CS 580: Algorithmic Game Theory, Fall 2023
HW 0

Instructions:

1. The purpose of this homework is to reacquaint you with topics, ideas and tools that will be needed in this course.
2. This homework is **NOT** for submission and will **NOT** be graded.

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1. Let $a_1, a_2, \ldots, a_n$ be fixed real numbers and X be a random variable that takes value a_i with some probability p_i. Define the set of probability distributions that maximize $E[X]$.

2. Consider throwing n balls into n bins where each ball is thrown independently and uniformly at random into a bin.

   (a) What is the probability that a given bin (say the first bin) is empty?

   (b) What is the probability that it contains exactly k balls?

   (c) What is the expected number of bins that are empty?

3. Consider the following linear program:

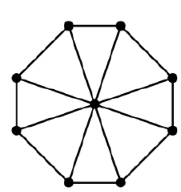
$$\begin{aligned} max \quad & c^T x \\ s.t. \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

   (a) Write the dual linear program of the above LP.

   (b) Write the corresponding complementary slackness conditions.

   (c) Using the complementary slackness conditions, derive the strong duality theorem. *(If x^* is an optimal solution to the primal LP and y^* is an optimal solution to the dual LP, then $c^T x^* = b^T y^*$.)*

4. A *wheel* of size k consists of a cycle on k vertices along with an additional vertex connected to every vertex in the cycle. As an example, you can see a wheel of size 8 in the figure below. The WHEEL problem is the following: Given an undirected graph $G = (V, E)$ and an integer k, does G contain a wheel of size k as a subgraph? Prove that WHEEL is NP-Complete.



(Hint: To show NP-hardness reduce from the Hamiltonian cycle problem.)