Spring 2010, CS 598CC: Topics in Combinatorial Optimization
Homework 2
Due: 3/18/2010 in class

Instructions and Policy: Each person should write up their own solutions independently. You need to indicate the names of the people you discussed a problem with. Solutions to most of these problems can be found from one source or the other. Try to solve on your own first, and cite your sources if you do use them.

Please write clearly and concisely. Refer to known facts. You should try to convince me that you know the solution, as quickly as possible.

Problem 1 Given a (multi)-graph $G = (V, E)$ an edge-coloring of G with k colors is an assignment of colors from $1, \ldots, k$ to the edges of G such that no two edges that share a vertex receive the same color. It is well-known that any bipartite graph can be edge-colored with $\Delta(G)$ colors where $\Delta(G)$ is the maximum degree. Show this via the integer decomposition property of the matching polytope of bipartite graphs. Give an example to show that the matching polytope for general graphs does not satisfy the integer decomposition property.

Problem 2 Derive the Tutte-Berge formula from the Cunningham-Marsh formula.

Problem 3 Recall that Gallai's theorem gives an easy way to compute a minimum cardinality edge cover via an algorithm for the maximum cardinality matching problem. Show how to reduce the problem of computing a minimum weight edge cover in a graph to the problem of computing a maximum weight matching. *Hint:* You may want to consider edge weights $c'_{uv} = w_u + w_v - c_{uv}$ where $w_v = \min\{c(e) \mid e \in \delta(v)\}$.

Problem 4 Let $G = (V, E)$ be an undirected graph with non-negative edge lengths $\ell : E \to R^+$. Given nodes s, t we wish to find a shortest s-t path with an odd number of edges. Show that this problem can be solved via matching techniques by following the hint below. Suppose s, t have degree 1. Consider the reduction of the maximum weight matching problem to the maximum weight perfect matching problem. Adapt this reduction and show how a perfect matching can be used to obtain a shortest odd length path from s to t. How can you get rid of the assumption that s, t have degree 1? Extend the ideas for finding a shortest even length s-t path.

Problem 5 [Extra credit] Petersen's theorem states that every bridgeless (no cut-edge) cubic graph has a perfect matching. This can be deduced easily from Tutte-Berge formula. Show that every bridgeless cubic-graph has a T-join of size at most $|V|/2$ for any even $T \subseteq V$.