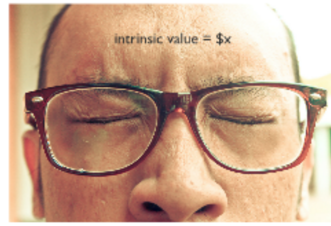
Auctions

hari sundaram
hs1@illinois.edu

introduction



will not buy an item above this value

one seller → multiple buyers bid

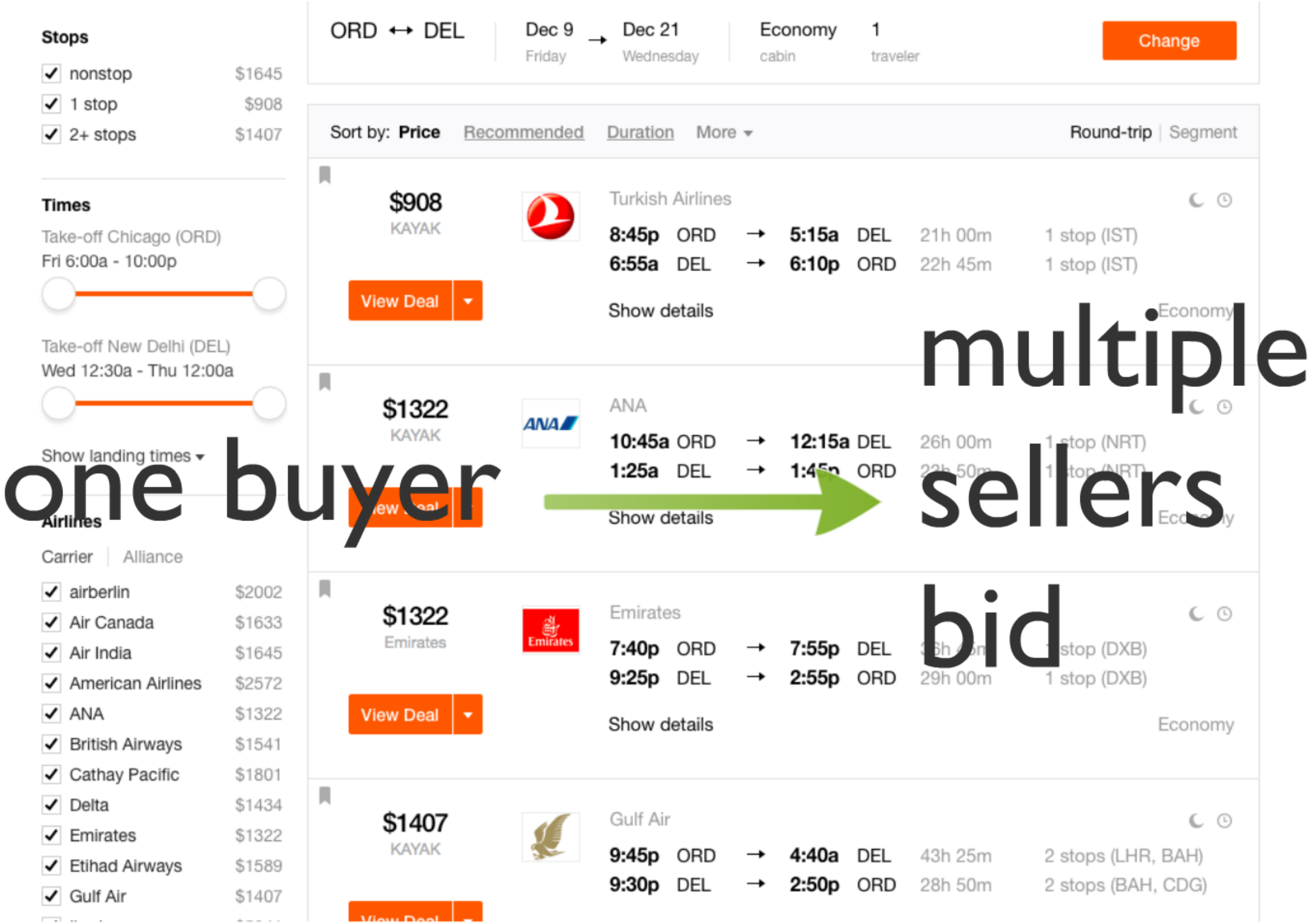
When are auctions appropriate?

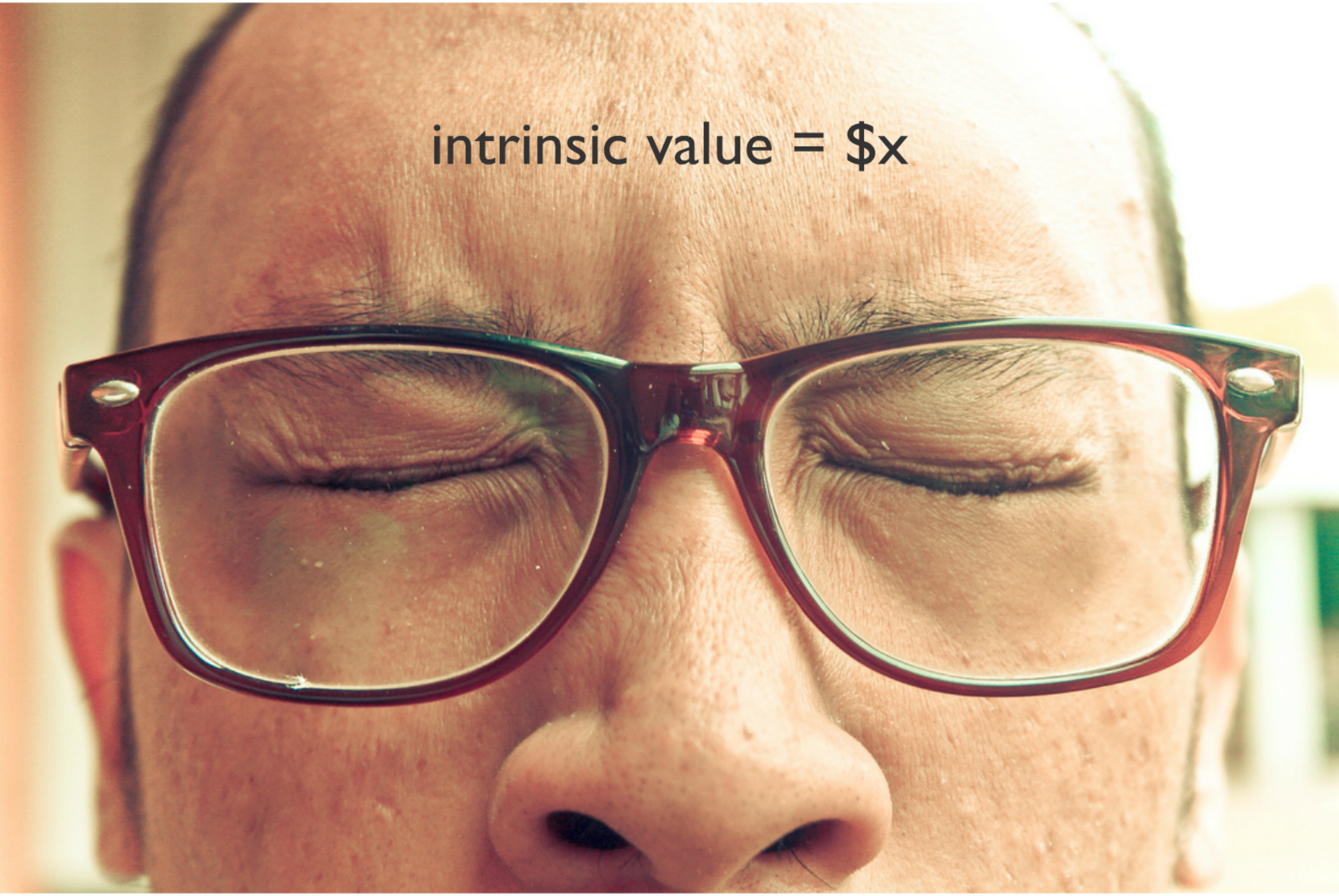
Equivalences

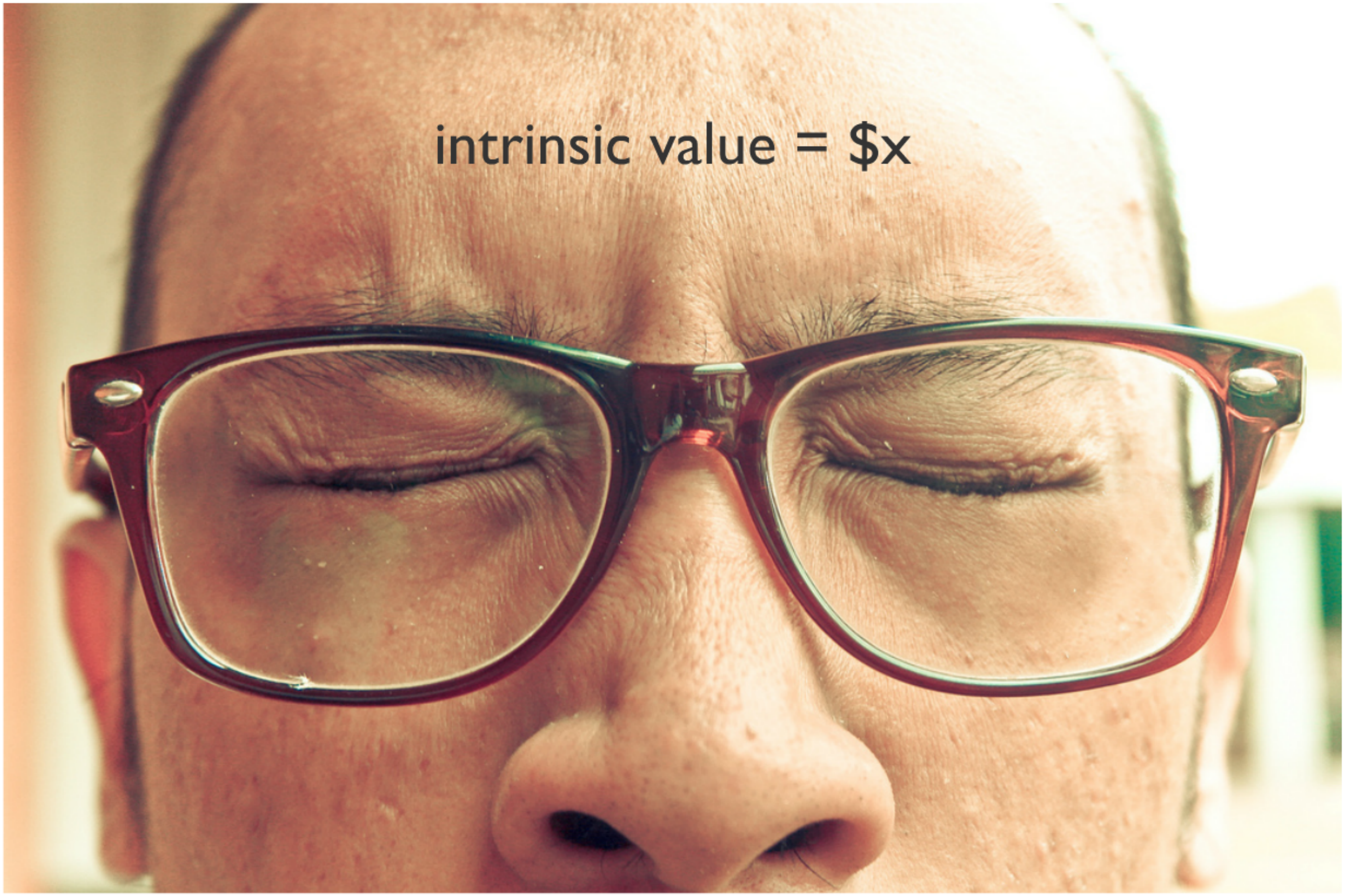
2 Descending bid

4 Second-price sealed bid

one seller → multiple buyers bid


one buyer
multiple sellers bid
Stops
nonstop $1645
1 stop $908
2+ stops $1407
Times
Take-off Chicago (ORD)
Fri 6:00a - 10:00p
Take-off New Delhi (DEL)
Wed 12:30a - Thu 12:00a
Show landing times
Airlines
Carrier
Alliance
airberlin $2002
Air Canada $1633
Air India $1645
American Airlines $2572
ANA $1322
British Airways $1541
Cathay Pacific $1801
Delta $1434
Emirates $1322
Etihad Airways $1589
Gulf Air $1407
ORD ↔ DEL
Dec 9 Friday → Dec 21 Wednesday
Economy cabin
1 traveler
Change
Sort by: Price
Recommended
Duration
More
Round-trip
Segment
$908 KAYAK
Turkish Airlines
8:45p ORD → 5:15a DEL 21h 00m 1 stop (IST)
6:55a DEL → 6:10p ORD 22h 45m 1 stop (IST)
View Deal
Show details
$1322 KAYAK
ANA
10:45a ORD → 12:15a DEL 26h 00m 1 stop (NRT)
Show details
$1322 Emirates
Emirates
7:40p ORD → 7:55p DEL
9:25p DEL → 2:55p ORD 29h 00m 1 stop (DXB)
View Deal
Show details
Economy
$1407 KAYAK
Gulf Air
9:45p ORD → 4:40a DEL 43h 25m 2 stops (LHR, BAH)
9:30p DEL → 2:50p ORD 28h 50m 2 stops (BAH, CDG)


intrinsic value = $x



will not buy an item above this value

there are four basic types of auctions

1 Ascending bid

English Auctions

2 Descending bid

Dutch auctions

3 First-price sealed bid

4 Second-price sealed bid

Vickrey auctions

Nobel 1996

Equivalences

2 Descending bid

Dutch auctions

there are four basic types of auctions

1 Ascending bid

English Auctions

3 First-price sealed bid

4 Second-price sealed bid

Vickrey auctions

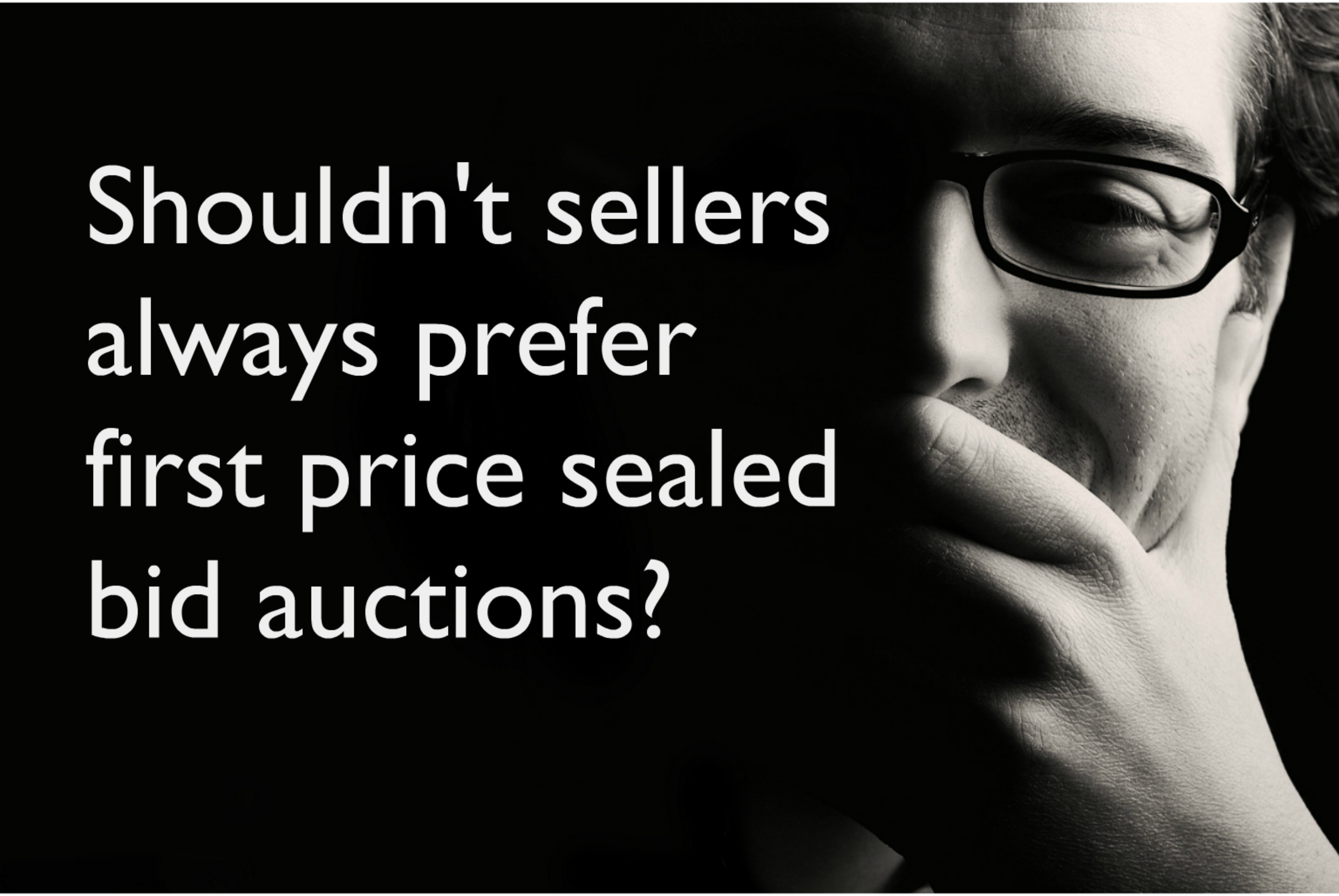
Nobel 1996

When are auctions appropriate?

independent,
private values

common values

(buying with the
intention to sell)


Shouldn't sellers always prefer first price sealed bid auctions?

types of auctions

true value: v_i

2nd highest bid

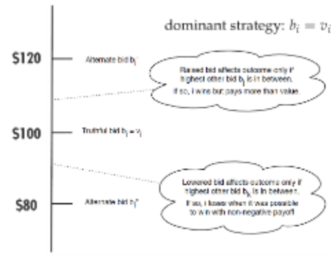
$$\text{payoff} = \begin{cases} v_i - b_j & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

bid: b_i

bid decides whether you win

view auctions as an n player game

dominant strategy: $b_i = v_i$



In a sealed-bid first-price auction, the value of your bid not only affects whether you win but also how much you pay.

1 you can bid close to your true value

2 you can bid far below your true value (shading)

Why shouldn't you bid your true value in a first price auction?

$$\text{payoff} = \begin{cases} v_i - b_i & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you bid your true value?

bidding your true value isn't a dominant strategy view auctions as an n player game

true value: v_i

payoff =

bid: b_i

2nd highest bid

true value: v_i

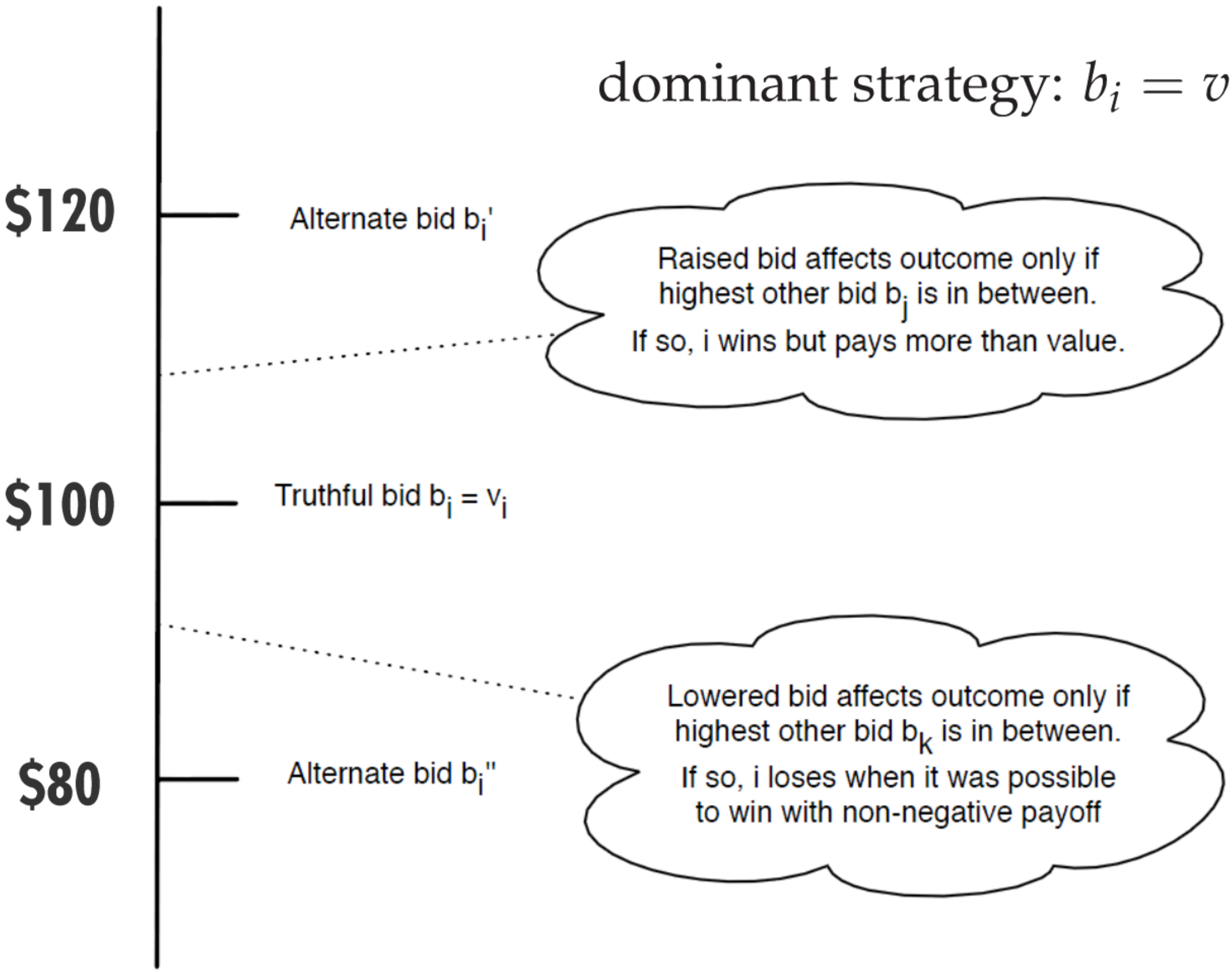
$$\text{payoff} = \begin{cases} v_i - b_j & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

bid: b_i

bid decides whether you win


bidding your true value is a dominant strategy in a second price auction

dominant strategy: $b_i = v_i$



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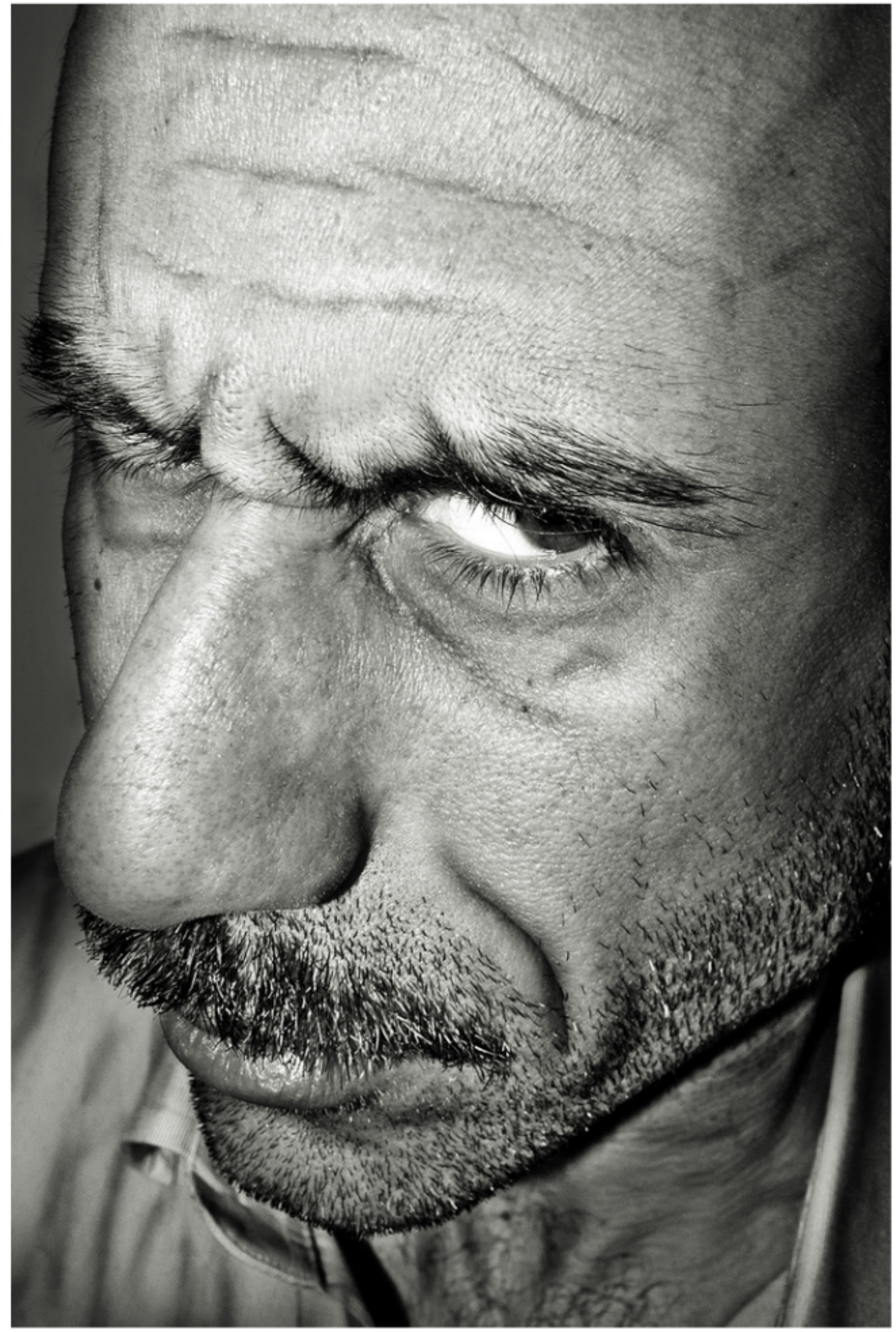
Why shouldn't you bid your true value in a first price auction?



1

you can bid close to your true value

2

you can bid far below your true value (shading)

$$\text{payoff} = \begin{cases} v_i - b_i & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you bid your true value?

$$\text{payoff} = \begin{cases} v_i - b_i & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you bid your true value?

bidding your true value isn't a dominant strategy

what if the goal was to resell?

the winners curse

There is an eventual common value for the object (the amount it will generate on resale) but it is not necessarily known.

is this a dominant strategy?

$$v_i = v + x_i$$

estimate — common value — error

Winner's curse

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There is an eventual common value for the object (the amount it will generate on resale) but it is not necessarily known.

is this a dominant strategy?

$$v_i = v + x_i$$

estimate

common value

error

is this a dominant strategy?

$$v_i = v + x_i$$

estimate — common value — error

Winner's curse


First noticed in oil exploration

shading in first and second price auctions

$$\dot{s} \geq 0$$

differentiable, increasing

strategy

$$b = s(v)$$

bid   value

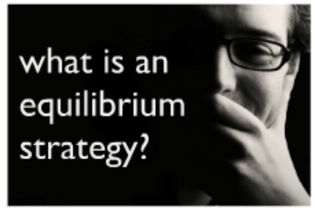
$$s(v) \leq v$$

bid always less than true value

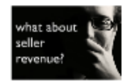
assume that both players use the same strategy s

Equilibrium

Let's examine the case of two bidders



From the seller's point of view, is there a preferred auction mechanism (i.e. type of auction, first or second)?



In first price auctions, individuals shade their bids, while in second price auctions, the seller gets the second highest bidder's bid.

Suppose that there are n bidders who draw their values independently from [0,1].

details!

$$v_i$$

Expected revenue of second price auction

$$\frac{n-1}{n+1}$$

$$\frac{n-1}{n} \times \frac{n}{n+1} = \frac{n-1}{n+1}$$

Expected revenue of first price auction

$$r = \frac{1+u}{2}$$

notice that the highest value results in the highest bid


Across a wide range of auctions, seller revenue remains the same!
In first price auctions, the degree of shading depends on the number of participants
The big picture

Let's examine the case of two bidders

$$v \in (0,1)$$

private values

strategy

$$b = s(v)$$

bid

value

$$v \in (0, 1)$$

private values

$$\dot{s} \geq 0$$

differentiable, increasing

strategy

$$b = s(v)$$

bid

$v \in (0,1)$

private values

value

$$s(v) \leq v$$

bid always less than true value

assume that both players use the same strategy s

what is an equilibrium strategy?


how will we
compare across
strategies?

The revelation principle

notice that the highest value results in the highest bid

value of
winning bid

$$v_i$$

probability
of winning

$$g(v_i) = v_i(v_i - s(v_i))$$

payoff

probability of winning

bid

instead of changing the strategy function pretend that we have a different value

$$v_i(v_i - s(v_i)) \geq v(v_i - s(v)) \, \forall v$$

for dominance

two bidders

$$g'(v) = v_i - s(v) - vs'(v)$$

$$\Downarrow$$

$$s'(v_i) = 1 - \frac{s(v_i)}{v_i}$$

n bidders

$$G(v_i) = v_i^{n-1}(v_i - s(v_i))$$

↓

$$s'(v_i) = (n-1)\left(1 - \frac{s(v_i)}{v_i}\right)$$

↓

$$s(v_i) = \left(\frac{n-1}{n}\right) v_i$$

Equilibrium

two bidders

$$g'(v) = v_i - s(v) - vs'(v)$$

$$s'(v_i) = 1 - \frac{s(v_i)}{v_i}$$

n bidders

$$G(v_i) = v_i^{n-1}(v_i - s(v_i))$$

$$s'(v_i) = (n-1)\left(1 - \frac{s(v_i)}{v_i}\right)$$

$$s(v_i) = \left(\frac{n-1}{n}\right)v_i$$


what about
seller
revenue?

In first price auctions, individuals shade their bids, while in second price auctions, the seller gets the second highest bidder's bid.

From the seller's point of view, is there a preferred auction mechanism (i.e. type of auction, first or second?)

Suppose that there are n bidders who draw their values independently from $[0,1]$.

Suppose n numbers are drawn independently from the uniform distribution on the interval $[0, 1]$ and then sorted from smallest to largest. The expected value of the number in the kth position on this sorted list is $k / n+1$.

Expected revenue of second price auction

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$$\frac{n-1}{n+1}$$

Expected revenue of second price auction

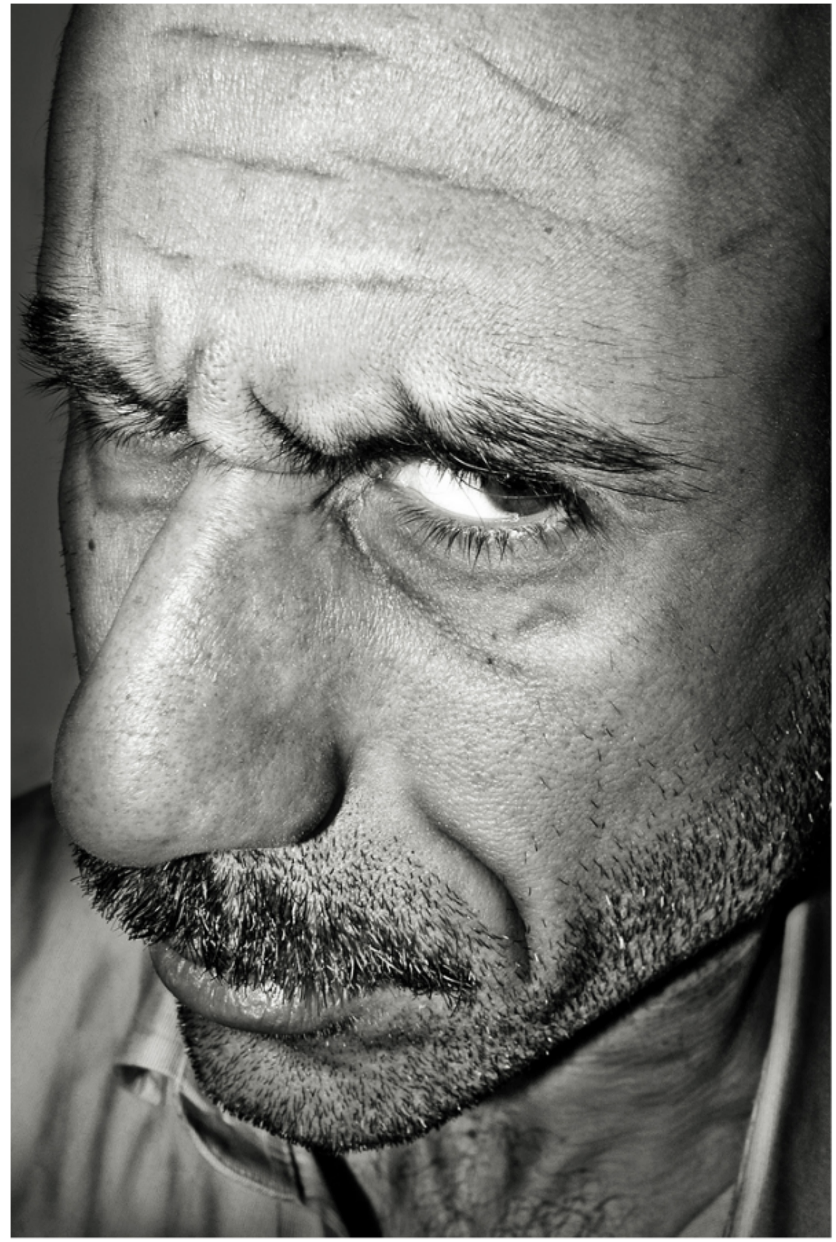
Suppose n numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the kth position on this sorted list is k / n+1 .

$$\frac{n-1}{n+1}$$

$$\frac{n-1}{n} \times \frac{n}{n+1} = \frac{n-1}{n+1}$$

Expected revenue of first price auction

What if I didn't want to sell below $x?



Clearly,

$$r \geq u$$

reserve price

seller's value

Is there any point in setting r to be different from u?

Clearly,

$$\underset{\text{reserve price}}{r} \geq \underset{\text{seller's value}}{u}$$

Expected revenue in the case of only one bidder

$$r(1-r)+ru$$

expected revenue

optimal reserve price

$$r = \frac{1+u}{2}$$

Expected revenue in the case of only one bidder

$$r(1-r) + ru$$

expected revenue