Discussion Summary:

Our presentation covered the “Price of Anarchy” lecture by Tim Roughgarden. Basically, Roughgarden proves that when considering selfish routing, the worst case Price of Anarchy (POA) is in Pigou-like networks, so we are able to bound the POA. He also shows that the main consideration for inefficiencies in networks is the cost function rather than topology or other factors. Our summary will cover clarifications of the paper brought up during our presentation, as well as questions we discussed.

In terms of clarifications of the paper, we first reinforced the different conceptualizations of price of anarchy -- when discussing welfare, the ratio is the reciprocal of the one when discussing cost. For the purposes of our presentation, we mainly focused on the cost definition. Next, we had many clarifications about the behavior of agents on the routing networks being discussed. In particular, we clarified that the cost function defines the cost experienced by everyone on a particular edge. We further discussed how agents choose among paths: for example, in the network where a zero-cost edge is added to augment the network, we can reason that individuals on node v will choose to teleport for free rather than pick the edge with cost 1. Similarly, in a Pigou network with an exponential cost function, we saw that since x is always a fraction, unless x=1, the bottom edge is always close to 0, so that agents will always choose the lower-cost edge.

A number of details of Rougharden’s proof were also explained. In particular, Hari emphasized the point that in equilibrium, all paths with nonzero traffic must cost the same, because if this were not the case, agents would have reason to switch to alternate paths, and thus the network would not be in equilibrium. Furthermore, we then discussed why “freezing” the cost of the network edges at that equilibrium flow is important. This is because we need to show that the error term bound is manageable. We can see that no other flow can be better than the equilibrium flow when we freeze the cost at equilibrium.

One critique brought up during the discussion was that the paper assumes the only thing people look for in a route is a low driving time. It was pointed out that factors like routine and scenic value can also influence decision making. People may prioritize sticking to a habit over having the shortest driving time which could be a factor in the “cost”. There were some questions about whether this affected the “fairness” of routes and Hari mentioned that optimization problems with more attributes could lead to a better best case scenario. For example, if one driver prefers scenic routes and the other prioritizes arriving quickly, both can be given different routes that satisfy them, resulting in a “fairer” network. This conversation came from discussing whether people would stick to habits or utilize external information like highway signs or Google Maps. There were also some brief thoughts about foregrounding using systems like Google Maps for ways to help drivers estimate the “cost” from a strictly time standpoint, which relates a bit to tradeoff thinking in scarcity. This relates to another point brought up, which was that the system Roughgarden discusses assumes that agents have perfect information with which to make

choices, but in real life this could only be achieved by using things like Google Maps or highway signs to foreground both the current state of the network and the norms being employed.

Another question brought up was whether using fractional amounts of traffic to demonstrate the effect of non-linear cost functions on PoA made sense, since intuitively x^3 should be a worse cost function than x^2, and is only better since we were using fractional amounts of traffic. This is true, but we could construct a similar scenario with >1 units of traffic, where an increasingly nonlinear cost function would still lead to an increasing price of anarchy -- e.g. taking x^(1/p)

We ended our discussion with a question to consider for social networks: “How can we incentivize not spreading fake news?” There were a few suggestions for what was already done (social reputation, etc), but it could be worthwhile to continue thinking on this question.