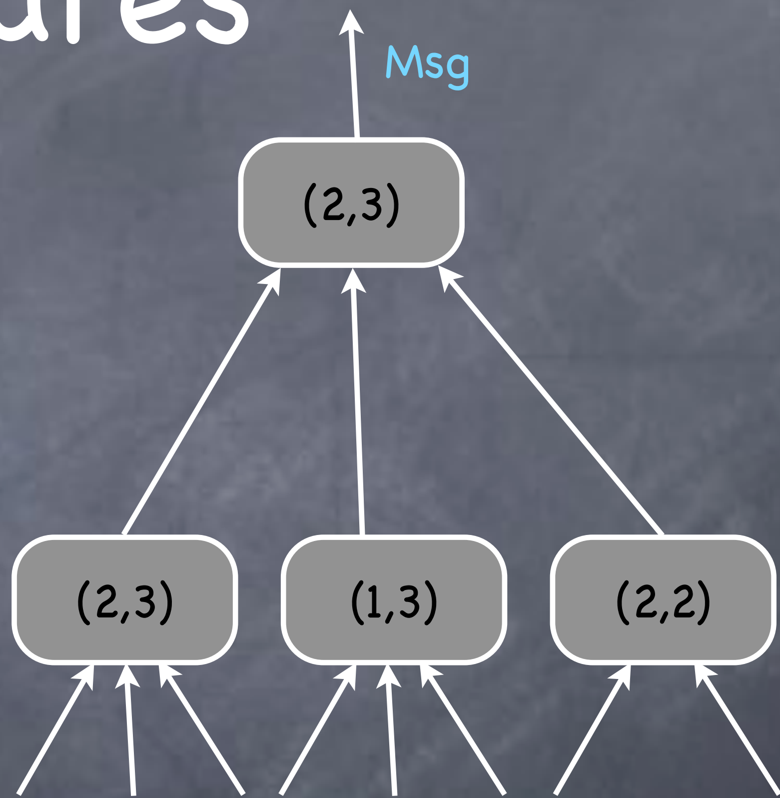

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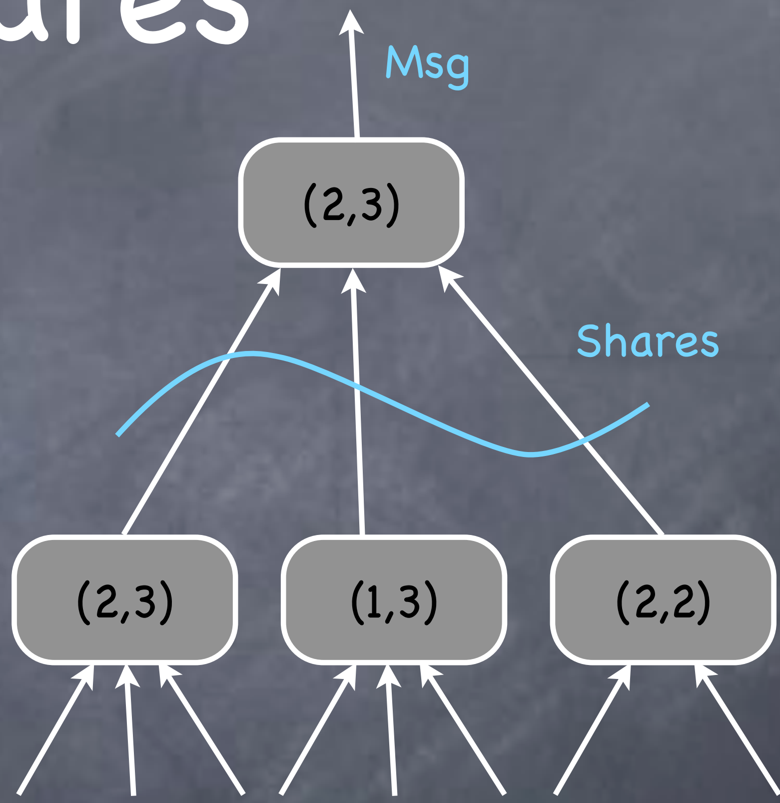

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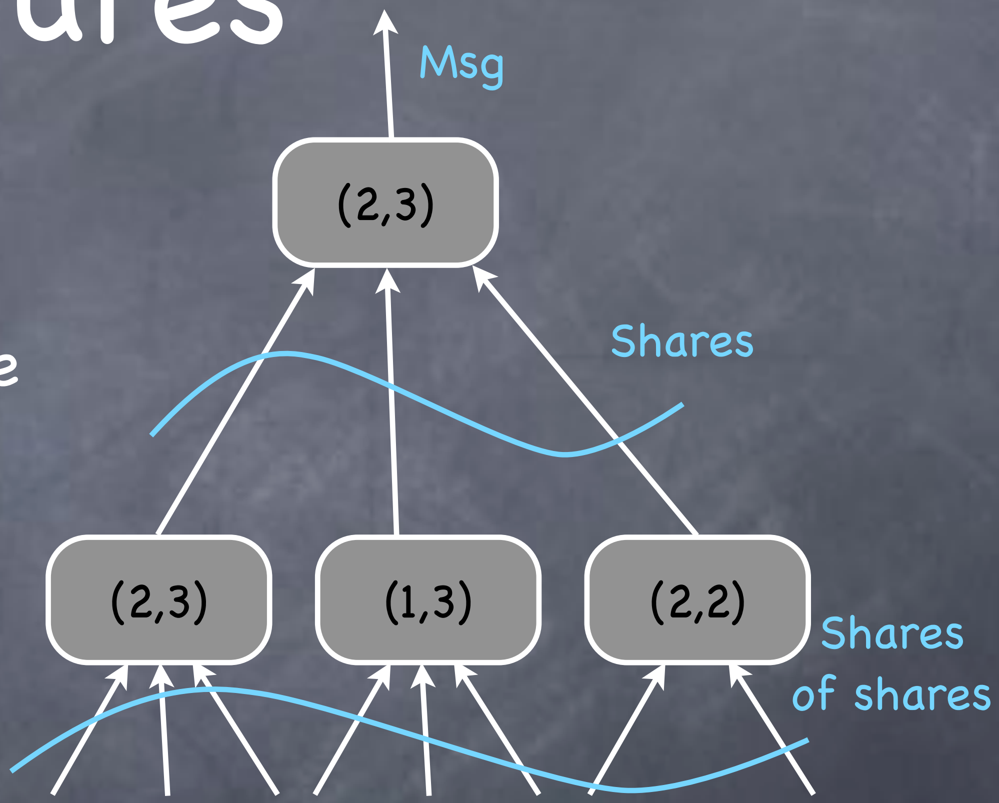

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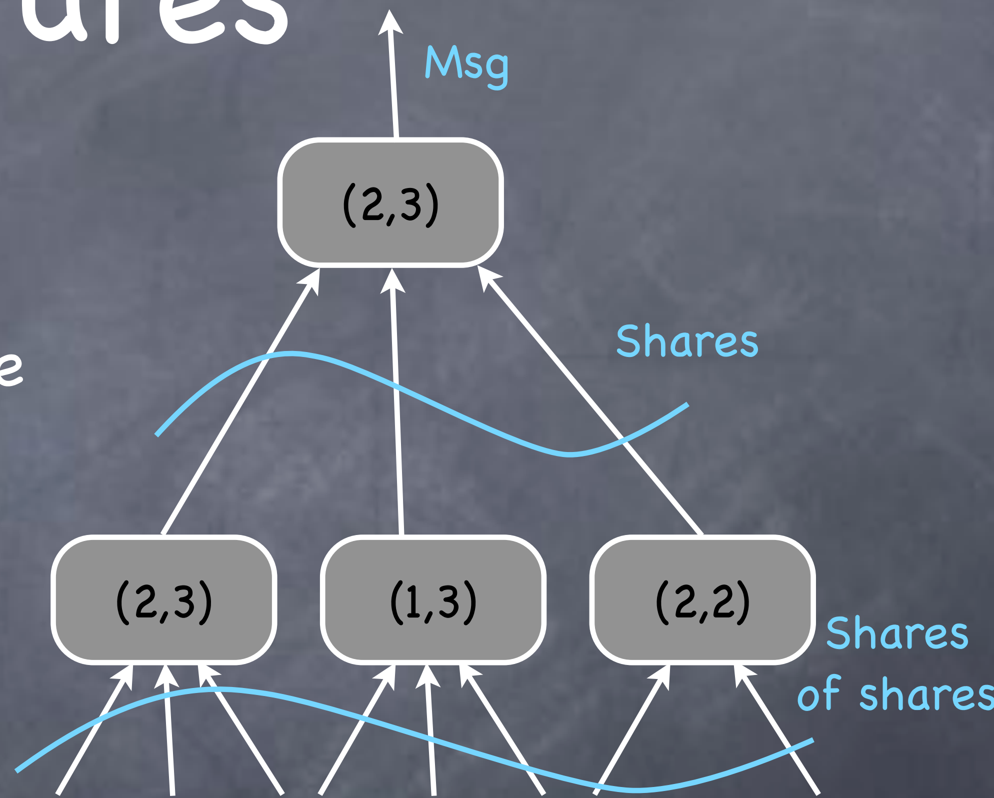

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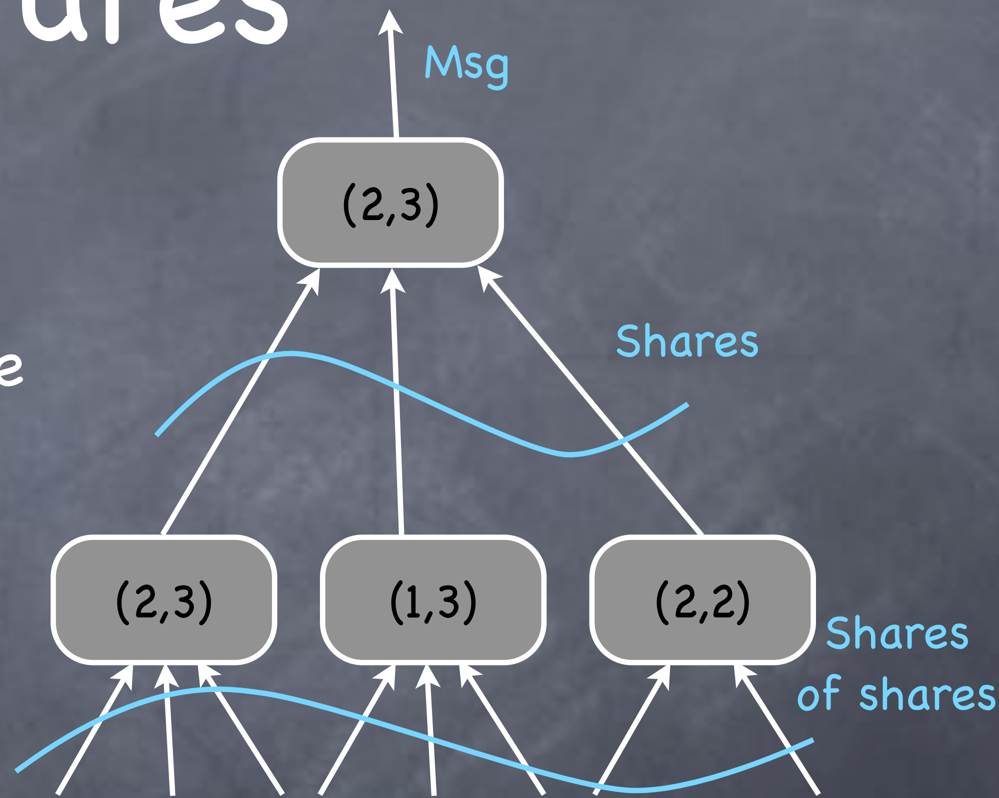

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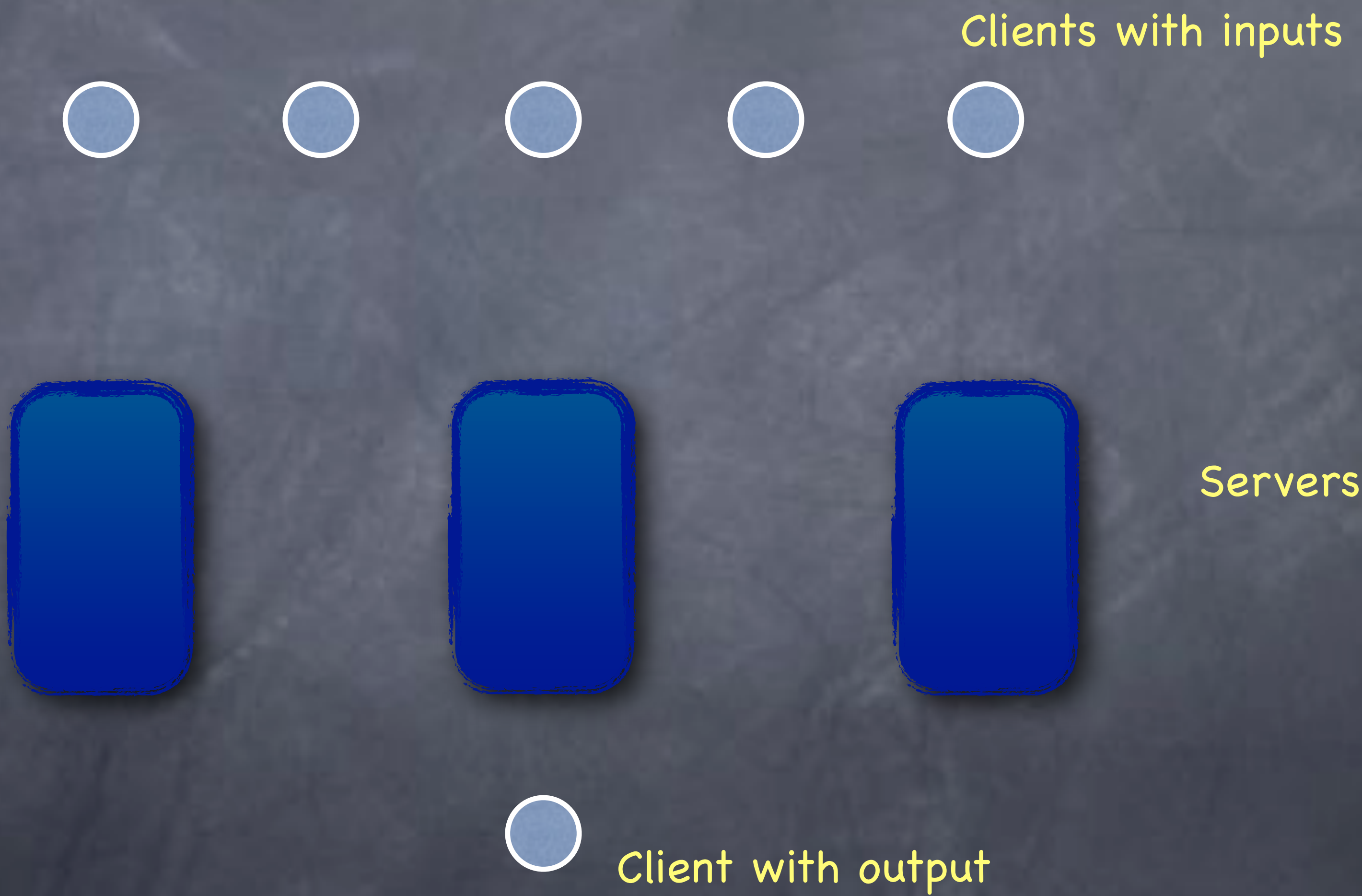

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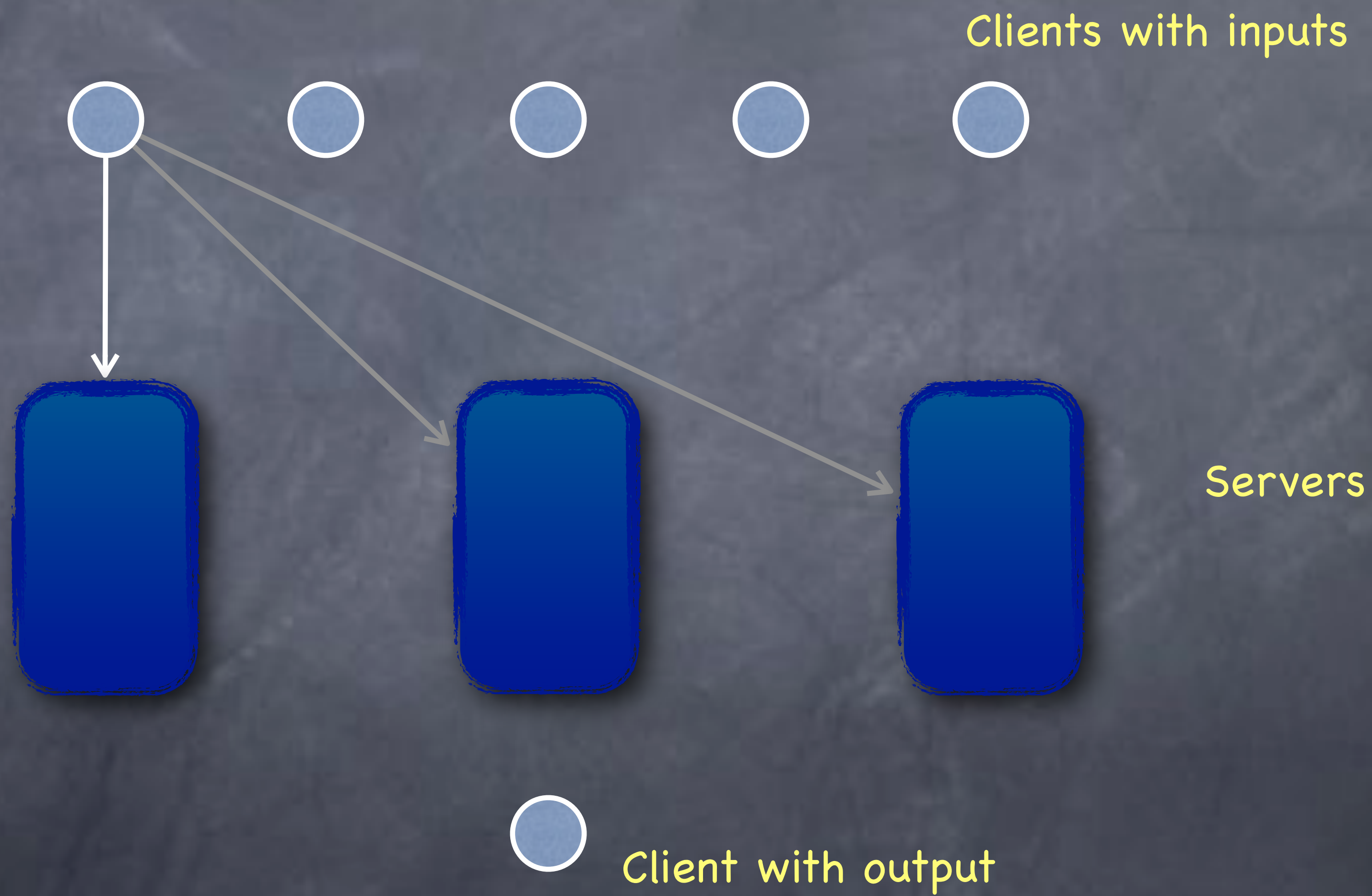

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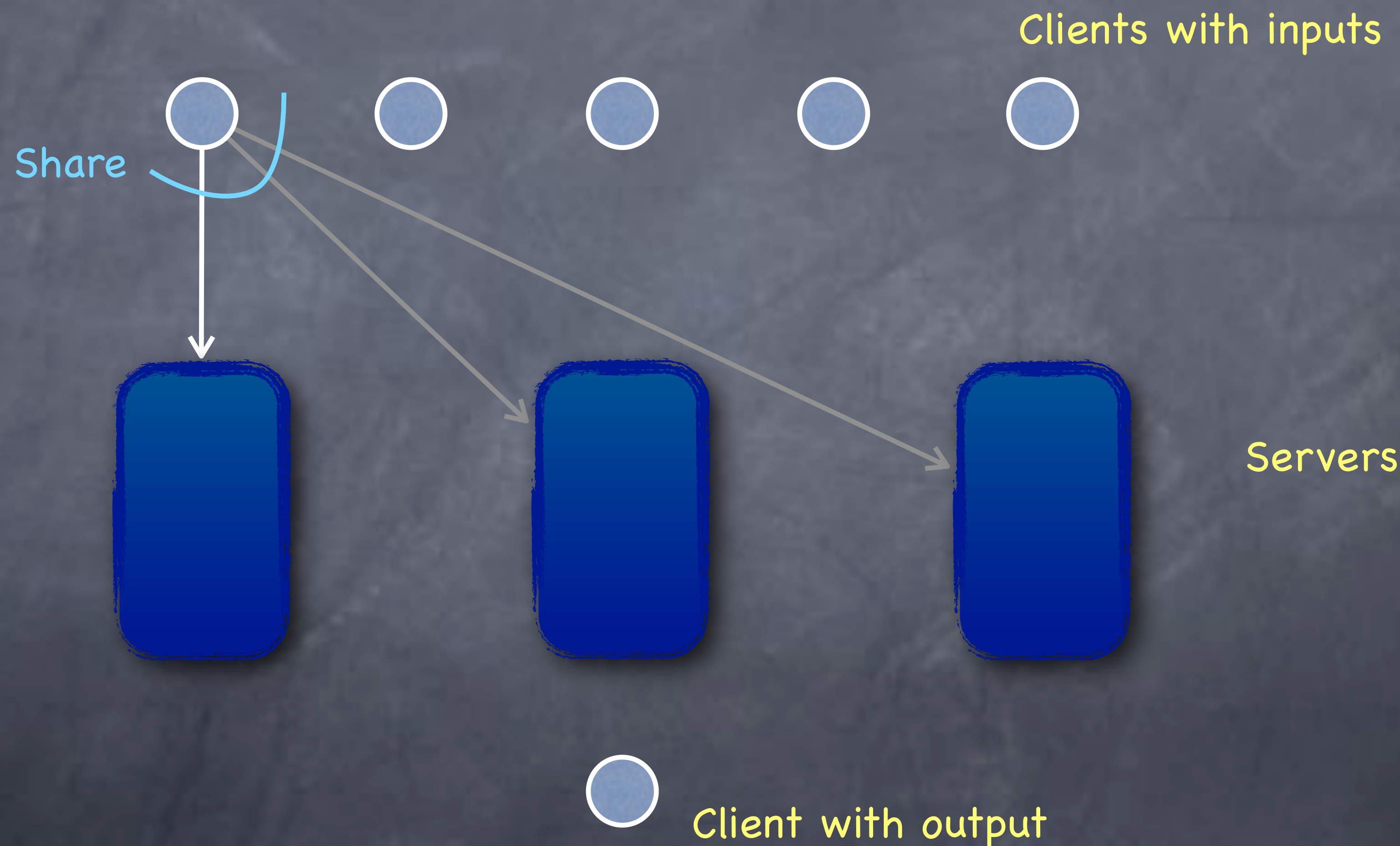

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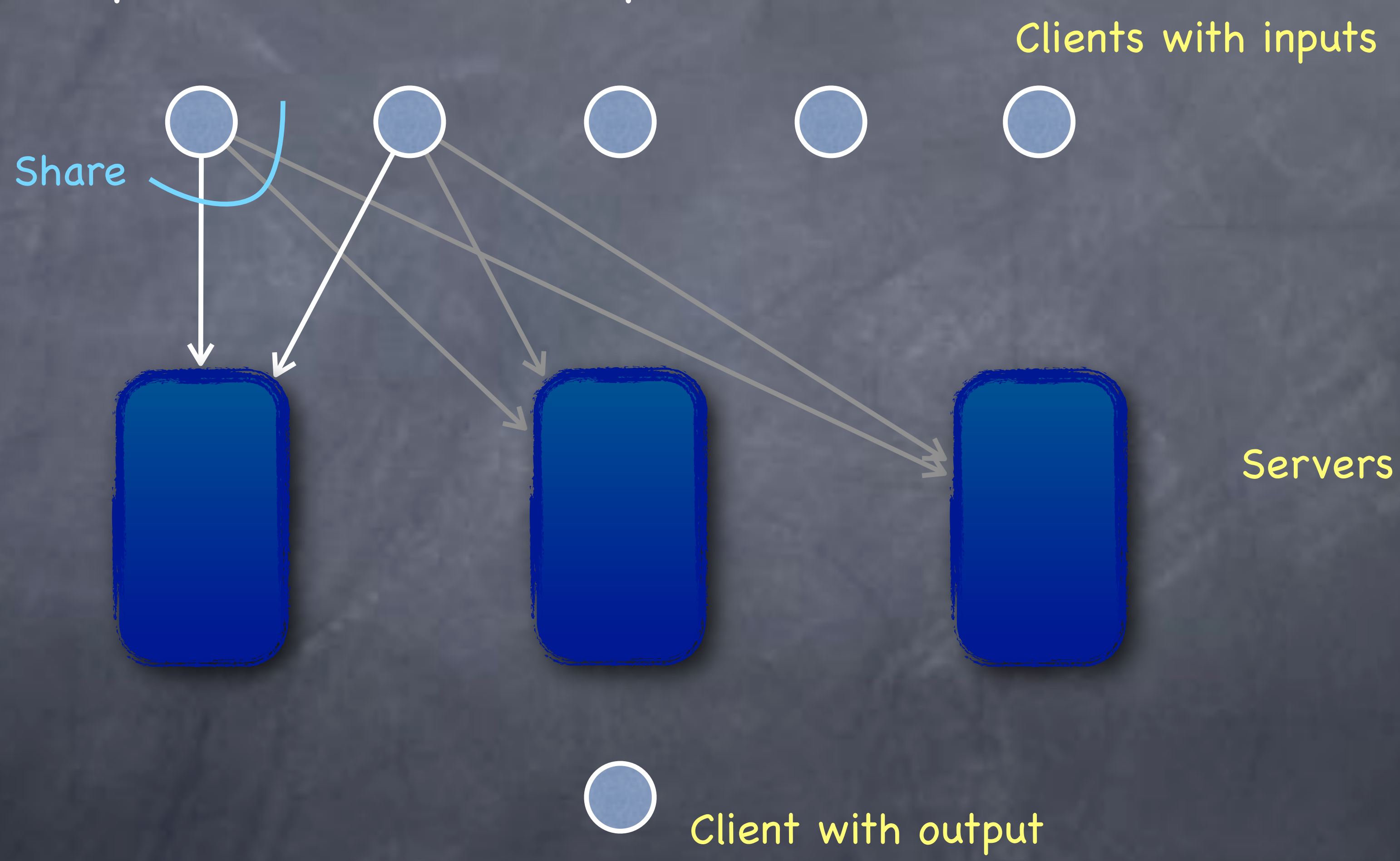

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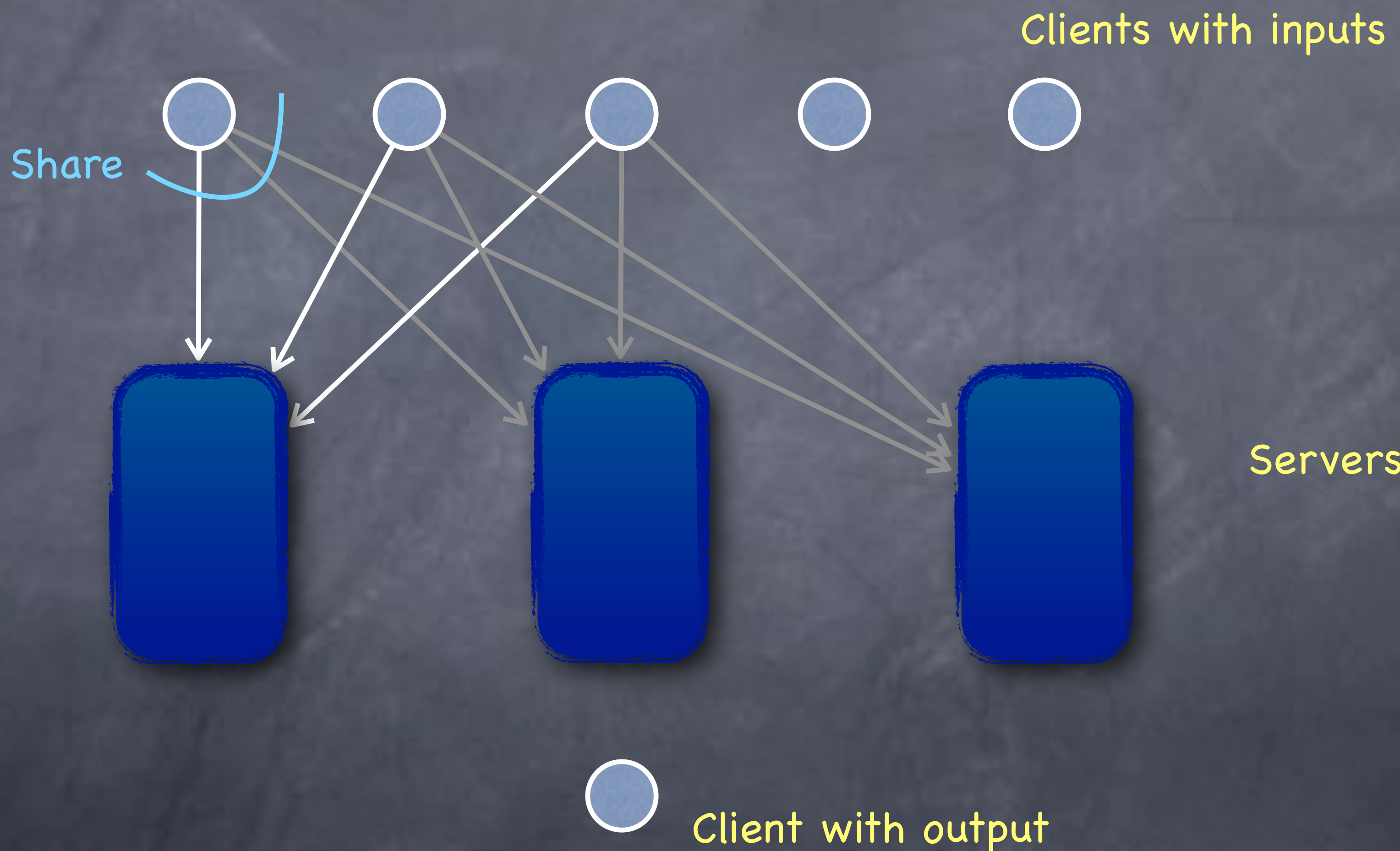

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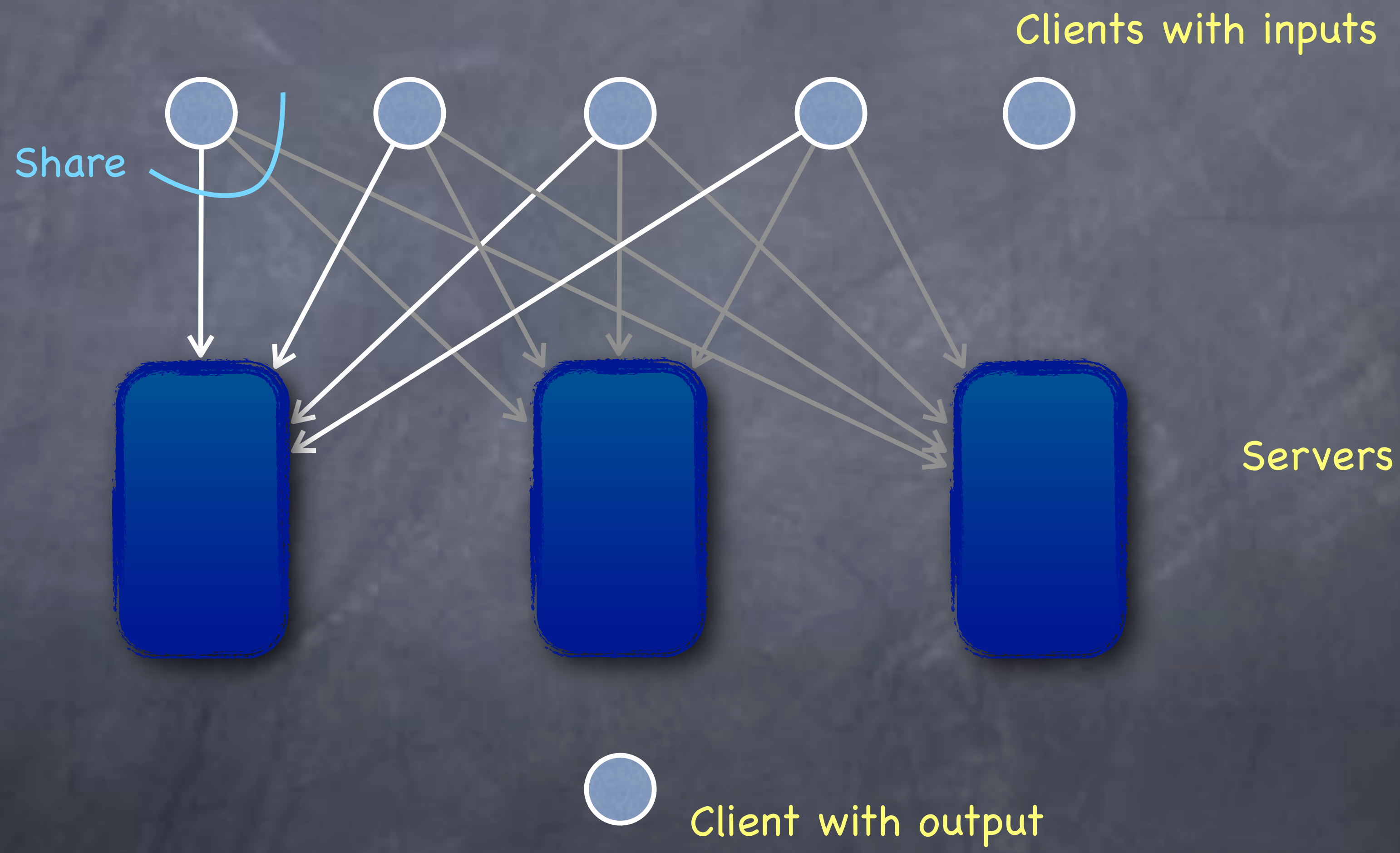

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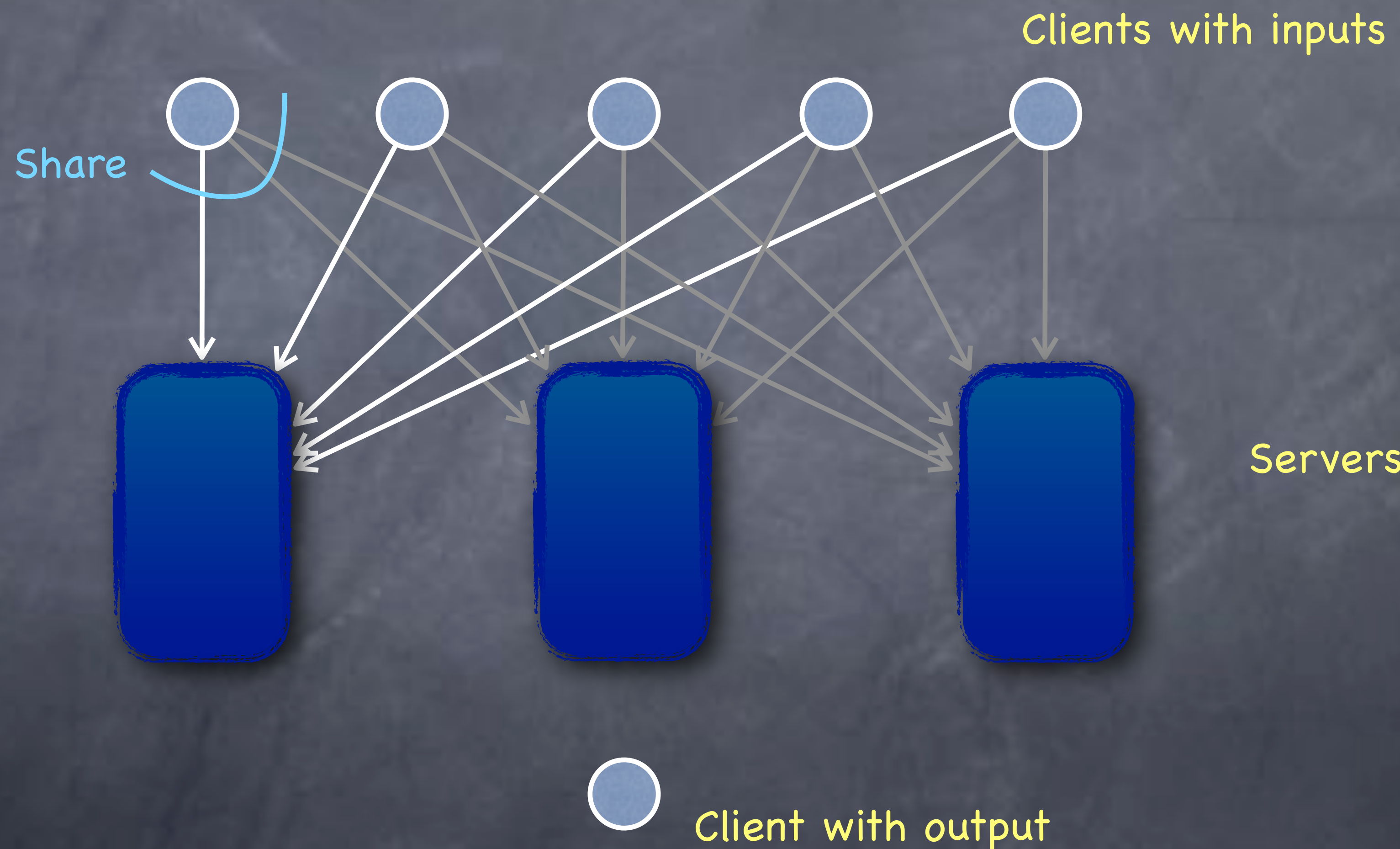

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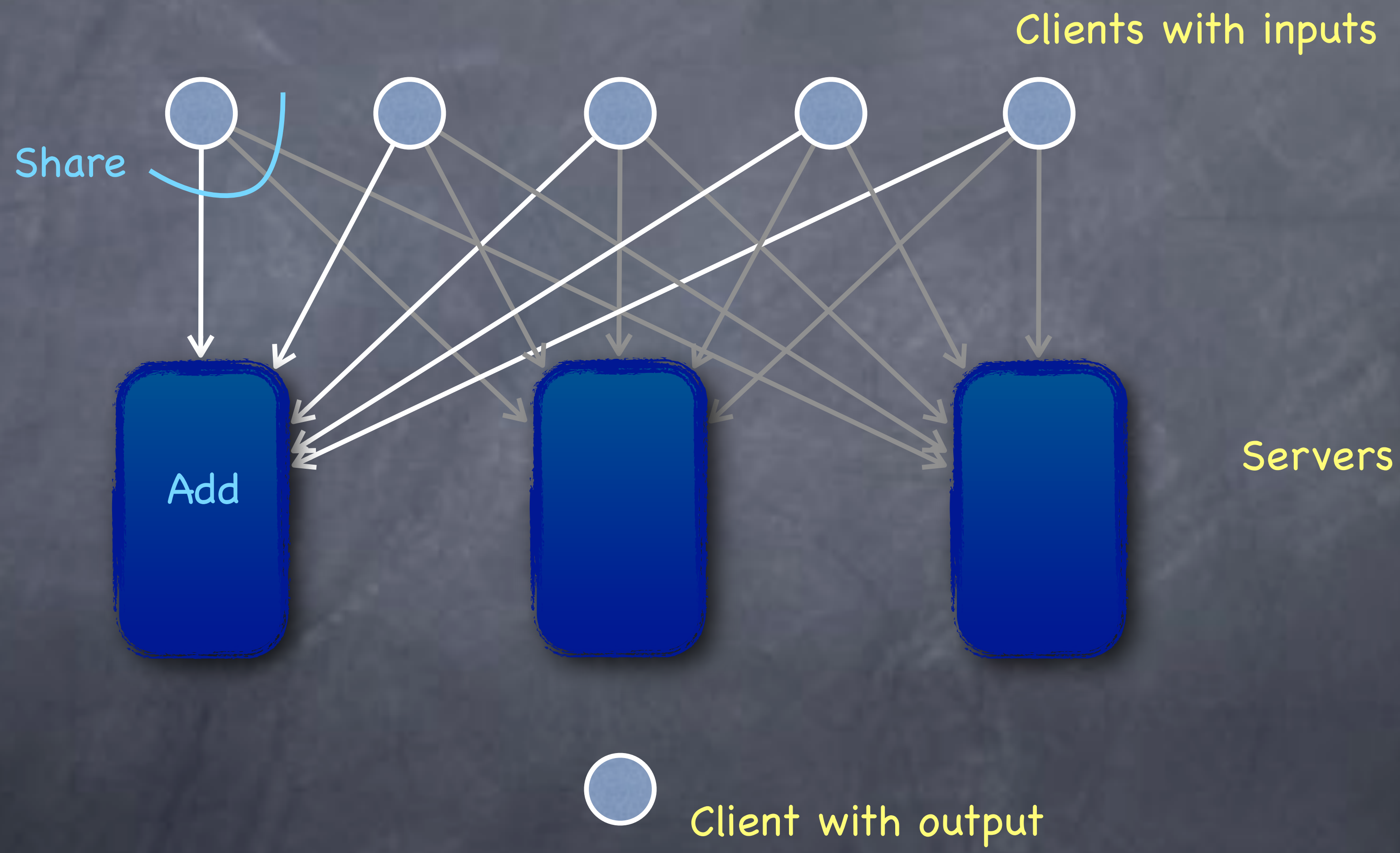

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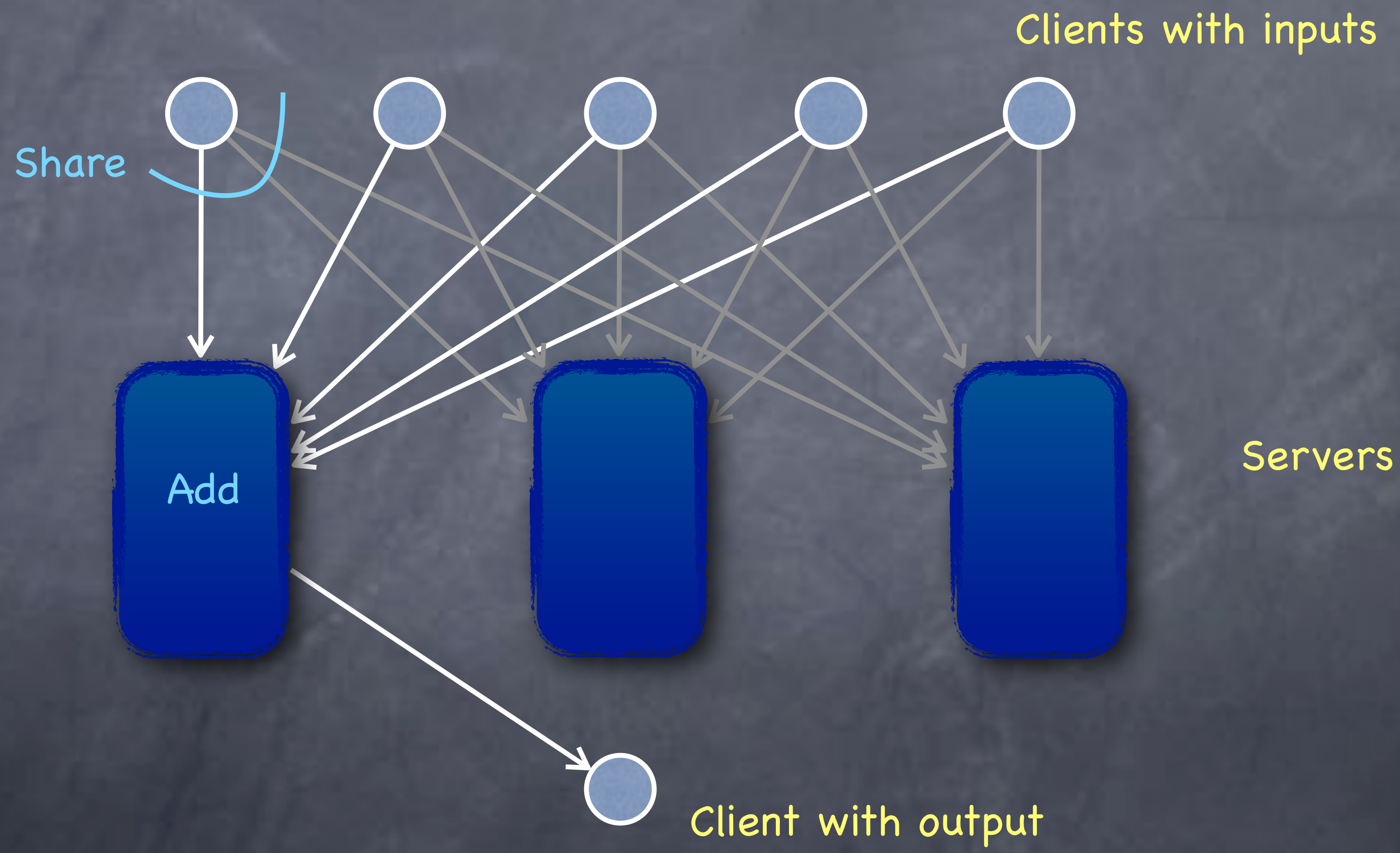

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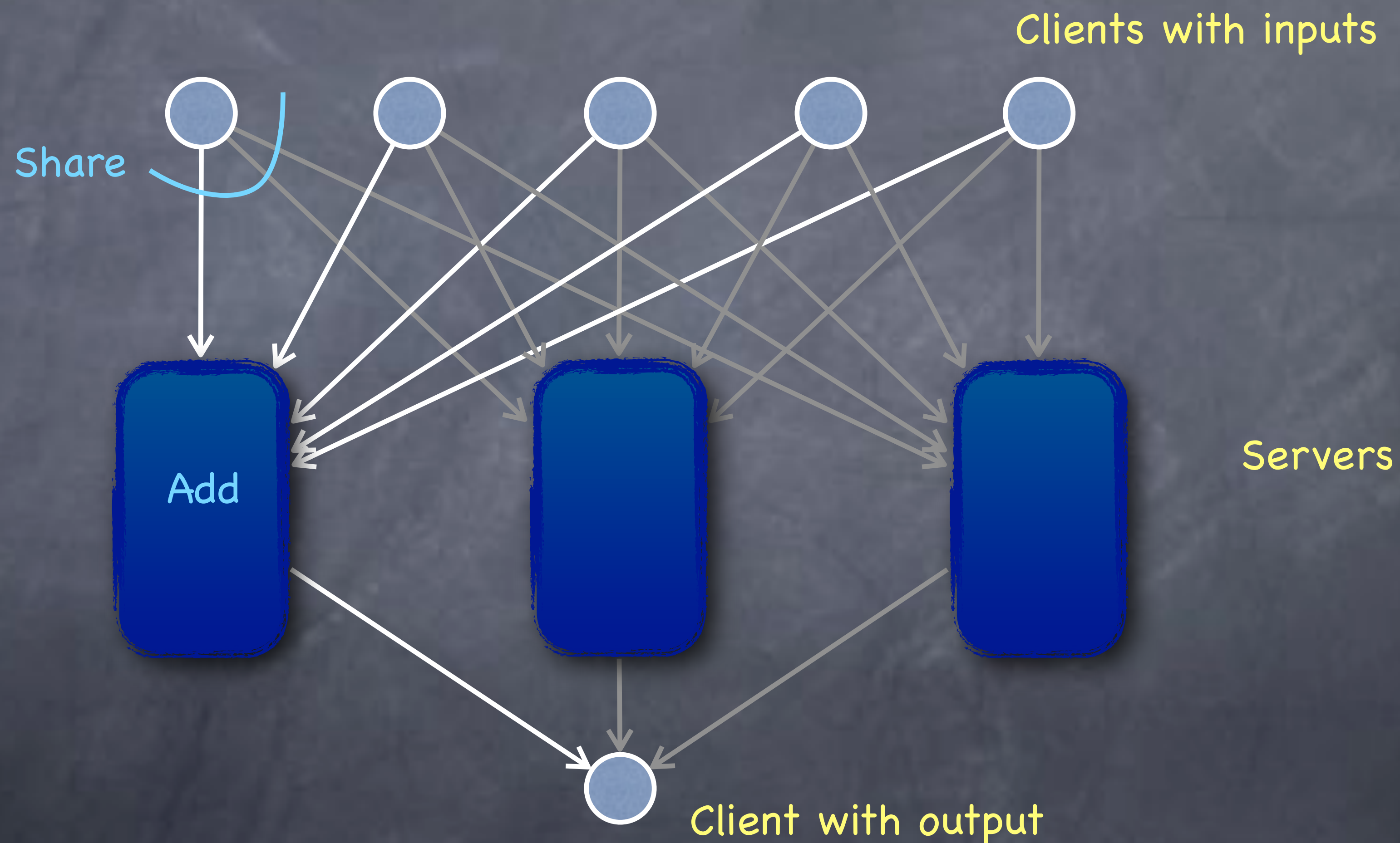

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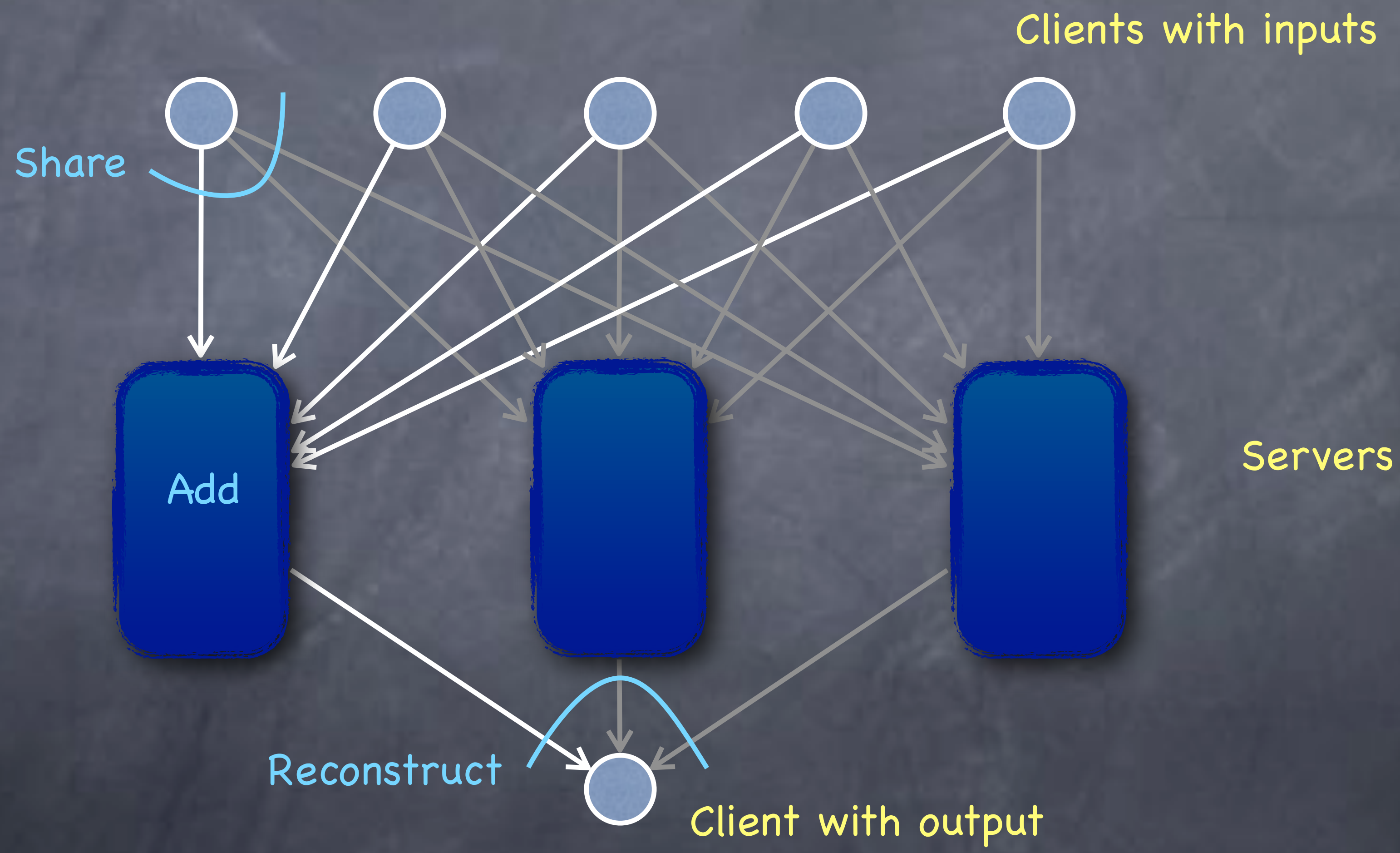

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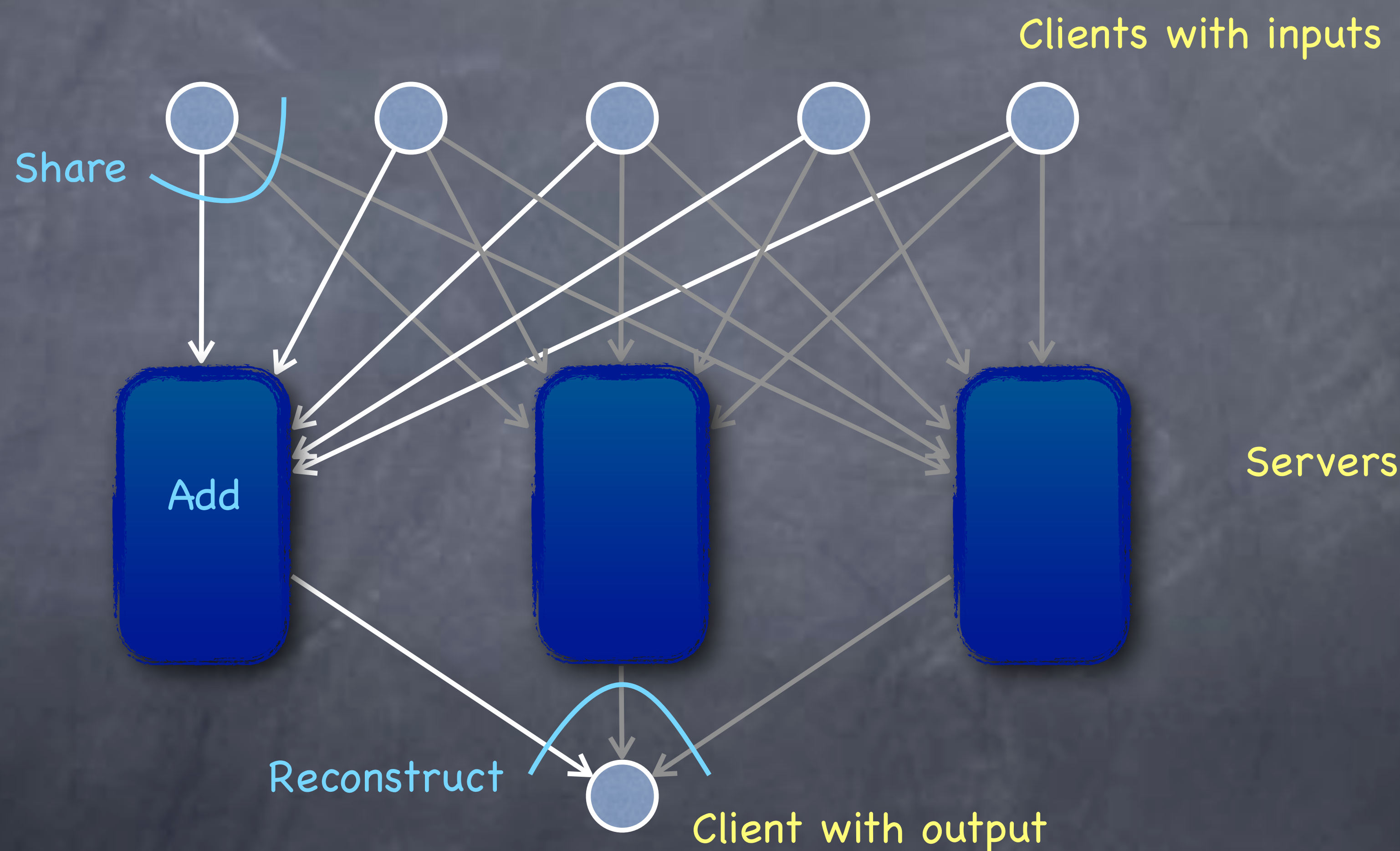

- Secure against passive corruption (no set of parties learn more than what they must) if at least one server is uncorrupted

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    - Otherwise malicious players can cause denial-of-service

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