Voting

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Introduction

markets vs. voting (similarities and distinctions)

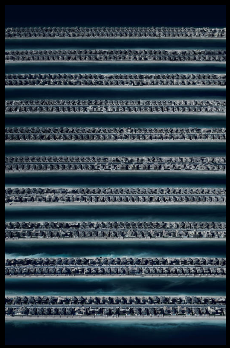
complete information (film critics) vs. genuine uncertainty (jury)

How should we produce a **single** ranking from the conflicting opinions provided by multiple voters?

Is some version of majority voting a **good** mechanism? Is there a better one?

And ultimately, what does it even mean for a voting system to be **good**?

Preferences



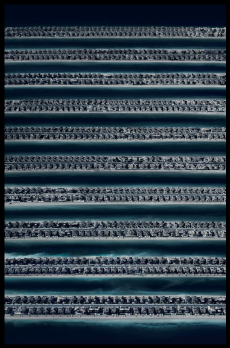
A group of people is evaluating a **finite** set of possible alternatives

These alternatives could correspond to political candidates, possible verdicts in a trial, amounts of money to spend on national defense, nominees for an award, or any other set of options in a decision.

The people involved wish to produce a **single group ranking** that orders the alternatives from best to worst and that in some sense reflects the collective opinion of the group.

$$X \succ_i Y$$

Individual i prefers X to Y




Completeness

Each individual has a preference between **any** pair of alternatives

for now, disallow ties and no preference cases

photos by Andreas Gursky

Jumeirah Palm

$\succ$

Shanghai

$\succ$

Dubai World

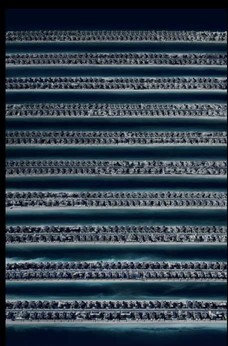
$\succ$

Baharain

Transitivity

If $X \succ_i Y$ and $Y \succ_i Z \implies X \succ_i Z$

Jumeirah Palm



$\succ$

Shanghai

$\succ$

Dubai World

$\succ$

Baharain

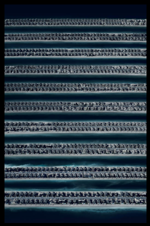
$\succ$

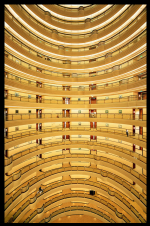
Shanghai

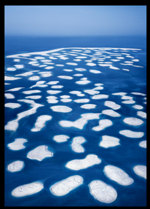
What might explain **intransitivity**?

we'll assume completeness and transitivity

A ranked list ≡ complete and transitive preference relation

1. 

2. 

3. 

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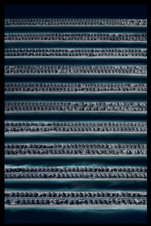
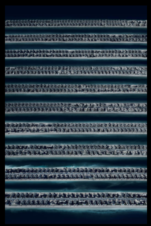
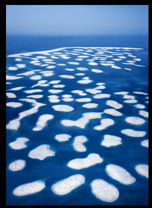
 $\succ$ 

 $\succ$ 

 $\succ$ 

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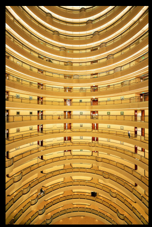
 $\succ$ 

A **voting** system

Any method that takes a collection of complete and transitive individual preference relations—or equivalently, a collection of individual rankings—and produces a group ranking.

Majority Rule

Let us assume an odd number of participants

Two alternatives

Things get remarkably **complicated** with more than two alternatives

Let's try by aggregating
pairwise preferences

For every X and Y

$X \succ Y$ or $Y \succ X$

that is, group pairwise preferences are **complete**

But, aggregate preferences may **violate transitivity** even if all individual preferences are transitive!

$$X \succ_1 Y \succ_1 Z$$

$$Y \succ_2 Z \succ_2 X$$

$$Z \succ_3 X \succ_3 Y$$

does the aggregate preserve transitivity?

Condorcet paradox

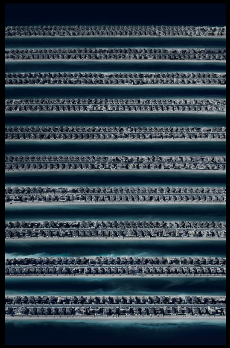
also arises in individual decisions!

here, the criteria function as individual voters

College	National Ranking	Average Class Size	Scholarship Money Offered
X	4	40	$3,000
Y	8	18	$1,000
Z	12	24	$8,000

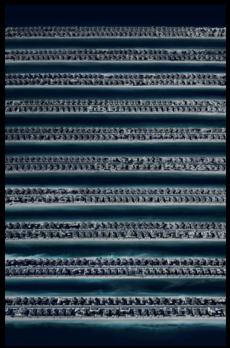
Figure 23.2. When a single individual is making decisions based on multiple criteria, the Condorcet Paradox can lead to nontransitive preferences. Here, if a college applicants wants a school with a high ranking, small average class size, and a large scholarship offer, it is possible for each option to be defeated by one of the others on a majority of the criteria.

why not **arrange** the alternatives in some order and eliminate?



susceptible to agenda setting

the order matters!



Positional Voting

rather than look at pairs, positional systems produce group ranking from the individual rankings

in this system, each alternative receives a weight based on its position

Borda count

used to determine Heisman trophy winners

If there are **k** alternatives, the weight to the top ranked preference is **k-1**, a weight of **k-2** to the second alternative and so on.

the total weight of each alternative is the sum of the weights from individual rankings

	3		2		1		0
Alice	A	$\succ_1$	B	$\succ_1$	C	$\succ_1$	D
	3		2		1		0
Bob	B	$\succ_2$	C	$\succ_2$	A	$\succ_2$	D
Final	B	$\succ$	A	$\succ$	C	$\succ$	D
	5		4		3		0

The Borda count **always** produces a complete, transitive ranking for a set of alternatives

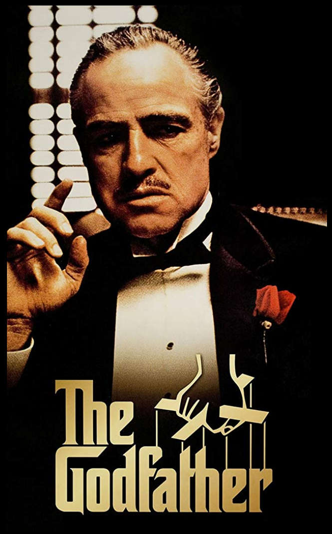
Pathologies in positional voting systems

Assume that a magazine asks a group of critics to identify "the greatest movie of all time," so that the magazine can run a story on that movie

The group of five critics start with Citizen Kane and The Godfather

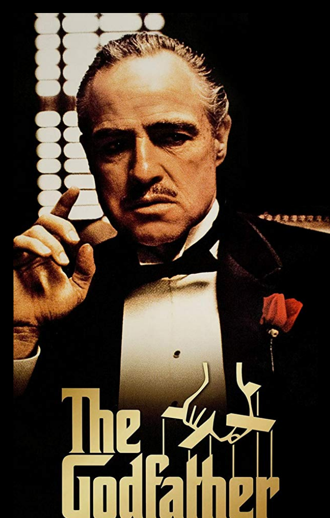
3

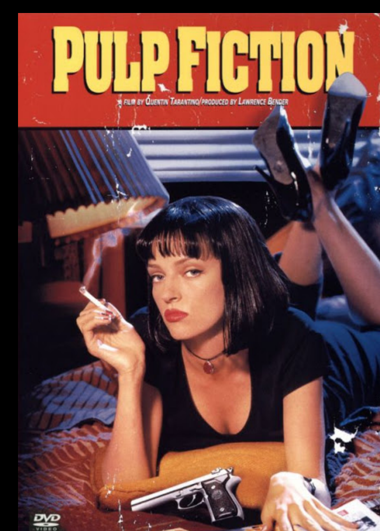
2



now the cretics feel that perhaps they should add a more modern option

they decide to add "Pulp Fiction"





three critics produce

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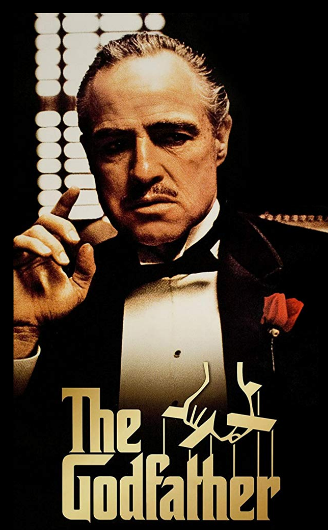

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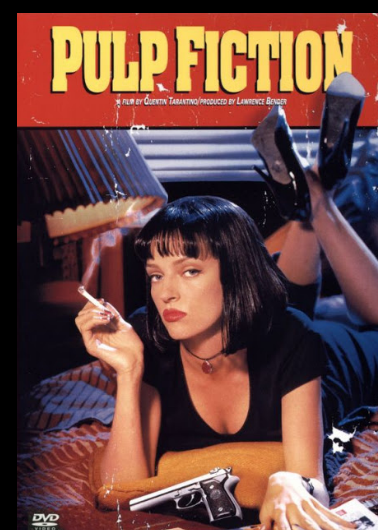
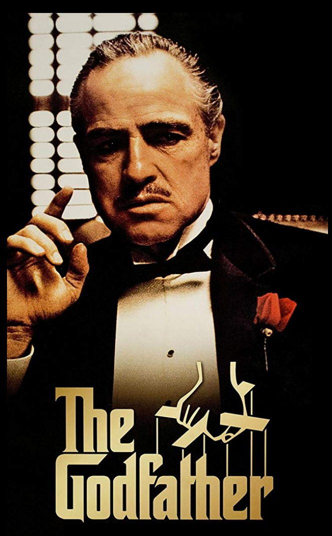
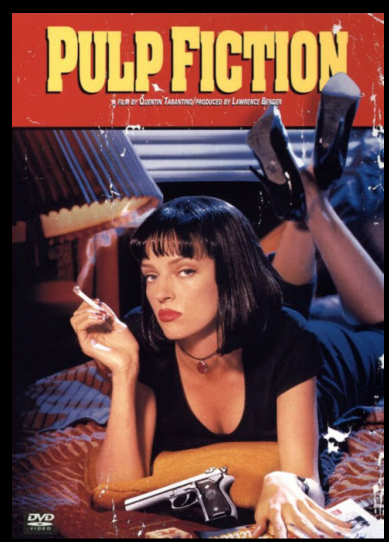


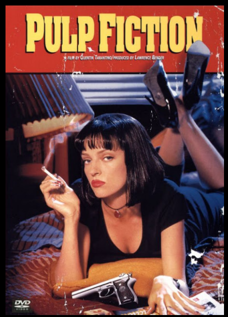
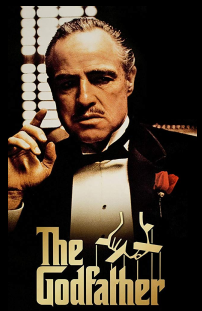
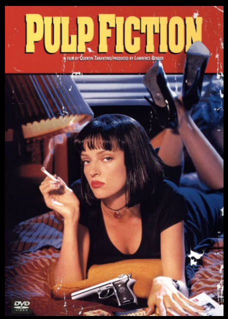
two critics produce



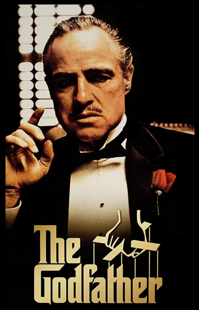
$\succ$



$\succ$

	2		1		0
Three critics		$\succ$		$\succ$	
	2		1		0
Two critics		$\succ$		$\succ$	

Group favorite:



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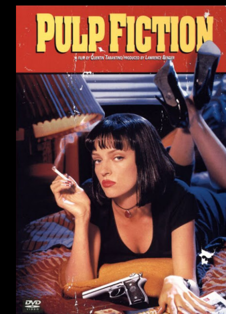
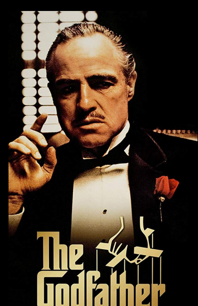
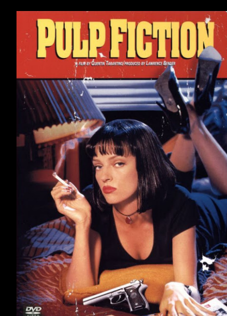
odd results?

without “Pulp fiction,” “Citizen Kane” wins

Pairwise, both “Citizen Kane” and “The Godfather” are preferred to “Pulp Fiction”

This seems to imply that strategic misreporting can change the outcome

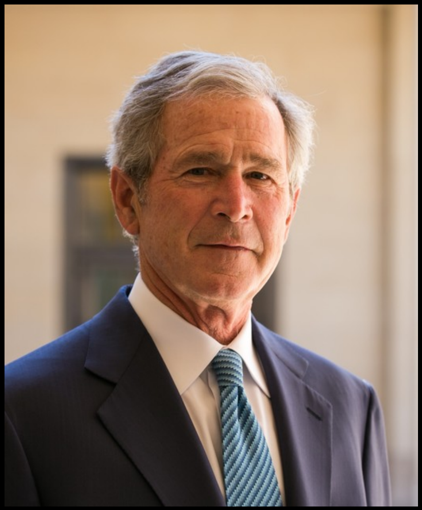
	2		1		0	
Three critics		$\succ$		$\succ$		
	2		1		0	
Two critics		$\succ$		$\succ$		under true reporting

Group favorite:

8

a choice that no one prefers changes the outcome!

This is an issue in presidential elections too



Arrow’s impossibility theorem

is there **any** voting system that produces a group ranking for **three** or more alternatives and avoids all of the pathologies we've seen thus far?

what would we like a voting system to satisfy?

First, if there is **any pair** of alternatives **X** and **Y** for which **X** $>_i$ **Y** in the rankings of **every** individual i, then the group ranking should also have X > Y

Unanimity

Second, we require that, for **each pair** of alternatives, the ordering of **X** and **Y** in the group ranking should depend only on how each individual ranks **X** and **Y** relative to each other.

With **two** alternatives, **majority voting** satisfies unanimity and IIA.

with **three** or more alternatives, **dictatorship** satisfies unanimity and IIA

If there are at least three alternatives, then there is no voting system that satisfies Unanimity, IIA, and Non-dictatorship.

Arrow's Impossibility theorem

Single peaked-preferences

Are there special cases when majority voting works with more than three alternatives?

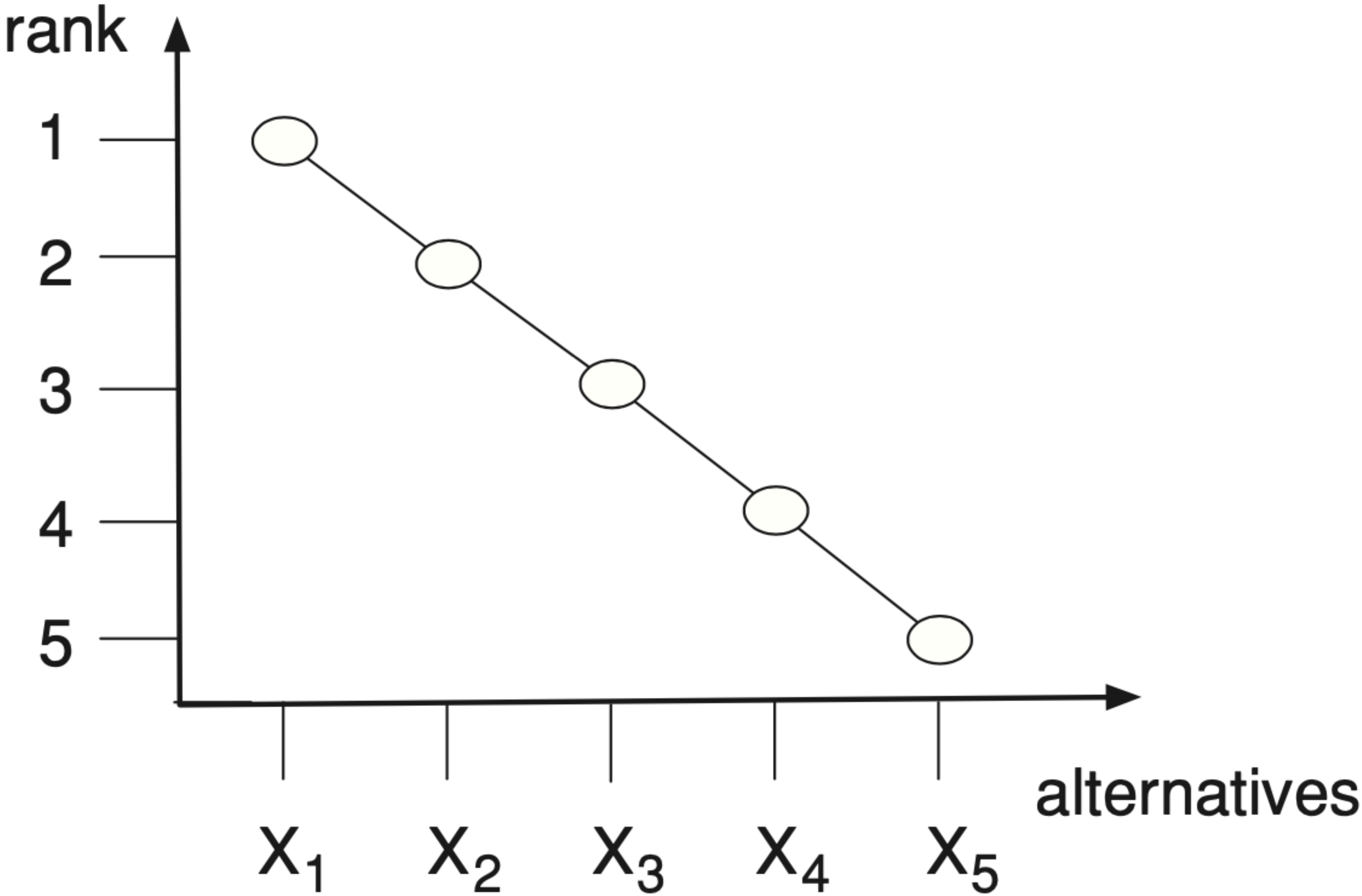
assume that the choices correspond to amounts of money to spend (say on education)

voter 1 $X \succ_1 Y \succ_1 Z$

voter 2 $Y \succ_2 Z \succ_2 X$

voter 3 $Z \succ_3 X \succ_3 Y$

voter 3 is a bit odd!


rank
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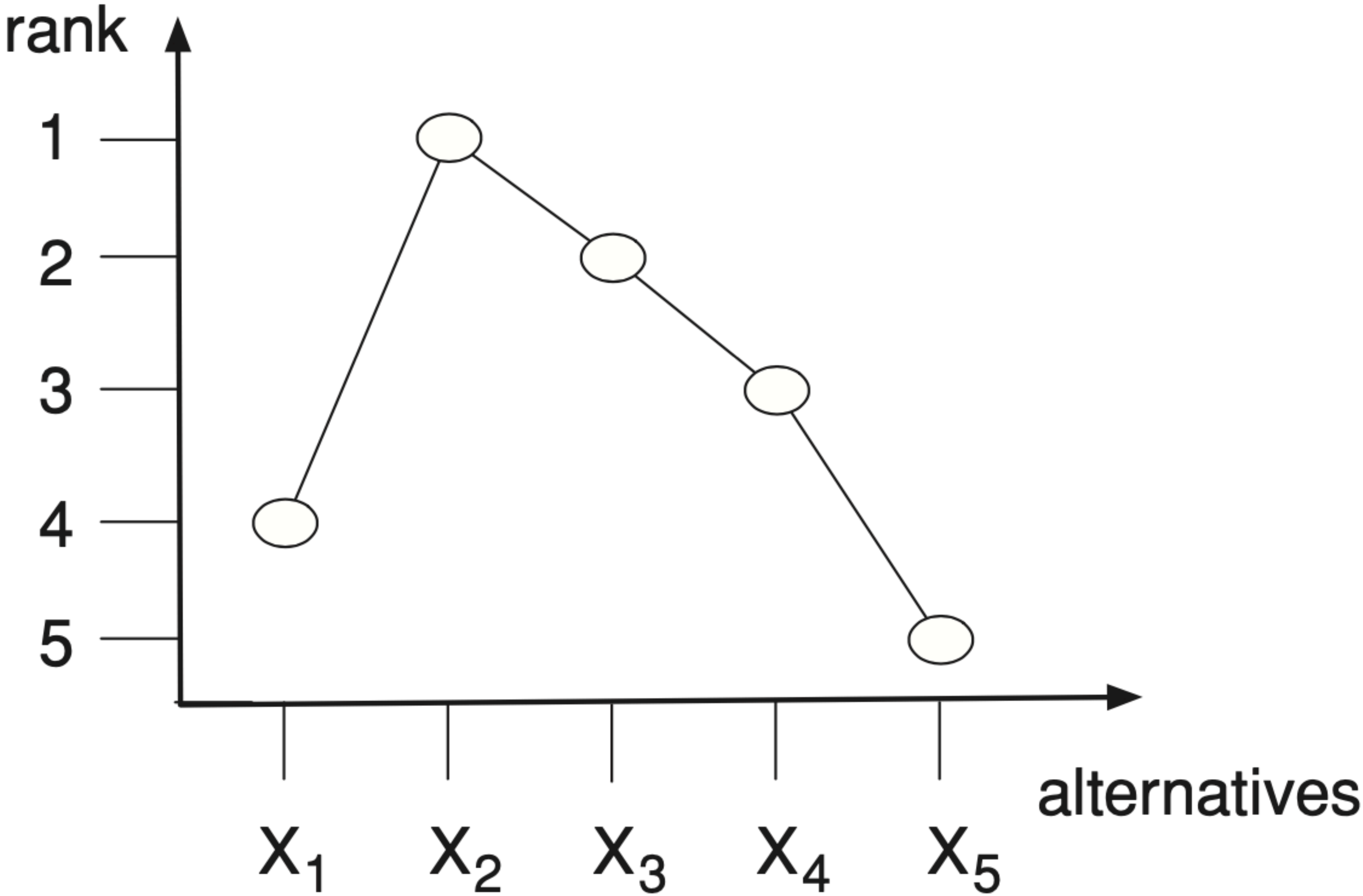
2
3
4
5
alternatives
x1
x2
x3
x4
x5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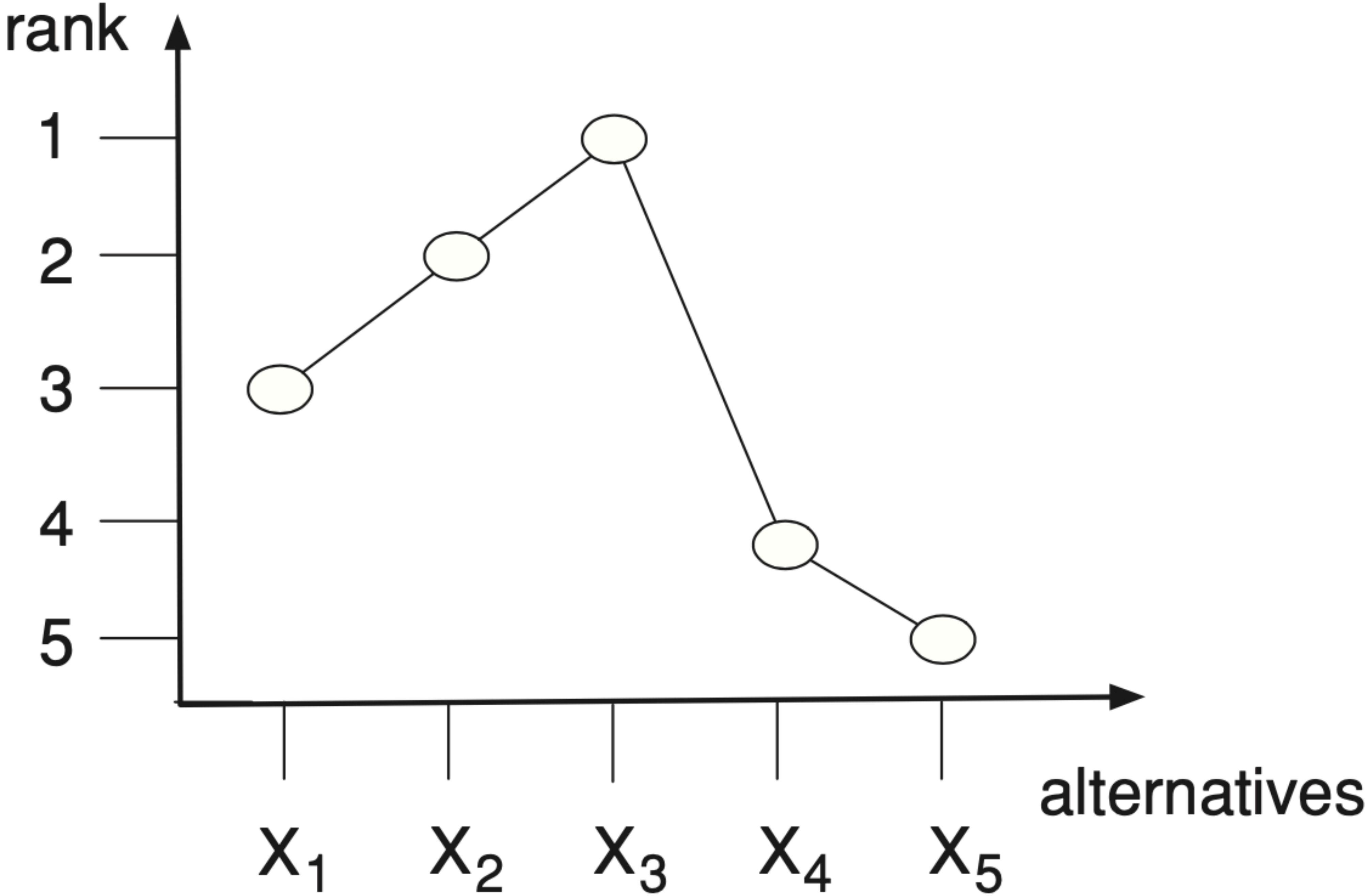
rank
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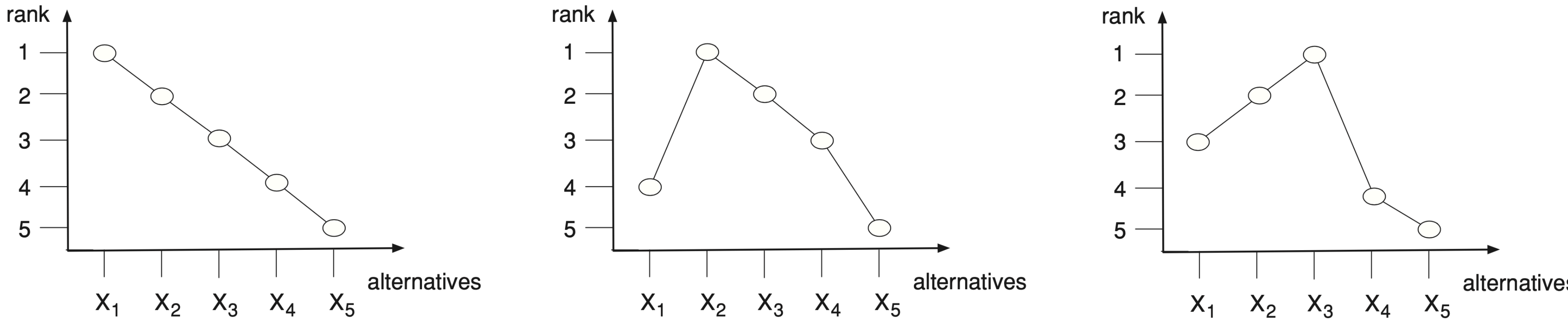
X_3
X_4
X_5



all three preferences are **singly** peaked: no case where **neighbors** are ranked above an option

If **all** individual rankings are single-peaked, then majority rule applied to all **pairs** of alternatives produces a group preference relation > that is **complete** and **transitive**.

With single-peaked rankings, the median individual favorite defeats every other alternative in a pairwise majority vote.

median voter theorem

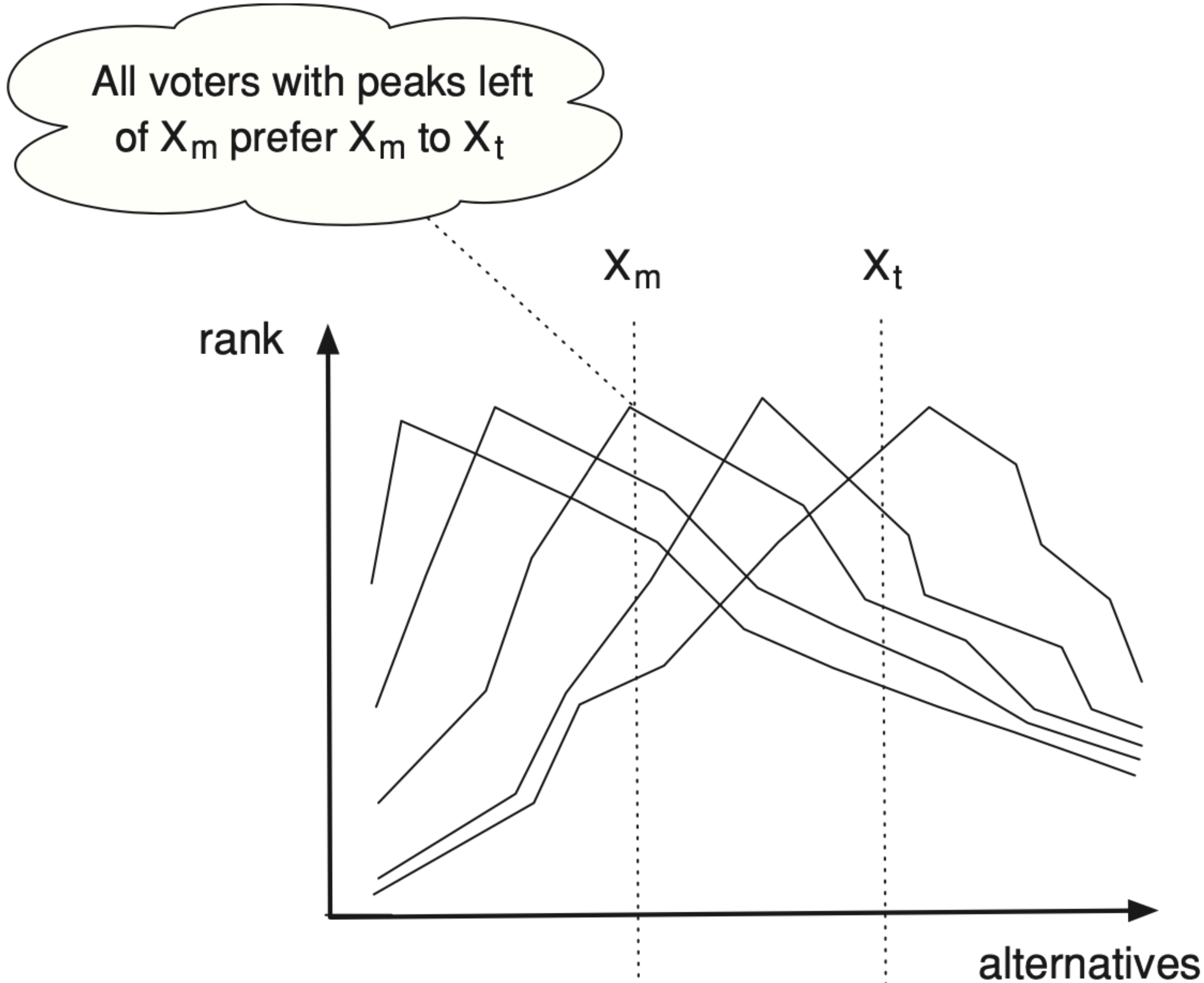


Figure 23.6. The proof that the median individual favorite X_m defeats every other alternative X_t in a pairwise majority vote: if X_t is to the right of X_m, then X_m is preferred by all voters whose peak is on X_m or to its left. (The symmetric argument applies when X_t is to the left of X_m.)

Ranked choice voting, quadratic voting

voting as information aggregation

what if the purpose of voting was to figure out true ranking?

Jury decisions

simultaneous, sincere decisions

assume equally likely priors

assume that everyone receives an **independent**, **private** signal

X-signal is observed

$$P(S_X \mid X = T) = q$$

$$P(S_Y \mid Y = T) = q$$

Y-signal is observed

$$q > \frac{1}{2}$$

X-signal is observed

$$P(X = T \mid S_X)$$

$$P(X = T \mid S_X) = \frac{P(S_X \mid X = T)P(X = T)}{P(S_X)}$$

$$P(S_X) = P(S_X \mid X = T)P(X = T) + P(S_X \mid Y = T)P(Y = T)$$

$$P(S_X) = q\frac{1}{2} + (1 - q)\frac{1}{2}$$

$$P(X = T \mid S_X) = \frac{P(S_X \mid X = T)P(X = T)}{P(S_X)}$$

↓

$$P(X = T \mid S_X) = q$$

↑

$$q > \frac{1}{2}$$

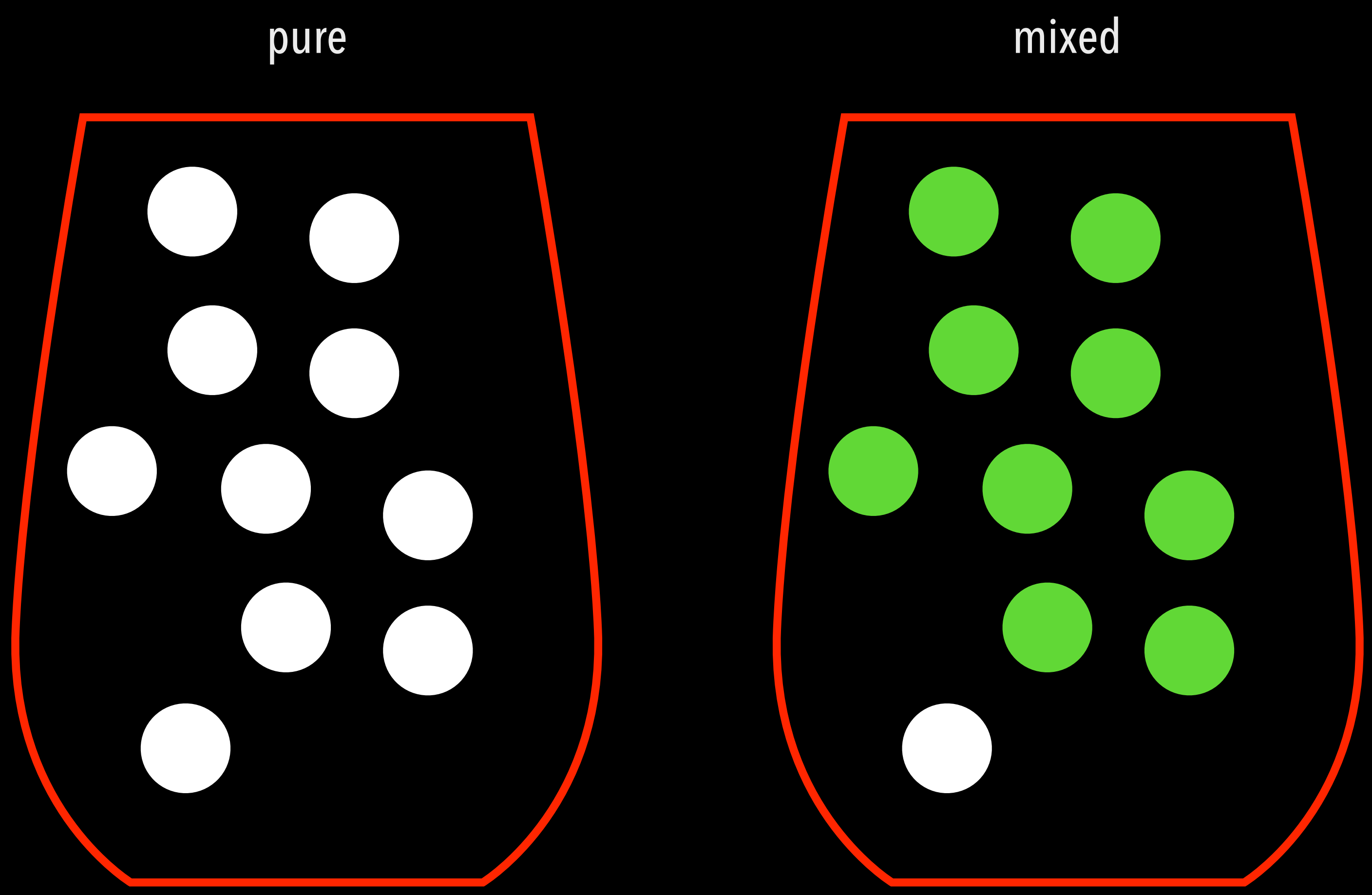
So what?

The probability that the voters will make the right decision goes to 1 as the number of voters becomes large.

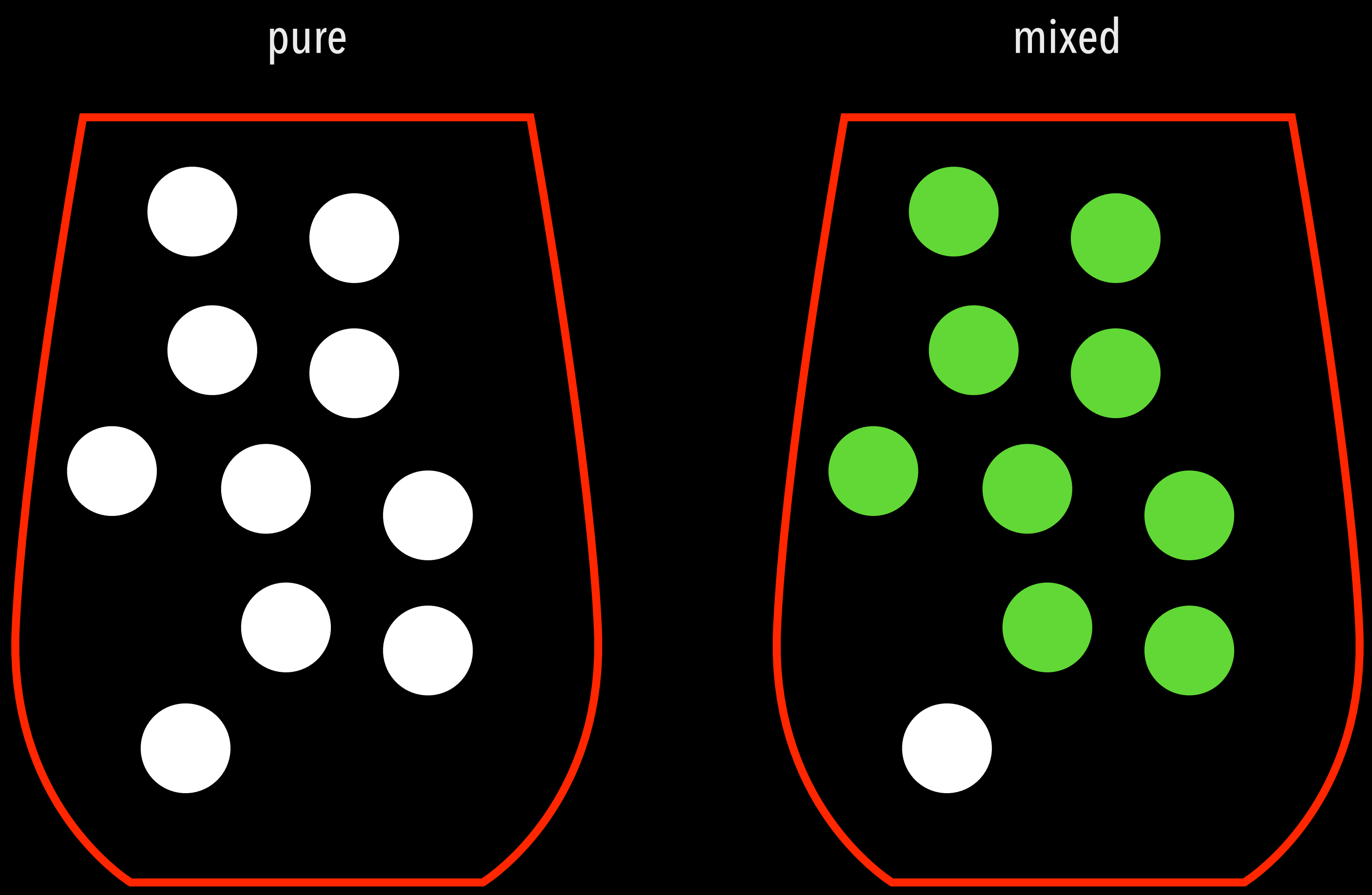
Insincere voting

Are there situations in which an individual should actually choose to vote **insincerely?**

That is, she favors the alternative she believes to be worse, **even though her goal is to maximize the probability that the group as a whole selects the best alternative.**



Imagine three people are asked to determine what kind of an urn they are picking balls from if a majority guesses correctly, all win a reward.



Imagine you pick a ball and it turns out to be white. What should you report?

Assume that you know that the other two will report sincerely.

What you report only matters when the other two disagree

Sincere reporting is not a Nash equilibrium!

a **key** methodological point in this analysis – the underlying principle in which you evaluate the consequences of your actions **only in the cases where they actually affect the outcome.**

insincere reporting is also an outcome in "the winner's curse"

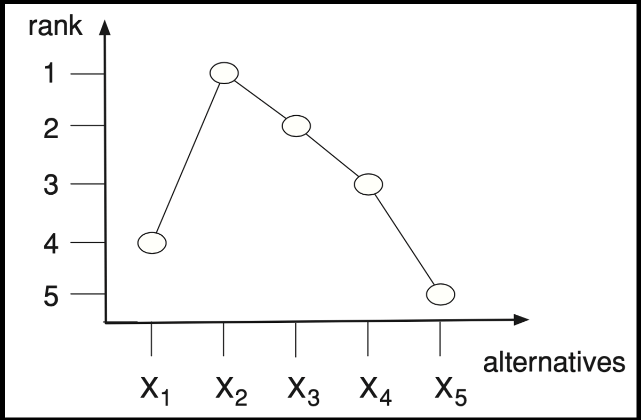
summary

$X \succ_i Y$

completeness transitivity

Condorcet paradox

Singly peaked preferences



Positional Systems

Borda Count

Arrow's theorem

Unanimity, IIA

Voting as information aggregation