The role of compatibility in the diffusion of technologies through social networks

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Overview

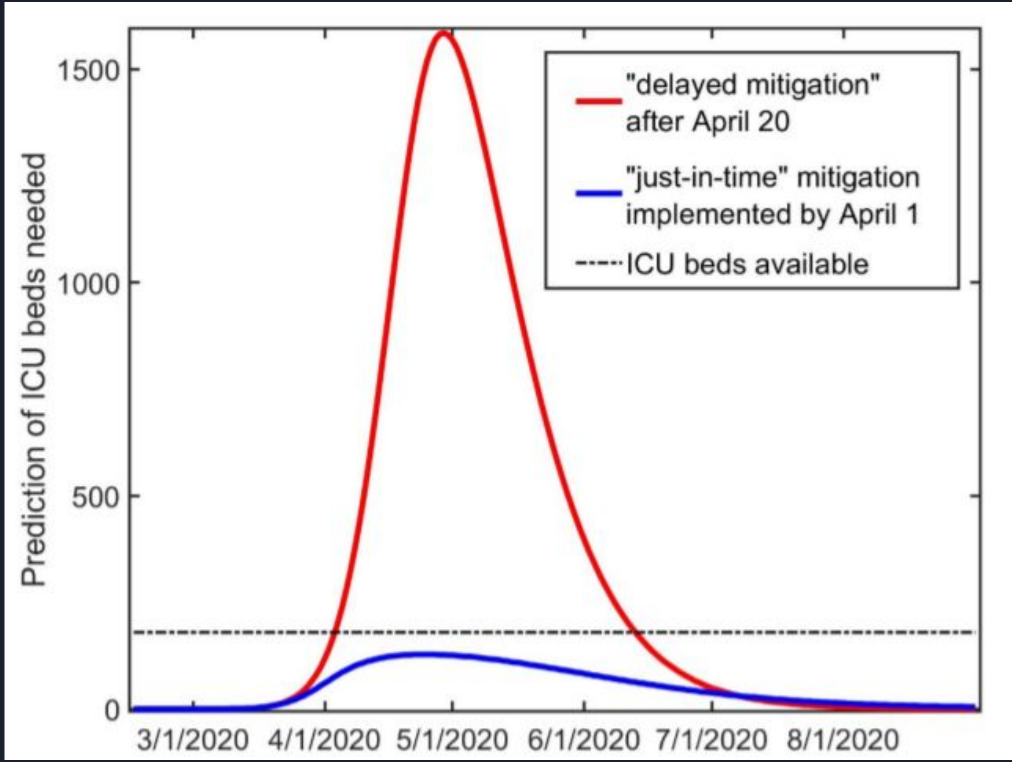
- Diffusion in networks
- Coordination games
- Introducing bilingualism to the game
- Characterization
- Non-Epidemic regions
- Limited compatibility and three technologies
- Strengths and weaknesses

Diffusion in Networks

Diffusion is a process by which information, viruses, ideas and new behavior spread over the network.

J. Leskovec. Diffusion and Cascading Behavior in Networks.
http://www.cs.cmu.edu/~jure/pub/diffusion-paper.pdf

Epidemics



S. Maslov and N. Goldenfeld. Window of Opportunity for Mitigation to Prevent Overflow of ICU capacity in Chicago by COVID-19. arXiv:2003.09564 [q-bio.PE]

Epidemics

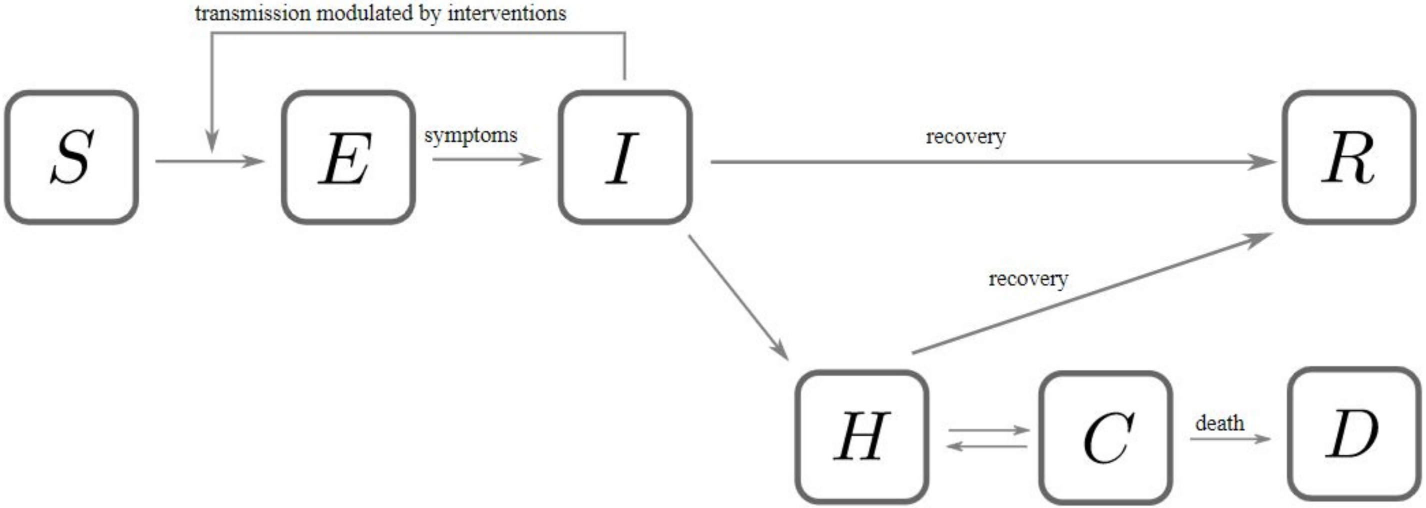
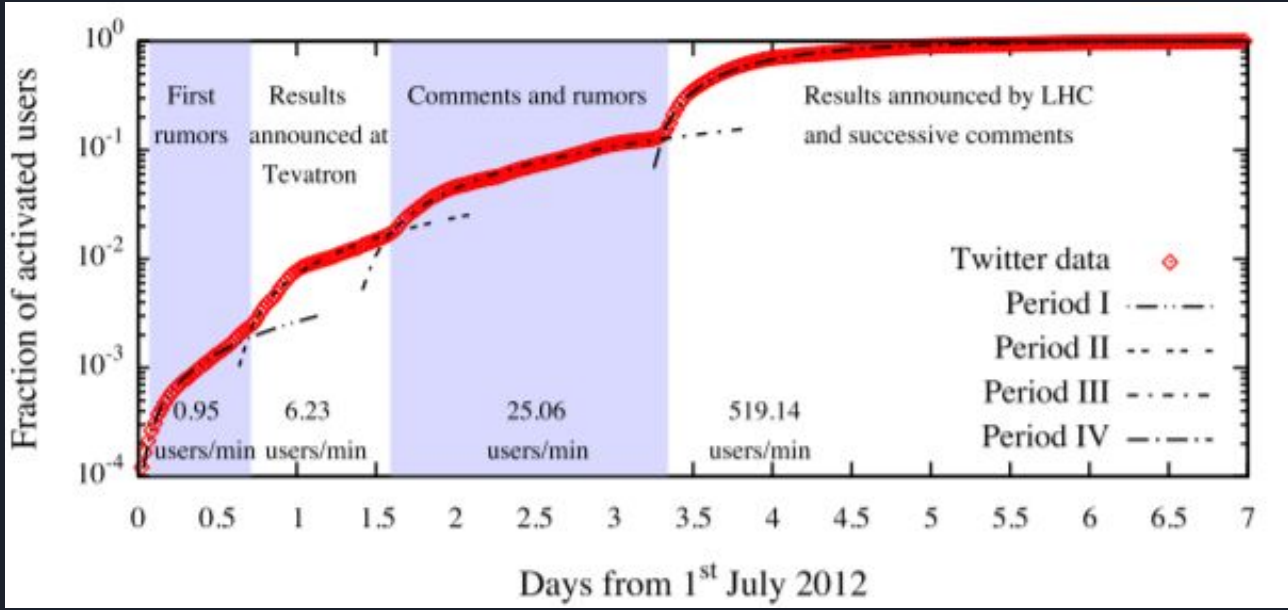


Figure 1. A schematic illustration of the underlying model. S corresponds to the 'susceptible' population, E is 'exposed', I is 'infectious', R 'recovered', H 'severe' (hospitalized), C 'critical' (ICU), and D are fatalities.

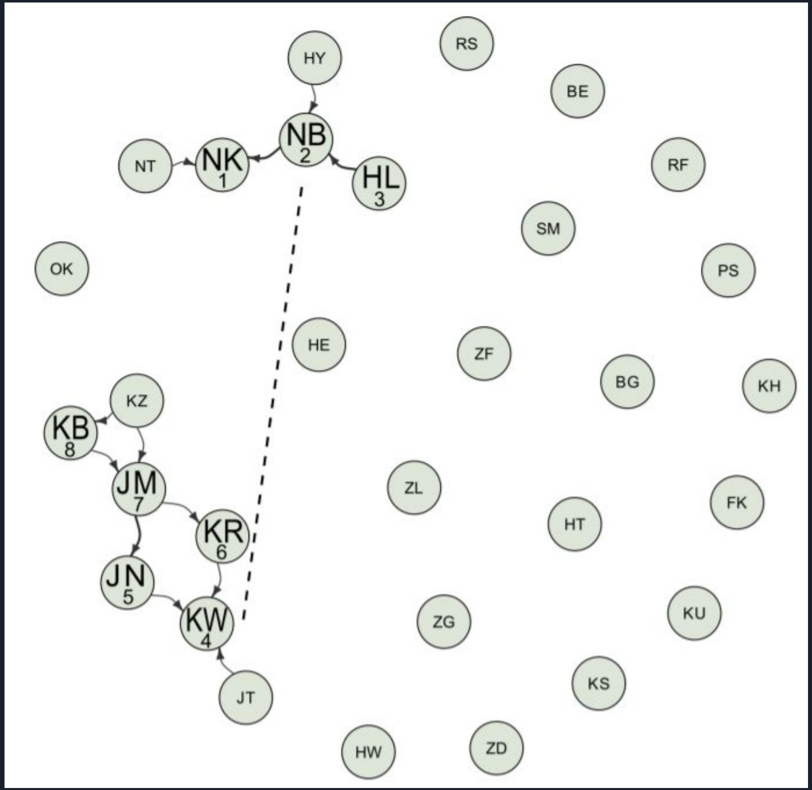
Ivan Aksamentov, N.N., Richard Neher. COVID 19 Scenario Simulator. 2020; Available from: https://neherlab.org/covid19/about.

Information



De Domenico, M., Lima, A., Mougel, P. et al. The Anatomy of a Scientific Rumor. Sci Rep 3, 2980 (2013). https://doi.org/10.1038/srep02980

Technology



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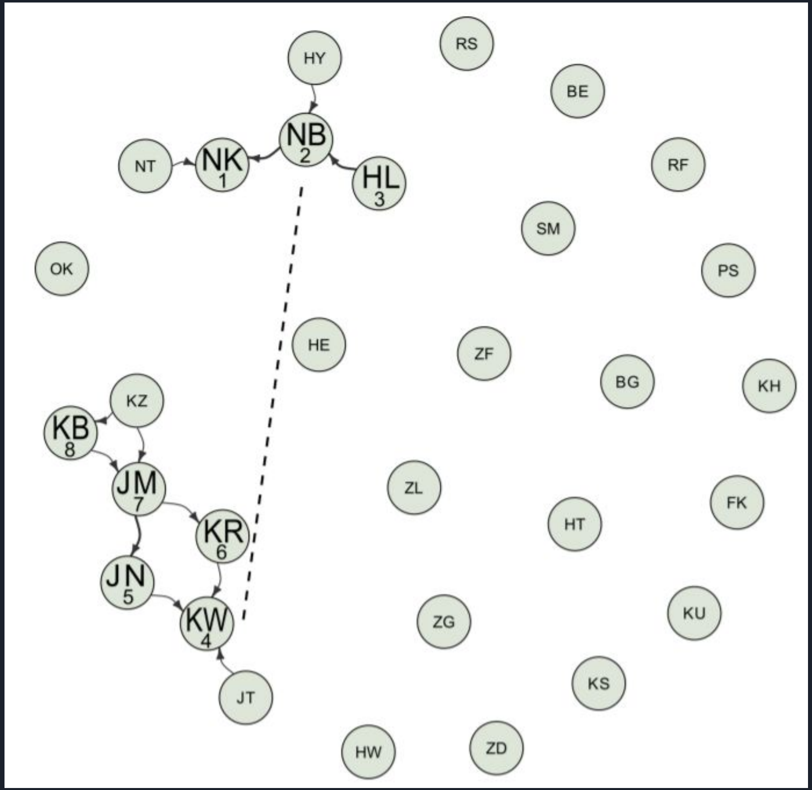
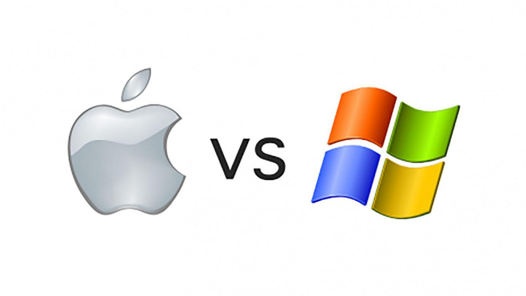
Technology



C. Hobaiter, et. al. Social Network Analysis Shows Direct Evidence for Social Transmission of Tool Use in Wild Chimpanzees. Plos Biology, 2014

Introduction

- Technologies are competing with each other



- Difficult for an individual to adopt several technologies at the same time
  - But not impossible
  - The difficulty in using multiple technologies is balanced somewhere between the two extremes of impossibility and easy interoperability. How?

Introduction - Diffusion and Networked Coordination Games

- Two instant messenger (IM) systems, A&B, which are not interoperable
  - Users can only communicate with each other in the same system

- A social network G on the users
  - G indicates who want to talk to whom
  - The endpoints of each edge (v, w) play a coordination game

The Coordination Game

- A Pure Coordination Game
  - Two users must simultaneously elect a technology to use
  - A choice of different technologies result 0 payoff
  - One technology is preferred over the other
- In this case
  - v, w each receive a payoff of q if they both choose B
  - v, w each receive a payoff of 1-q if they both choose A

	A	B
A	1-q, 1-q	0, 0
B	0, 0	q, q

S. Morris. Contagion. Review of Economic Studies, 67:57–78, 2000

The Coordination Game

- A is ‘better’ technology is if $q < ½$ while A is worse if $q > ½$

- All nodes in a network G are initially play B
  - A small number of nodes begins updates to adopting A
  - Play a best-response updates to nodes in the network
    - switch to A if enough of your network neighbors have already adopt A
  - **A network-wide equilibrium is reached - all nodes adopt A**
  - **Or coexistence - the nodes partitioned into a set adopting A and a set adopting B**

S. Morris. Contagion. Review of Economic Studies, 67:57–78, 2000

Compatibility, Interoperability & Bilinguality

- In previous model of diffusion, user can either choose A or B, but not both

- Coexistence is a typical outcome in real world
  - Financial Industry uses Windows while entertainment industry uses MacOS
  - WeChat is the dominant IM app in China while Messenger is the one in North America

- What happens on the coexistence boundary?

Bilinguality & Diffusion with Bilingual Behavior

- Bilinguality is a essential feature of interaction
  - People with the ability to work in multiple computer systems to collaborate with people embedded in each
  - Inhabitants who live in a boundary region between two different language areas tend to be speak both
- Diffusion with Bilingual Behavior
  - The IM Systems A & B, with the same payoff structure as before
  - Each node now can adopt a bilingual strategy, AB
  - An adopter of AB interacting with B, both receive payoff q
  - An adopter of AB interacting with A, both receive payoff 1-q
  - An adopter of AB interacting with another adopter of AB, both receive max(q, 1-q)
  - An adopter of AB pays a fixed penalty of c, a cost of maintaining both technologies

Diffusion with Bilingual Behavior

- Two Parameters
  - The relative qualities of the two technologies q
    - A: 1 - q
    - B: q
  - The cost of being bilingual, c

- The social network graph G
  - G is infinite with each node having degree Δ
  - $r = c/\Delta$, denoting the fixed penalty for adopting AB, scaled to per edge cost

Model - Diffusion with Bilingual Behavior

- Question: whether the new technology A can spread through a network where almost everyone is initially using technology B

- Technology A can become *epidemic* if:
  - All nodes in a finite set S adopt technology A in the starting state
  - All the other nodes in G adopt B
  - A sequence of best-response update in G - S causes every node to eventually adopt A

Model - Diffusion with Bilingual Behavior

- Two dimensional parameter space (q, r)
  - An epidemic region Ω(G), which is the subset of (q, r) plane for which A can become epidemic.

- Result:
  - A can become epidemic if r is sufficiently small or sufficiently large, but cannot take a value in between

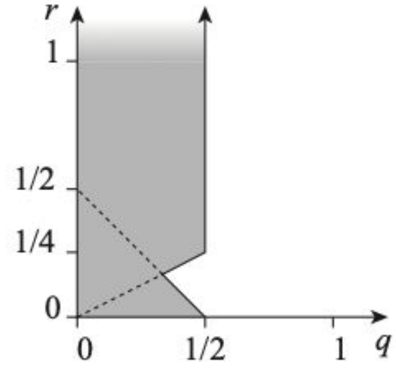


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Model - Diffusion with Bilingual Behavior

- Interpretations of the result

<u>r is small</u>, it is cheap to adopt AB. So AB spreads everywhere. Then the best-response updates cause all nodes to switch to A to avoid the penalty r.

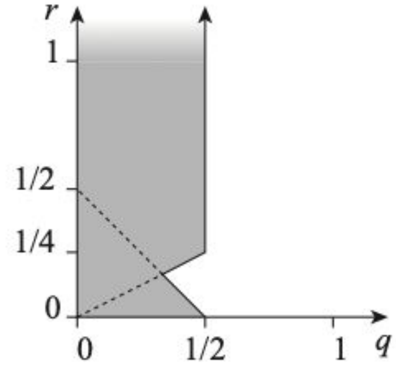


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Model - Diffusion with Bilingual Behavior

- Interpretations of the result

r is too big, it is too expensive to adopt AB. So the nodes at the interface will choose A, the better technology. A will spread step-by-step through the network.

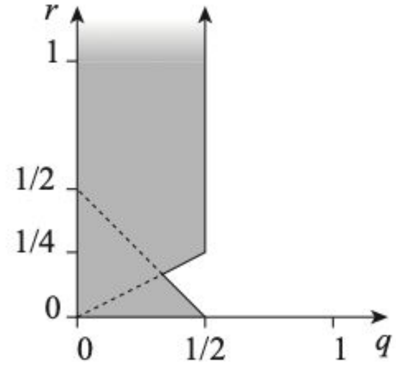


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Model - Diffusion with Bilingual Behavior

- Interpretations of the result

<u>r is in the middle</u> - tend to adopt AB. Nodes at the interface adopt AB. But nodes playing B lack the incentive to switch. As a result, the bilingual AB nodes form a boundary.

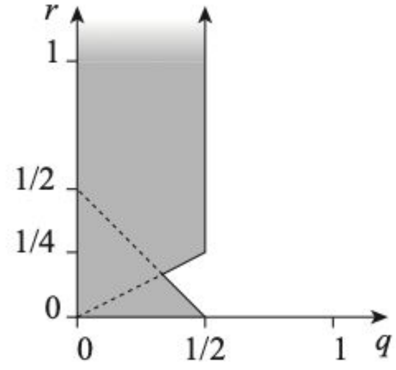


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Example

Thick Line Graph LΔ

- Groups of vertices
- Each group has Δ/2 vertices
- Edges between vertices in neighboring groups

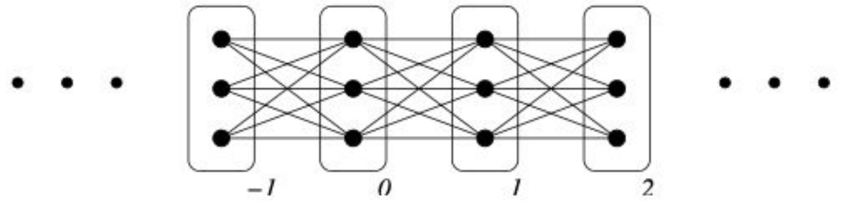


Figure 2: The thick line graph

Example

- Endowed all agents in group 0 with strategy A
- Group 1 payoff with various responses
  - Strategy B: $q\Delta/2$
  - Strategy A: $(1-q)\Delta/2$
  - Strategy AB: $\Delta/2 - r\Delta$
- We want make group 1 take strategy A
  - $(1-q)\Delta/2 >= q\Delta/2 \rightarrow q \leq 1/2$
  - $(1-q)\Delta/2 >= \Delta/2 - r\Delta \rightarrow q \leq 2r$
- Group 1, -1, 2, -2 will all change to A

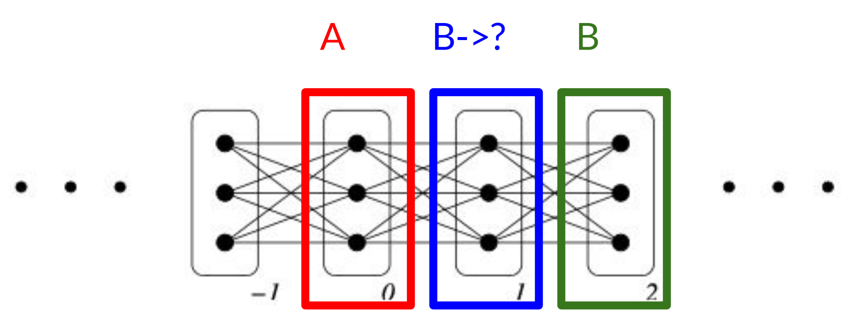


Figure 2: The thick line graph

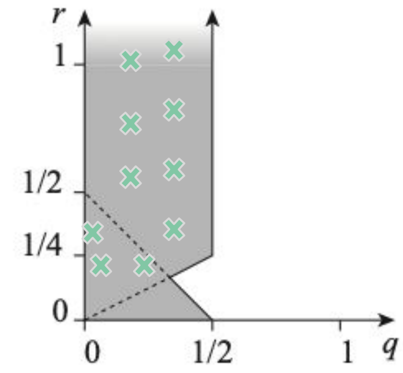


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Example

- q ≤ 1 - 2r and q > 2r, one side A, one side B change to AB -> group 1 to AB
- Group 2 payoff with various responses
  - Strategy B: qΔ
  - Strategy A: (1-q)Δ/2
  - Strategy AB: (q+max(q,1-q))Δ/2-rΔ
- We want make group 2 take strategy AB
  - (q+max(q,1-q))Δ/2-rΔ ≥ 1-q ->   2r ≤ q
  - (q+max(q,1-q))Δ/2-rΔ ≥ 2q -> q + r ≤ ½
- Group 2 takes AB, group 2,-2, 3, -3 will also take AB
- Group 1, -1, 2, -2 changes strategy again, A becomes epidemic

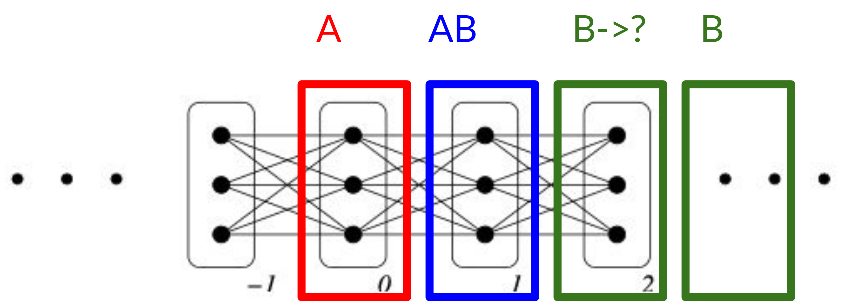


Figure 2: The thick line graph

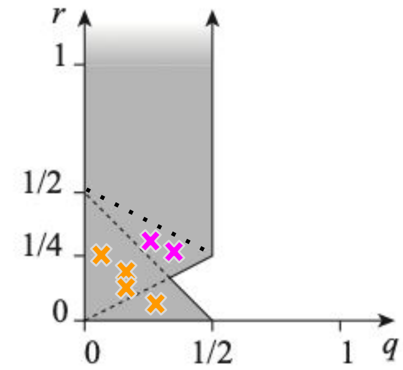


Figure 1: The region of the (q, r) plane for which technology A can become epidemic on the infinite line.

Characterization

The model discussed above is based on two assumptions:

- The outcome of a game is well-defined and unique (the equilibria is stable)
- The outcome is also invariant to the sequence of best-response moves under certain mild conditions.

Characterization: Lemma and Theorem

If the outcome is unique (Lemma 4.1):

- Once an agent decides to adopt technology A, she never discards it
- Once she decides to discard technology B, she never re-adopts it

Characterization: Lemma and Theorem

If the outcome is unique (Lemma 4.1):

$$s_i(v_{i+1}) = B \textit{ and } s_{i+1}(v_{i+1}) = A,$$

$$s_i(v_{i+1}) = B \textit{ and } s_{i+1}(v_{i+1}) = AB,$$

$$s_i(v_{i+1}) = AB \textit{ and } s_{i+1}(v_{i+1}) = A,$$

$$s_i(v_{i+1}) = s_{i+1}(v_{i+1}).$$

If an agent prefers X than Y in state k, A $>^k$ B, AB $>^k$ B, and A $>^k$ AB.

This statement is constantly true (see the detailed proof in the paper)

Characterization: Lemma and Theorem

If outcome is also invariant to the sequence of best-response moves, under certain mild conditions (two theorems):

Theorem 4.3:

T is a subset of V(G); S is a schedule of Vertices in V(G) \ T → the outcome of the game is all-A equilibrium

If we have S’, the out come of the game is still all-A equilibrium

Characterization: Lemma and Theorem

If outcome is also invariant to the sequence of best-response moves, under certain mild conditions (two theorems):

Theorem 4.4:

Under the same definition in Theorem 4.3, we have S and S’, the outcomes for both S and S’ are the same.

Characterization: Blocking Structures

DEFINITION 4.5. *Consider a contagion game* (G, q, r)*. A pair* (S_{AB}, S_B) *of disjoint subsets of* $V(G)$ *is called a* blocking structure *for this game if for every vertex* $v \in S_{AB}$*,*

$$\deg_{S_B}(v) > \frac{r}{q}\Delta,$$

and for every vertex $v \in S_B$*,*

$$(1-q)\deg_{S_B}(v) + \min(q, 1-q)\deg_{S_{AB}}(v) > (1-q-r)\Delta,$$

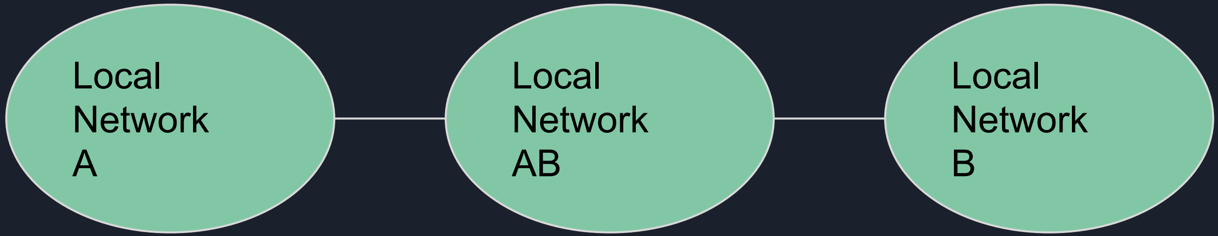
and

$$\deg_{S_B}(v) + q\deg_{S_{AB}}(v) > (1-q)\Delta,$$

where $deg_S(v)$ *denotes the number of neighbors of* v *in the set* S*.*

Characterization: Blocking Structures

State j



A very simple example graph: the transmission of A is blocked

Non-Epidemic Regions in General Graphs Theorem 5.1

For every Δ-regular graph G and parameters q and r, the technology A cannot become an epidemic in the game(G, q, r) if $q>1/2$.

By intuition, since technology A brings less payoff than technology B, it is impossible for a network to only adopt technology A when there is only a very few proportion of nodes with technology A.

Non-Epidemic Regions in General Graphs Theorem 5.2

There exist $q<½$ and r such that for every contagion game(G,q,r), A cannot become epidemic.

Potential function: $qX_{A,B}+cn_{AB}$

Where $q=1/2-1/64\Delta$ and $c= r\Delta = \alpha$

α is any irrational number strictly between 3/64 and q

Non-Epidemic Regions in General Graphs
Theorem 5.2

Case 1: $B \rightarrow AB$

$P_B = q(z_{AB}+z_B)$

$P_{AB} = (1-q)(z_{AB}+z_A)+qz_B - c$

$P_{AB} - P_B = (1-2q)z_{AB}+(1-q)z_A - c \geq 0$

$z_A \geq \lceil c/(1-q) - (1-2q)z_{AB}/(1-q)\rceil$

Non-Epidemic Regions in General Graphs Theorem 5.2

Case 2: $AB \rightarrow A$

$P_{AB} = (1-q)(z_{AB}+z_A) + qz_B - c$

$P_A = (1-q)(z_{AB}+z_A)$

$P_{AB} - P_A = qz_B - c \leq 0$

Non-Epidemic Regions in General Graphs Theorem 5.2

Case 3: $B \rightarrow A$

$P_A = (1-q)(z_{AB}+z_A)$

$P_B = q(z_{AB}+z_B)$

$P_A - P_B \geq 0$

$z_A \geq \lfloor qz_B/(1-q) + (1-2q)z_{AB}/(1-q) \rfloor$

Model So Far...

	A	B	AB
A	$(1 - q; 1 - q)$	$(0; 0)$	$(1 - q; 1 - q - r)$
B	$(0; 0)$	$(q; q)$	$(q; q - r)$
AB	$(1 - q - r;$ $1 - q)$	$(q - r; q)$	$(\max(q, 1 - q) - r;$ $\max(q, 1 - q) - r)$

Limited Compatibility

	A	B	AB
A	$(1 - q; 1 - q)$	$(x; x)$	$(1 - q; 1 - q - r)$
B	$(x; x)$	$(q; q)$	$(q; q - r)$
AB	$(1 - q - r;$ $1 - q)$	$(q - r; q)$	$(\max(q, 1 - q) - r;$ $\max(q, 1 - q) - r)$

assume special case that $x<q \leq 1 - q$

Limited Compatibility

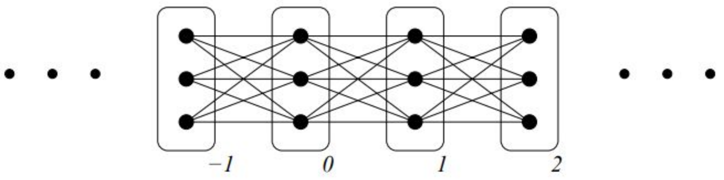
$((1-q)-x)/(1-2x)$
$= (1-q-2x+x)/(1-2x)$
$=(1-2x-(q-x))/(1-2x)$
$=1-((q-x)/(1-2x))$
$=1-q'$

$((q-r)-x)/(1-2x)$
$= \dots$
$= q'-r/(1-2x)$
$=q'-r'$

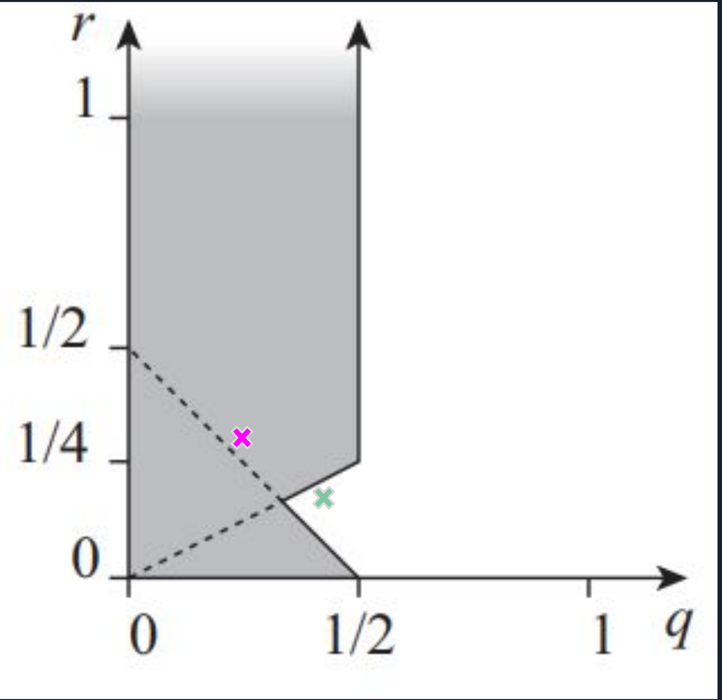
	A	B	AB
A	$(1-q; 1-q)$	$(x; x)$	$(1-q; 1-q-r)$
B	$(x; x)$	$(q; q)$	$(q; q-r)$
AB	$(1-q-r;$ $1-q)$	$(q-r; q)$	$(\max(q, 1-q)-r;$ $\max(q, 1-q)-r)$

assume special case that $x<q \leq 1-q$

Example



Game played on a thick line graph with $r = 5/32$ and $q = 3/8$



With limited compatibility $x=1/4$,
$r' = 5/16$ and $q' = 1/4$

Limited compatibility: Blocking structures

Let (G, q, r) be a game without compatibility and (G', q, r,x) the limited compatibility version.

We know: (G', q, r,x) ~ (G', q', r')

Is a blocking structure in G' also one in G?

Is a blocking structure in G' also one in G?

$$qd_{SB}(v) > (q - x)\, d_{SB}(v)$$

$$= q'(1 - 2x)\, d_{SB}(v)$$

$$> r'(1 - 2x)\Delta$$

$$= r\Delta$$

Three technologies

THEOREM 6.3. *For any even* $\Delta \geq 12$, *there is a* Δ*-regular graph* G*, an initial state* s*, and values* q_A, q_B, q_C, *and* q_{BC}, *such that*

- s *is an equilibrium in both* $(G, q_A, q_B, q_C, 0)$ *and* $(G, q_A, q_B, q_C, q_{BC})$,
- *neither* B *nor* C *can become epidemic in either* $(G, q_A, q_B, q_C, 0)$ *or* $(G, q_A, q_B, q_C, q_{BC})$ *starting from state* s,
- A *can become epidemic* $(G, q_A, q_B, q_C, 0)$ *starting from state* s, *and*
- A *can not become epidemic in* $(G, q_A, q_B, q_C, q_{BC})$ *starting from state* s.

But not always ...

Conclusion

An application of game theory in network simulation

- Complete information
- Full rationality
- Without too much external influence
- Homogeneous utility/payoff
- Deductive approach (theory-driven)

Strengths

- Straightforward language to present the ideas
- Detailed explanation on intuitions of the result
- Clear assumptions and proofs that theorems build upon

Weakness

Do you really start to use a new messenger because you know its fixed payoff in terms of technology itself?

Weakness

Internal factors:

- Most people adopt a new messenger just because their friends are using it without calculating payoffs.
- The model only considers the neighbors of one node, but the neighbors of neighbors may also matter (potential friends, business partners, etc.).

Weakness

External factors:

- Marketing strategies (promotion, free trial, Ads, etc.)
- Basis of existing users (e.g. Tech A from Company P has a huge number of users. P and a startup Q develop an identical Tech B_P and B_Q. Which one is more likely to be adopted?)

What computer science can do to improve the competitivity of technology they design?

Marketing strategies? NO

Improve the payoff of technology? YES

Change the technology diffusion game? DIFFICULT

Improve Compatibility? IMPORTANT!