

Q: What if the auctioneer has no idea about the D_i 's, the distribution of the values of the bidders?

★ i.i.d. $D_i = D$ regular $\Rightarrow \phi_i = \phi$ monotone.

Thm: $E \left[\begin{array}{l} \text{Rev of} \\ \text{Vickrey-second-price} \\ \text{Auction} \\ \text{w/ } (n+1) \text{ bidders} \end{array} \right] \geq E \left[\begin{array}{l} \text{Rev of} \\ \text{OPT} \\ \text{w/ } n \text{ bidders} \end{array} \right]$

pf: Suppose "HAVE TO" sell the item.

$$\tilde{X} = \left\{ x \in \{0,1\}^n \mid \sum_{i=1}^n x_i = 1 \right\}$$

$$x^*(b) = x^*(v) = \underset{x \in \tilde{X}}{\text{argmax}} \sum_{i=1}^{n+1} \phi(v_i) x_i$$

↑
virtual value.

$$i^* = \underset{i \in N}{\text{argmax}} \phi(v_i) = \underset{i \in N}{\text{argmax}} v_i$$

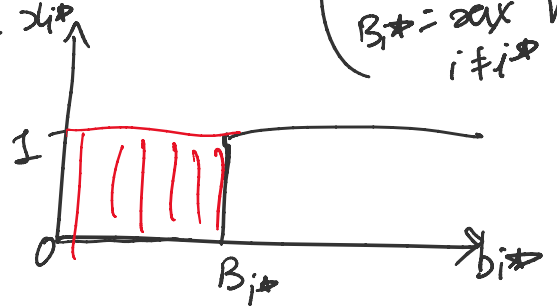
$$(v_{i^*} \geq v_i, \forall i)$$

$$B_{i^*} = \max_{i \neq i^*} v_i$$

OPT Auction when have to sell the item.

$P_{i^*} = B_{i^*} =$ second-highest bid

Second-Price!
Vickery Auction



Always gives out the item

Auction A: $(n+1)$ bidders.

Step-1. Run OPT on 1 to n bidders.

Step-2. If item not sold in step-1, then give it to $(n+1)$ th bidder for free.

$$\mathbb{E} \left[\begin{array}{l} \text{Rev of} \\ \text{Vickrey} \\ \text{w/ } (n+1) \text{ bidders} \end{array} \right] \geq \mathbb{E} \left[\begin{array}{l} \text{Rev of} \\ \text{Auction A} \\ \text{w/ } (n+1) \text{ bidders} \end{array} \right] = \mathbb{E} \left[\begin{array}{l} \text{Rev of} \\ \text{OPT} \\ \text{w/ } n \text{-bidders} \end{array} \right]$$

