

Auctions

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introduction



will not buy an item above this value



independent, private values 

common values  everybody has the same value for the item

When are auctions appropriate?



Equivalences



one seller



multiple
buyers
bid



Stops

- nonstop \$1645
- 1 stop \$908
- 2+ stops \$1407

Times

Take-off Chicago (ORD)
Fri 6:00a - 10:00p



Take-off New Delhi (DEL)
Wed 12:30a - Thu 12:00a



Show landing times ▾

Airlines

Carrier | Alliance

- airberlin \$2002
- Air Canada \$1633
- Air India \$1645
- American Airlines \$2572
- ANA \$1322
- British Airways \$1541
- Cathay Pacific \$1801
- Delta \$1434
- Emirates \$1322
- Etihad Airways \$1589
- Gulf Air \$1407

ORD ↔ DEL

Dec 9 → Dec 21
Friday → Wednesday

Economy 1
cabin traveler

Change

Sort by: **Price** Recommended Duration More ▾

Round-trip | Segment

\$908
KAYAK



Turkish Airlines

8:45p ORD → 5:15a DEL 21h 00m 1 stop (IST)
 6:55a DEL → 6:10p ORD 22h 45m 1 stop (IST)

View Deal ▾

Show details

\$1322
KAYAK



ANA

10:45a ORD → 12:15a DEL 26h 00m 1 stop (NRT)
 1:25a DEL → 1:45p ORD 22h 50m 1 stop (NRT)

View Deal ▾

Show details

\$1322
Emirates



Emirates

7:40p ORD → 7:55p DEL 26h 00m 1 stop (DXB)
 9:25p DEL → 2:55p ORD 29h 00m 1 stop (DXB)

View Deal ▾

Show details

\$1407
KAYAK



Gulf Air

9:45p ORD → 4:40a DEL 43h 25m 2 stops (LHR, BAH)
 9:30p DEL → 2:50p ORD 28h 50m 2 stops (BAH, CDG)

View Deal ▾

one buyer



multiple sellers

bid

intrinsic value = \$x





intrinsic value = $\$x$

will not buy an item above this value

there are **four**
basic types of
auctions



1 Ascending bid

English Auctions

2 Descending bid



Dutch auctions

3 First-price sealed bid

4 Second-price sealed bid

Vickrey auctions

Nobel 1996

Equivalences

2 Descending bid



3 First-price sealed bid

there are **four** basic types of auctions

1 Ascending bid

English auctions

4 Second-price sealed bid

Vickrey auctions
Harsanyi 1955



independent,
private values



common values

(buying with the
intention to sell)

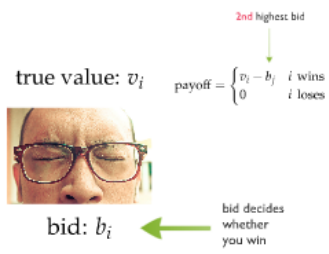


When are
auctions
appropriate?

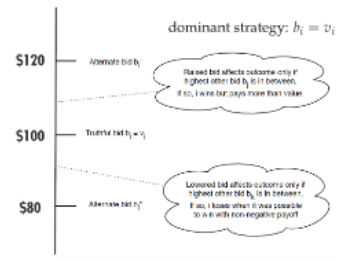


Shouldn't sellers
always prefer
first price sealed
bid auctions?





view auctions as an n player game



types of auctions

1 you can bid close to your true value

2 you can bid far below your true value (shading)

In a sealed-bid **first-price** auction, the value of your bid not only affects whether you win but also how much you pay.

$$\text{payoff} = \begin{cases} b - v & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you bid your true value?



bidding your true value isn't a dominant strategy

Why shouldn't you bid your true value in a first price auction?



view auctions
as an **n** player
game

true value: v_i

payoff =




bid: b_i



true value: v_i

2nd highest bid



$$\text{payoff} = \begin{cases} v_i - b_j & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$



bid: b_i



bid decides
whether
you win



bidding your true
value is a
dominant strategy
in a second price
auction

dominant strategy: $b_i = v_i$

\$120

Alternate bid b_i'

Raised bid affects outcome only if highest other bid b_j is in between.
If so, i wins but pays more than value.

\$100

Truthful bid $b_i = v_i$

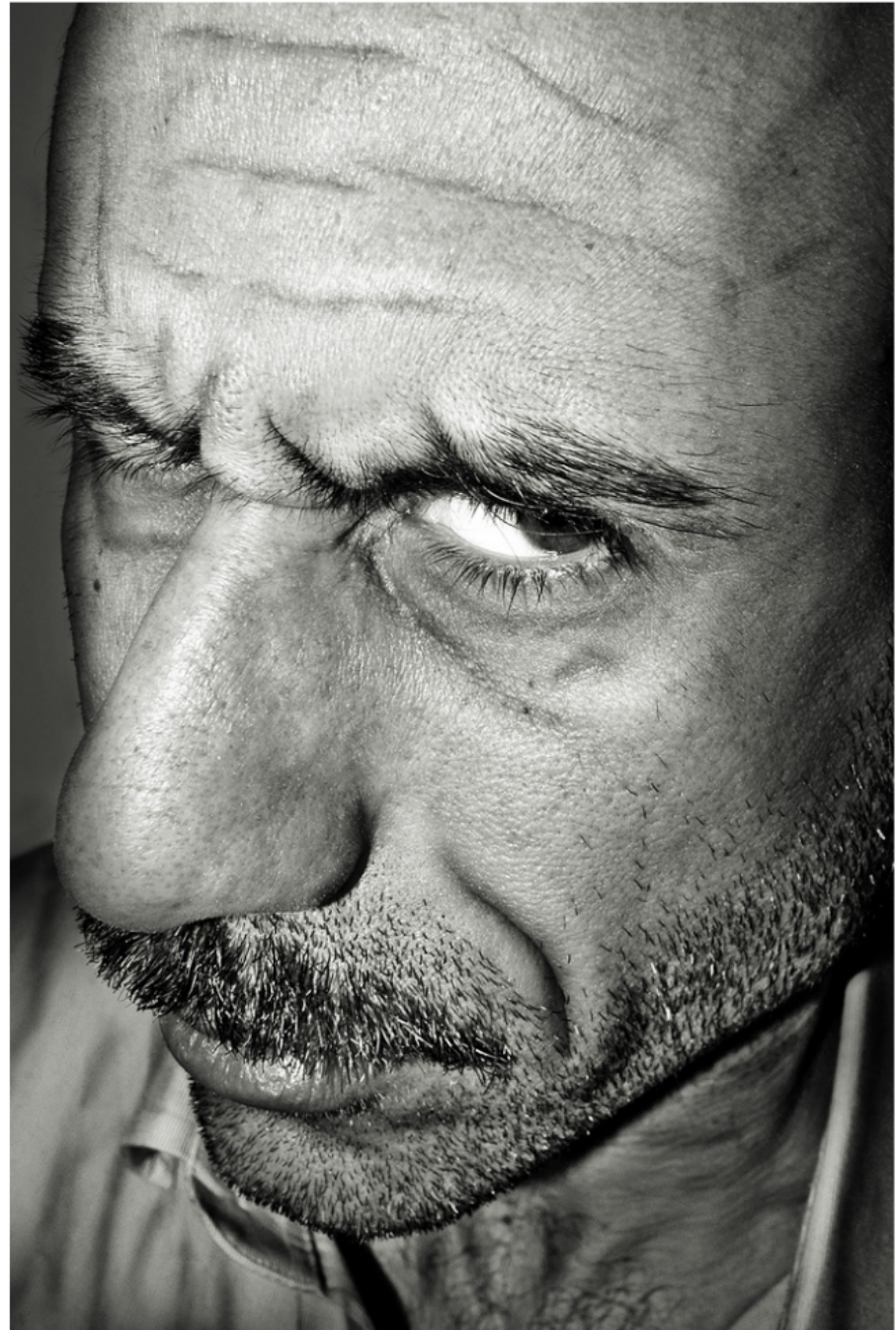
Lowered bid affects outcome only if highest other bid b_k is in between.
If so, i loses when it was possible to win with non-negative payoff

\$80

Alternate bid b_i''

In a sealed-bid **first-price** auction, the value of your bid not only affects whether you win but also how much you pay.

Why shouldn't you
bid your true value
in a first price
auction?



1 you can bid
close to your
true value

2 you can bid far
below your true
value (**shading**)

$$\text{payoff} = \begin{cases} v_i - b_i & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you
bid your true
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$$\text{payoff} = \begin{cases} v_i - b_i & i \text{ wins} \\ 0 & i \text{ loses} \end{cases}$$

should you
bid your true
value?



bidding your true value isn't a dominant strategy



what if the goal was to resell?

are the private values independent with other private values irrelevant?

the winners curse

There is an eventual common value for the object (the amount it will generate on resale) but it is not necessarily known.

is this a dominant strategy?

$$v_i = v + \epsilon_i$$

estimate common value error



Winner's curse





Monet's water lilies sell for \$208M

what if the goal was to resell?

There is an eventual common value for the object (the amount it will generate on resale) but it is not necessarily known.

is this a dominant strategy?



$$v_i = v + x_i$$

↑
estimate

↑
common value

↑
error

is this a dominant strategy?

$$v_i = v + x_i$$

estimate common value error

Winner's curse



First noticed in oil exploration

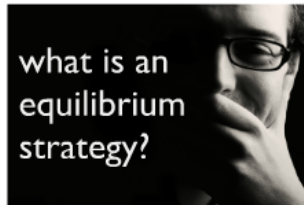
shading in first
and second
price auctions



details!

Let's examine the case of two bidders

what is an equilibrium strategy?



$$b = s(v)$$

strategy
↑
value

bid $v \in (0,1)$ value

$$\dot{s} \geq 0$$

differentiable, increasing

$$s(v) \leq v$$

bid always less than true value

assume that both players use the same strategy s

Equilibrium



how will we compare across strategies?



From the seller's point of view, is there a preferred auction mechanism (i.e. type of auction, first or second)?

what about seller revenue?

In first price auctions, individuals shade their bids, while in second price auctions, the seller gets the second highest bidder's bid.

Suppose that there are n bidders who draw their values independently from $[0,1]$

$$v_i$$

instead of changing the strategy function general idea we have a different value for distribution

Expected revenue of second price auction

$$\frac{n-1}{n+1}$$

$$\frac{n-1}{n} \times \frac{n}{n+1} = \frac{n-1}{n+1}$$

Expected revenue of first price auction


$$g(v_1) = v_1(v_1 - s(v_1))$$



notice that the highest value results in the highest bid



$$r = \frac{1+u}{2}$$

A person is sitting on a rock in the foreground, looking out over the vast, layered landscape of the Grand Canyon. The sky is filled with large, white, fluffy clouds, and the lighting suggests a late afternoon or early morning setting. The canyon's red and orange rock formations are visible in the distance.

In first price auctions,
the degree of shading
depends on the number
of participants

Across a wide range
of auctions, seller
revenue remains the
same!

The big picture

Let's examine
the case of two
bidders

$$v \in (0, 1)$$

private values

strategy



b

$=$

S

(v)



bid

$v \in (0, 1)$

private values



value

$$b = S(v)$$

↑ bid strategy ↓ value ↑
 $v \in (0, 1)$
 private values

$$\dot{S} \geq 0$$

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bid always less than true value

assume that
both players
use the same
strategy **s**

what is an
equilibrium
strategy?





how will we
compare across
strategies?

The revelation principle



notice that the
highest value results
in the highest bid

value of
winning bid



The image features two large, black, calligraphic letters, 'v' and 'i', positioned centrally. The 'v' is on the left and the 'i' is on the right. Two green diagonal lines are placed around the 'v': one above it and one below it, both slanted downwards from left to right.

probability
of winning

$$g(v_i) = v_i (v_i - s(v_i))$$

payoff

probability of
winning

bid

instead of changing the
strategy function pretend that
we have a different value

$$v_i(v_i - s(v_i)) \geq v(v_i - s(v)) \quad \forall v$$

for dominance

two bidders

$$g'(v) = v_i - s(v) - v s'(v)$$



$$s'(v_i) = 1 - \frac{s(v_i)}{v_i}$$

n bidders

$$G(v_i) = v_i^{n-1} (v_i - s(v_i))$$



$$s'(v_i) = (n-1) \left(1 - \frac{s(v_i)}{v_i} \right)$$




$$s(v_i) = \left(\frac{n-1}{n} \right) v_i$$

Equilibrium


two bidders


$$g'(v) = v_i - s(v) - v s'(v)$$


$$s'(v_i) = 1 - \frac{s(v_i)}{v_i}$$

n bidders

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seller
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Suppose that there are n bidders who draw their values independently from $[0, 1]$.

Suppose n numbers are drawn independently from the uniform distribution on the interval $[0, 1]$ and then sorted from smallest to largest. The expected value of the number in the k th position on this sorted list is $k / (n+1)$.

Expected revenue of second price auction

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$$\frac{n - 1}{n + 1}$$


Expected revenue of second price auction

Suppose n numbers are drawn independently from the uniform distribution on the interval $[0, 1]$ and then sorted from smallest to largest. The expected value of the number in the k th position on this sorted list is $k / (n+1)$.

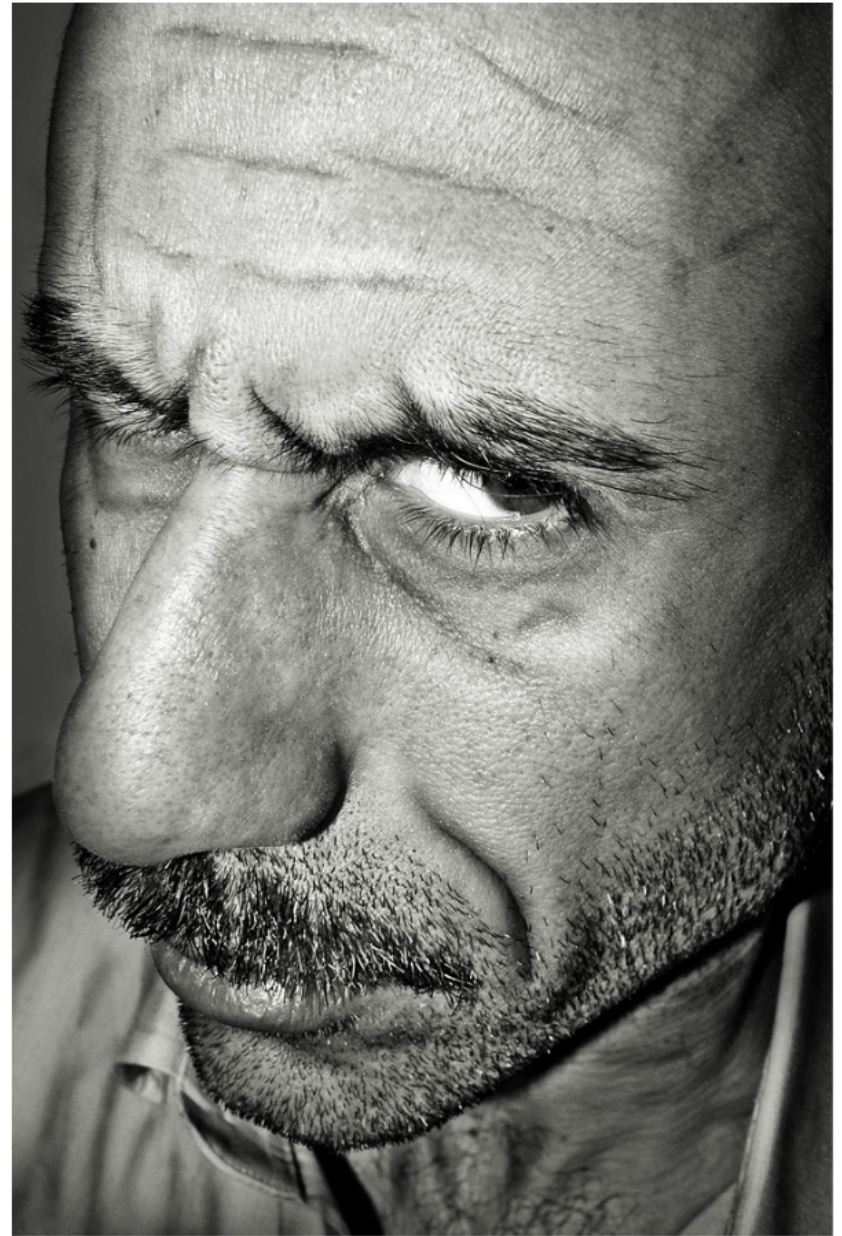
$$\frac{n-1}{n+1}$$



$$\frac{n-1}{n} \times \frac{n}{n+1} = \frac{n-1}{n+1}$$

Expected revenue of first price auction

What if I didn't
want to sell
below \$**x**?



Clearly,

$$r \geq u$$

reserve price

seller's value

Is there any
point in
setting r to be
different from
 u ?

Clearly,

$$r \geq u$$

reserve price seller's value



Expected revenue in the case of only one bidder



$$r(1-r) + ru$$

expected revenue

optimal reserve price

$$r = \frac{1 + u}{2}$$

Expected revenue in the case of only one bidder

