Signatures

CS/ECE 598MAN: Applied Cryptography

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Digital Signature Properties

Signature Properties

- Authentication
- Third-party verifiability
- Non-repudiation

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- Third-party verifiability
- Non-repudiation

But what if we don't want all of these properties?

Crypto Real Estate

Alice Bank

Bob

I'd like a mortgage

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Alice Bank	Bob
	I'd like a mortgage
You're pre-approved for $100,000	

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$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $100,000")	

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Alice Bank	Bob	Charlie Financial
	I'd like a mortgage Can I get that in writing?	
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$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $100,000")		
	Charlie, can you do better?	

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$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $100,000")	Charlie, can you do better?	$\mathrm{Sign}_{\mathrm{Charlie}}$("Bob is pre-approved $100,001")

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$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $100,000")		$\mathrm{Sign}_{\mathrm{Charlie}}$("Bob is pre-approved $100,001")
	Charlie, can you do better?	
	Alice, can you beat that?	

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	I'd like a mortgage Can I get that in writing?	
You're pre-approved for $100,000		
$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $100,000")		$\mathrm{Sign}_{\mathrm{Charlie}}$("Bob is pre-approved $100,001")
	Charlie, can you do better?	
$\mathrm{Sign}_{\mathrm{Alice}}$("... $100,002")	Alice, can you beat that?	

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	Charlie, can you do better?	
$\mathrm{Sign}_{\mathrm{Alice}}$("... $100,002")		
	Alice, can you beat that?	
$\mathrm{Sign}_{\mathrm{Alice}}$("Bob is pre-approved for $1,000,000")		
	...	

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	Charlie, can you do better?	
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	...	
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How can Bob and Charlie prevent this?

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	Is Bob pre-approved for a mortgage?	

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I'm not telling you!		

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But on closing day, Alice changes her mind and disavows any promises to David or Bob!

Undeniable Signatures

Undeniable signatures:

- Can only be verified with help of signer
- Cannot later be disavowed

Other applications: resisting blackmail.

Construction

[Chaum,van Antwerpen, CRYPTO'89; Chaum, EUROCRYPT'90]

Setup

Generate private key $x \in \mathbb{Z}_q^*$, public key $y = g^x \bmod p$, where g is a generator of the group of order q in $\mathbb{Z}_p$.
A signature on a message m is $m^x \bmod p$

Construction

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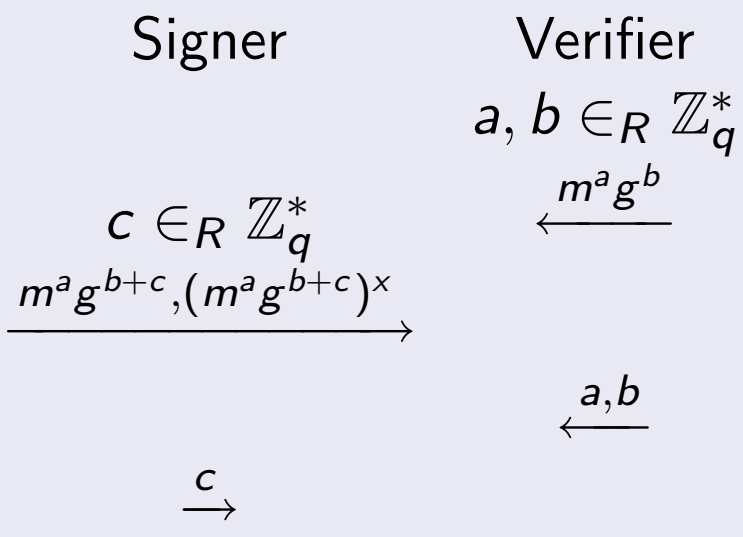
Verification

Verifier has z, wants to check whether $z = m^x \bmod p$



Note: $(m^a g^{b+c})^x = m^{ax} g^{(b+c)x} = z^a y^{b+c}$.

Construction

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Disavowal

Signer proves that $z \neq m^x \bmod p$

Signer		Verifier
		$a \in_R \mathbb{Z}_q, s \in_R \mathbb{Z}_k$
	$\xleftarrow{m^s g^a, z^s g^{xa}}$	
$s' = (m^s g^a)^x / (z^s g^{xa})$		
$s = \log_{(m/z)^x} s'$		
$\xrightarrow{\text{Commit}(s,r)}$		
	$\xleftarrow{a}$	
$\xrightarrow{r}$		

Note: k must be small enough for a brute-force search. Can repeat protocol l times to get $1/(k+1)^l$ chance of cheating.

Man-in-the-middle Attack

Bob will want to verify Alice's protocol himself.
Bob can *relay* messages to Charlie, letting Charlie pick the random numbers. Alice think she's talking with Bob. (Adaptation of [Desmedt et al., CRYPTO'87])

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Extensions:

- Bob can mix in his own randomness, so that both are convinced
- n people can combine randomness to simultaneously verify a message through a single interaction with Alice ([Desmedt,Yung, EUROCRYPT'91])
- Alice does not know which message she is verifying ([Jakobsson, EUROCRYPT'94])

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Need to tie verification to verifier!

Designated Verifier Undeniable Signatures

Recall: verify protocol is a ZK-proof that $z = m^x$.

$$PK\{(\xi) : y_{\text{Alice}} = g^{\xi} \wedge z = m^{\xi}\}$$

A designated verifier proof adds another clause to the proof:

$$PK\{(\xi, \xi') : (y_{\text{Alice}} = g^{\xi} \wedge z = m^{\xi}) \vee (y_{\text{Bob}} = g^{\xi'})\}$$

Bob can always build such a proof, so he cannot convince anyone else.

Trap-door Commitments

A trap-door commitment is defined over a public/private key pair. Given only public key, the commitment scheme is secure, but it can be broken given the secret key.

Syntax

- $\text{Gen}(1^k)$: Generate (sk, pk)
- $\text{Commit}(pk, w, r)$: Generate a commitment c to w
- $\text{Trapdoor}(pk, sk, w, r, w')$: Generate r' such that $\text{Commit}(pk, w, r) = \text{Commit}(pk, w', r')$

Example

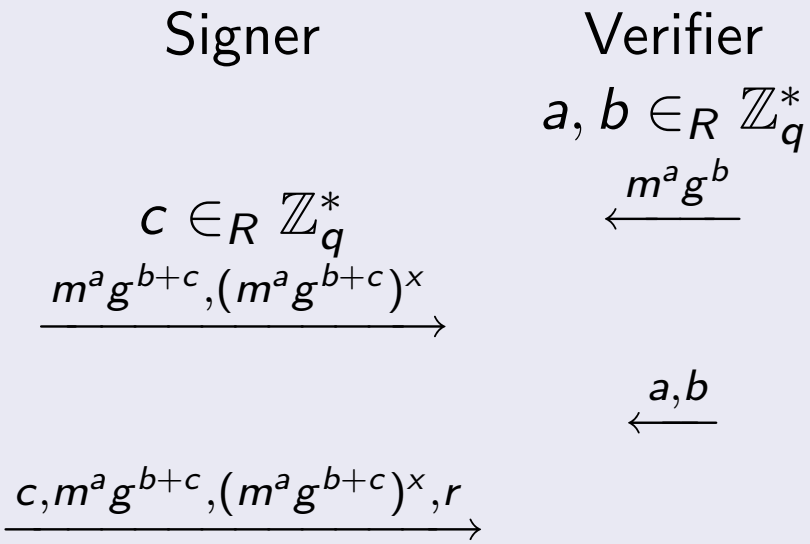
[Brassard,Chaum,Crépeau, JCSS'88]

- Gen(1^k): Generate an ElGamal public private keypair $(x, y = g^x \bmod p)$ (where g is the generator of the subgroup of order q in $\mathbb{Z}_p$).
- Commit$(y, w, r) = g^w y^r \bmod p$
- Trapdoor$(x, y, w, r, w') = (w - w')/x + r \bmod q$

$$g^{w'} y^{(w-w')/x+r} = g^{w'+(w-w')+xr} = g^w g^{xr} = g^w y^r \bmod p$$

Designated Verifier Protocol
[Jakobsson et al.,EUROCRYPT'96]

Protocol



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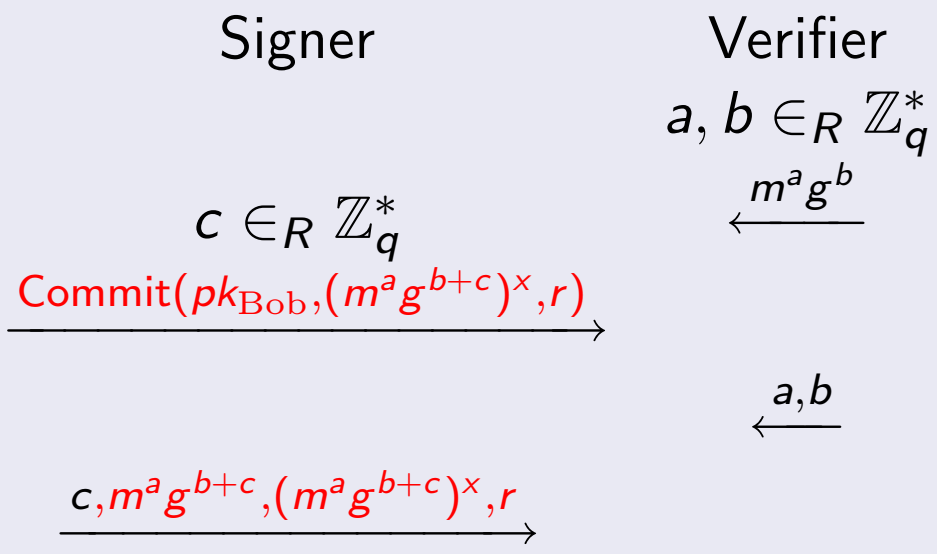

Alice uses a trapdoor commitment that Bob can break, and only opens it after seeing the challenges a, b.

Designated Verifier Non-interactive Proofs

Can use the same trick with non-interactive proofs.

- Three-move proof: (Commit,Challenge,Response) $= (c, a, r)$
- Fiat-Shamir: $a' = H(c)$
- Designated-Verifier: $a' = H(\mathrm{TD-Commit}(pk_{\mathrm{Bob}}, c, s))$, $r' = (r, s)$

Bob can break the commitment and generate new proof
Note: can be used to generated designated-verifier signatures

How to Leak a Secret
[Rivest et al., ASIACRYPT'01]

Problem

Cabinet member wants to leak a story to a journalist. But journalist's editor must verify the source of the leak, and cabinet member does not want the editor to know his name.

Cannot solve this with DV signatures (why not?)

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Ring Signatures

A ring signature on a message m shows that one of a set ("ring") of members has signed a message, but not which one.

Construction

Consider simple RSA:

RSA

$$\text{Sign}(m, n, d) = H(m)^d \bmod n$$

$$\text{Verify}(m, \sigma, n, e) : H(m) \stackrel{?}{=} \sigma^e \bmod n$$

Ring signature

Set of public keys (n_i, e_i) and private keys d_i for $i = 1, \ldots k$.
Assume wolog that the first person is actually signing.

1. Pick random values $\sigma_2, \ldots, \sigma_k \in \mathbb{Z}_n$
2. Compute $x_i = \sigma_i^{e_i} \bmod n_i$
3. Let $x_1 = H(m) \oplus x_2 \oplus \cdots x_k$
4. Compute $\sigma_1 = x_1^{d_1} \bmod n_1$
5. Signature: $(\sigma_1, \ldots, \sigma_n)$

Note

- A ring signature with ring $\{\mathrm{Alice}, \mathrm{Bob}\}$ is similar to a signature by Alice designated for Bob.
- Indeed, $\mathrm{Commit}(pk, c, r) = E_{pk}(c) \oplus E_{pk}(r)$ is a trapdoor commitment scheme.

Mesh Signatures
[Boyen, EUROCRYPT'07]

- Ring signatures prove that "Alice signed m" OR "Bob signed m" OR ...
- Mesh signatures extend this to more complicated clauses

Definition

Mesh Signatures

$$
\begin{aligned}
E ::= & [VK : M] \\
& | E_1 \vee E_2 \\
& | E_1 \wedge E_2 \\
& | \geq_t \{E_1, \ldots, E_m\} \qquad (t \text{ out of } m \text{ threshold})
\end{aligned}
$$

Mesh Signature Examples

- Ring signatures:

$$[VK_1 : M] \vee [VK_2 : M] \vee \ldots [VK_k : M]$$

- Messages can be different:

$$[VK_1 : M_1] \vee [VK_2 : M_2]$$

- Threshold

$$2\text{-out-of-}3\{[CEO : memo], [CFO : memo], [COO : memo]\}$$

- Certificate chains

$$[VK_1 : M_1] \vee ([VK_{CA} : VK_2] \wedge [VK_2 : M_2])$$

Group Signatures

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- Ring signatures provide absolute anonymity

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Definition

Ring Signatures

- $\text{Gen}(1^k) : (gpk, gmsk, gsk[i])$. $gmsk$ is master secret key, $gsk[i]$ is a secret key assigned to each member of group.
- $\text{Sign}(gpk, gsk[i], M)$: Produce a signature σ on M using one of the group keys
- $\text{Verify}(gpk, M, \sigma)$: Verify the signature, given a public key
- $\text{Open}(gpk, gsk, M, \sigma)$: Trace a signature to member i