

## Voting Systems

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## Paper Discussed

Quadratic Voting: How Mechanism Design Can Radicalize Democracy


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## Paper Discussed

## Knapsack Voting



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## Voting

Method for a group to form a collective decision on certain issue.
Used for election, budgeting, policy or law passing, etc.
Types of Voting Mechanisms:

- Classical:
- One-person-one-vote (1p1v)
- Ranked-Choice Voting
- Quadratic voting
- Knapsack voting


## Problems with One-Person-One-Vote

Each voter is rationed a single unit of influence and people can show direction of interest but not intensity of the direction.

- Tyranny of the Majority
- results in the oppression of the minority
- Condorcet Paradox


## Prior Work

- The primary motivation for the creation of QV is based on VCG Mechanism.
- VCG is an optimal voting mechanism for decisions involving public goods, created in the 60's and 70's by Vickrey, Clarke, and Groves (VCG).
- Groves and Ledyard, Optimal Allocation of Public Goods: A Solution to the Free-Rider Problem (1977).
- They argued that the price individuals should pay for influencing public goods should not be proportional to the degree of influence and individual has, but to its square.
- Glen Weyl re-discovered and refined QV into its present form in 2012.


## Vickrey Clarke Groves Mechanism

Each voter has a valuation for each proposal.
The proposal which maximizes the social welfare (in terms of valuation) is chosen as the optimum proposal.

The utility of a voter consists of his valuation of the selected proposal and the payment.

## Collusion in VCG

Each voter pays for the selected issue based on:

- Clarke pivot rule: calculate valuation of other agents on the selected issue when the voter is not present
- Total valuation of other agents on the selected issue if the voter's vote is present

|  | Issue 1 | Issue 2 |
| :--- | :--- | :--- |
| Voter 1 | $\$ 5$ | $\$ 0$ |
| Voter 2 | $\$ 5$ | $\$ 0$ |
| Voter 3 | $\$ 0$ | $\$ 20$ |
| Voter 4 | $\$ 0$ | $\$ 20$ |

Voter 3 will pay: $\$ 20-\$ 00=\$ \$ 0$

## QV - Motivation

Classical mechanisms are based on the notion that everybody is exactly the same and cares the same amount.

Issues for minority groups will be "ignored"

- Plight of African Americans in the United States
- Drug war
- LGBT rights

With quadratic voting, you can vote harder on what's closer to home.

## Quadratic Voting

Preference intensity: multiple votes
Bounded expenditure: voice credit
Each voter pays certain amount of voice credits for her votes on each issue

$$
\text { Cost }=(\# \text { Votes })^{2}
$$

## Price-Theoretic Model Assumptions

- N voters each with a large stock of voice credits (VC)
- Binary decisions (issue A vs. issue B)
- Bank Voice Credits: retain VC for future votes instead of spending everything on a single issue
- Voters receive values in terms of voice credits
- Eg: Tom prefers issue A over issue B
- A > B: Tom receives positive value $2\left|u_{i}\right|$
- $\mathrm{A}<\mathrm{B}$ : Tom receives negative value $-2\left|u_{i}\right|$
- Voice credits are distributed in a fair manner considered by the society and maximizing them leads to social optimality


## QV - Mechanism

Community votes to determine which issue is implemented with each voter choosing a continuous number of votes $\boldsymbol{V}_{\boldsymbol{i}}$ (positive or negative)

A is implemented when $\sum_{i} v_{i} \geq 0$ (B otherwise)
Each voter pays a cost $C\left(v_{i}\right)$ voice credits for their votes where c is differentiable, convex, even, and strictly increasing in $\left|v_{i}\right|$
$C($.$) : Vote Pricing Rule$

## QV - Mechanism

Price-taking assumption: all voters agree on the marginal pivotality (p) of votes on an issue

Marginal pivotality: perceived chance that an additional vote will be pivotal in swinging the election

Voter chooses $v_{i}$ to maximize $2 u_{i} p v_{i}-c\left(v_{i}\right)$
Robustly optimal: for every $\mathrm{p}>0, \mathrm{~N}$, and vector u , each voter i chooses votes $v_{i}^{*}$ so that $\sum_{i} v_{i}^{*}$ has the same sign as $\sum_{i} u_{i}$

## QV - Why quadratic?

Vote pricing rules $c(x)=x^{a}$ for $a>1$
$2 p u_{i}=a\left(v_{i}\right)^{a-1} \rightarrow$
$v_{i}=\operatorname{sign}\left(u_{i}\right)\left(\frac{2 p}{a}\right)^{\frac{1}{a-1}}\left|u_{i}\right|^{\frac{1}{a-1}}$

Proportional if and only if $\mathrm{a}=2$

## QV - Extreme Cases

$$
v_{i}=\operatorname{sign}\left(u_{i}\right)\left(\frac{2 p}{a}\right)^{\frac{1}{a-1}}\left|u_{i}\right|^{\frac{1}{a-1}}
$$

- $a=1$, dictatorship of most intense voter
- $a=\infty$, reduced to 1 p 1 v
- QV is an optimal intermediate point between the extremes of dictatorship and majority rule.


## QV - Real life application



Fig. 1 Voting distributions on two proposals (paying women equally and banning abortion) in the Likertonly ( $n=1095$ ) and QV-only ( $n=1048$ ) test conditions

## QV - Real life application (Colorado, 2019)

Colorado State House of Representatives vote for bills to fund first
Use virtual tokens to buy votes
Difference among preferences for different proposals clearly
highlighted.

## QV - Summary

QV may also be viewed as individuals receiving votes equal to the square root or radical of the voice credits they spend.

QV ensures the state serves the general happiness of the people maximally.

QV radically expands the rights of citizens to fully and freely express their political views.

## Participatory Budgeting

Participatory budgeting (PB) is a process of decision-making, in which citizens decide how to allocate part of a public budget.

Example PB on Brazil: local government asks residents to vote on proposals for how a certain fraction of their total budget should be spent [Cabannes, 2004]

Participatory budgeting allows citizens to identify, discuss, and prioritize public spending projects, and gives them the power to make real decisions about how money is spent.

## Participatory Budgeting

## DESIGN

THE PROCESS
A steering committee that represents the community creates the rules and engagement plan.




DEVELOP PROPOSALS
Volunteer "budget delegates" develop the ideas into feasible proposals.

## Participatory Budgeting Problem

## V : Set of voters

P: Set of proposals, each with cost of $\boldsymbol{C} \boldsymbol{j}$
B: Fixed total budget
$\boldsymbol{v}_{i, j}$ : Benefit a voter i gets from proposal $j$
$\underset{W \subseteq \mathcal{P}}{\arg \max } \sum_{j \in W} \frac{1}{|\mathcal{V}|} \sum_{i \in \mathcal{V}} v_{i, j} \quad$ subject to $\quad \sum_{j \in W} c_{j} \leq B$

Decide How To Spend Your District's

## K-Approval

https://pbstanford.org/diepp e2015/approval?locale=en

## Budget

## - You may cast up to 5 votes

If you make a mistake, ak for a new ballot

- Do not fold the ballot

Х
EXAMPLE
Make an $X$ in the bax to select an option

## ants, Cuiture a Community Facilitien

1. Solar Panels at Douglaston/Little Neck Library $-249-01$ Northern Bivd (Douglaston/Little Neck) ( $\$ 380,000$ ) Project would install solar panels at the Douglaston/Littie Neck Library, creating a greener library community for all
2. Technology Upgrade at the Poppenhusen Library - 121.23 14th Avenue (College Point) (SiO0,000) Project would outfit the Poppenhusen Lbrary meeting room with high tech smart boards and ADA computers for public use.
3. Meeting Room Upgrade at East Flushing Libran - 196.36 Northem Boulevard (Flushing) (\$250,000) Project would upgrade the East Flushing Library's meeting room, which is used for public events and meetings.

## ducation

- 4. P.S. 129 Gymnasium Renovation
120.02 7 Ah Avenue (Coliege Poin) (\$350,000) henovate gym space by replacing backboarde, adding safety padding. new sound system, scoreboard \& electrical upgrade.

5. P.S. 79 Computer Lab Redesign \& Renova $-147-271$ 15th Drive (Whitestone) ( $\$ 200,000$ ) Project would redesign and renovate the current computer lab, as well as replace computers and other technology needs.
6. J.H.S. 194 Gymnasium Renovation 6. J.H.S. 194 Gymnaslum Renovation
-154.60 17th Avenue (Whitestone) ( $\$ 300,000$ )
10.P.S. 193 Gymnasium Rencration
$-152-20$ 11th Avenue (Whitestone) ( $\$ 400,000$ )
Renovate gym space, refinish floor \& add school name, basketball hoops, celling tiles, safety padding \& add scoreboard.
7. M.S. 67 State-of-the-Art S.T.E.A.M. Lab - 51.60 Marathon Parkway (Luttle Neck) ( $\$ 600,000$ ) Project would create state-of-the-art S.T.E.A.M. Lab, including experiment equipment and S.T.E.A.M. technology
8. P.S. 41 Gymnatium Renovation - 214-43 35th Avenue (Bayaide) ( $\$ 600,000$ ) Project would renovate gym apace, including new basketball hoops scoreboard with PA gyutem \& add new retractable wall.
13.P.S. 94 Makerspace for Engineering and Robotics $-41-77$ Lttle Neck Parkway (Litile Neck) $(\$ 200,000$ ) -41-7) would create a Makerspace for engineering and robotics, including robotics kits and other technologies.
9. P.S. 21 Makerspace: Full S.T.E.A.M. Ahead - 147.36 26th Avenue (Flushing) ( 5200,000 ) Project would renovate computer lab into a state-ot-the-art, fully engaging interactive Makerspace for student use.

Parks \& Recreation
15. Basketball Court Renovations at MacNell Park

- MacNeil Park (College Point) (\$650,000)

Project would renovate the MacNell Park basketball court, including repaving and rim/backboard replacement.
16. Adult Fitress Center at Littue Bay Park - Littie Bay Park (Bayside/Whitestone) (\$200,000) Project would install sdult fitness equipment within Littie Bay Park to promote good health and exercise.
17. Adult Fitness Center at MacNeil Park - MacNeil Park (Callege Point) ( $\$ 200,000$ ) Project would install adult fitness equipment within MacNell Park to promote good health and exercise.
18. Adult Fitness Center At Powell's Cove Part

- Poweif's Cove Park (College Point) (\$200,000)

Project would install adult fitness equipment within Powell's Cove Park to promote good heelth and exercise.

## K-Approval

Problems with k-approval voting: don't require the voter to take the cost of proposals into account [Brams and Fishburn 2007]

Assume government has: $\$ 1000$ budget

|  | P1 (cost: 700) <br> Park | P2 (cost: 400) <br> Fab Lab | P3 (cost: 500) <br> Playground |
| :--- | :--- | :--- | :--- |
| Alice | 1 | 1 | 0 |
| Bob | 1 | 0 | 1 |
| Charles | 1 | 1 | 0 |

## Participatory Budgeting Problem - Challenge

Voters may not have a precise knowledge of their valuations
No established unit for their valuations
Impossibility theorem [Arrow, 2012]

Allow voters to compare proposals according to their benefit per dollar in order to optimize the knapsack capacity

## Knapsack Voting

- Voters allocate their preferences among a set of proposals
- Each voter benefits differently from each proposal
- Each voter is constrained by a fixed budget. For e.g. \$1000.

|  | P1 (cost: 700) <br> Park | P2 (cost: 400) <br> Fab Lab | P3 (cost: 500) <br> Playground |
| :--- | :--- | :--- | :--- |
| Alice | 1 | 0 | 0 |
| Bob | 0 | 1 | 1 |
| Charles | 0 | 1 | 1 |

## Knapsack Mechanism

- Vote aggregation
- Each proposal earns a score equal to the number of voters that include it in their votes
- Budget is filled by choosing proposals in descending order of their scores
- Best response
- Considering other voters have already fixed their votes
- Best response for a voter is to choose the subset of proposals that maximizes the total benefits


## Partial Strategy-Proofness

Budget constraint allows partial strategy-proofness in the best response of the voter responding to the votes of all other voters.

Partial strategy-proofness: when a voter has to vote for a certain proposal j , it is in her best interest to also vote for those that she prefers more than j from among the ones that are winning

- Preference between proposals based on benefit per dollar

Sincerity: no benefits of not selecting a better proposal present in the winner set

## Voting Using Comparisons

Value-for-money Vote: for each pair of proposals, a voter chooses a winner with higher benefit per dollar

$$
w_{i}(j, k)=\operatorname{argmax} x_{t \in\{j, k\}} \frac{v_{i, t}}{c_{t}}
$$

Weighted directed graph

- Weight $w_{j \rightarrow k}$ : number of people who prefer proposal j over k


## Aggregation Problem

Aggregate votes by constructing a strict order which minimizes the number of disagreements with respect to the elicited comparisons


## Knapsack Example

Comparison: https://pbstanford.org/nyc27/comparison
Knapsack: https://pbstanford.org/nyc8/knapsack
https://pbstanford.org/boston16internal/knapsack

## Knapsack Summary

This voting scheme can intuitively elicit fine-grained user preference based on value-for-money.

Applying budget constraint leads to strategy-proofness.
Can be implemented using interactive digital tools

## Discussion: QV \& Knapsack

Disadvantages:

- Difficult to conduct on paper compared to 1 p1v (require difficult computations)
- QV implementation requires a mechanism to accumulate the Voice Credits for the individuals.
- Not so easy to understand.

What do you guys think of these two voting mechanisms?

