Auction: Single-Item

Tuesday, October 24, 2023 8:44 AM

- Auctioneer/ seller wants sell one item.
- N: set of buyers./bidders. $i \in N$ has value v_i (private)

☆ Sealed Bid Auction:

(1) Auctioneer solicit "bids" from agents. in sealed envelop.

agent i bids b_i (need not be v_i)

(2) Auctioneer open all the bids & decides the winner for the item & payment.

Goal: to maximize social welfare ≡ give item to the agent who values it the most!

winner $i^* = \arg\max_i b_i$

payment = p

$u_i(v_i; b_1, \dots, b_n) = v_i - p$ if $i = i^*$

$= 0$ o.w.

$= u_i(b_1, \dots, b_n)$

	1	2	3	4	
Private → v_i	300	500	72	200	
pay your bid → b_i	250	300	60	160	(First price)
			10	200	(second price)

pay your bid → b_i | 250 300 80

pay second highest bid → b_i | 300 400 72 200 (second price)

☆ First - Price Auction: Highest bidder wins & pay the bid.

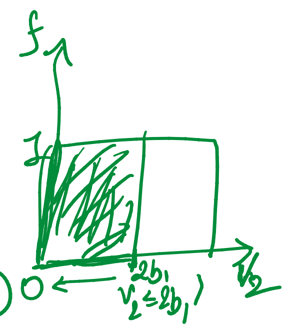
Example: $N = \{1, 2\}$ $\quad v_1, v_2 \sim U[0, 1]$

Suppose we fix $\boxed{b_2 = \frac{v_2}{2}}$

$$u_1(v_1; b_1, b_2) = (v_1 - b_1) \Pr[b_1 \geq b_2] + 0 \Pr[b_1 < b_2]$$

$$= (v_1 - b_1) \Pr\left[b_1 \geq \frac{v_2}{2}\right]$$

$$\Pr[v_2 \leq 2b_1]$$

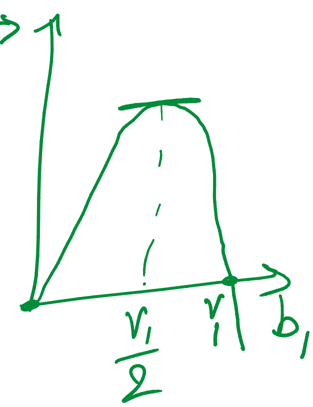


$$= (v_1 - b_1)\,(2b_1) = 2(v_1 b_1 - b_1^2)$$

$\frac{d}{db_1}$

$$= 2(v_1 - 2b_1) = 0$$

$$\Rightarrow \boxed{b_1 = \frac{v_1}{2}}$$



← Best response of agent 1 when $b_2 = \frac{v_2}{2}$.

Fix agent 1 to $b_1 = \frac{v_1}{2}$ then B.R. of agent 2 will be $b_2 = \frac{v_2}{2}$

$\left(\frac{v_1}{2}, \frac{v_2}{2}\right)$ is a Bayes NE. (BNE)

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n agents $\left(\frac{n-1}{n} v_1, \frac{n-1}{n} v_2, \ldots, \frac{n-1}{n} v_n\right)$

- ✯ what if $v_1, v_2 \sim D$ complex?
- ✯ " " $v_1 \sim D_1$, $v_2 \sim D_2$ & D_1 D_2 are complex?
- ✯ Coordination issues if none BNE.
- ✯ what if some agents are not rationally.

✯ Second Price : Highest bidder wins, pays second highest bid.

winner $i^* = \arg\max_i b_i$

payment $p_i = \max_{k \neq i} b_k$ " B_i critical bid of agent i. if $i = i^*$

$= 0$ o.w.

Thm (Vickrey '61): Under Second Price Auction, for each $i \in N$, $b_i = v_i$ is optimal / dominant strategy

NO MATTER how others are bidding.

$\shortparallel$

... dominant strategy

$\Updownarrow$

$b_i = v_i$ is a dominant strategy

$\Updownarrow$

$\forall i, \forall b_{-i} \quad u_i(v_i; b_{-i}) \geq u_i(b_i, b_{-i}), \quad \forall b_i \in \mathbb{R}$

[Also, called Dominant strategy Incentive Compatible (DSIC) ←
Truthful Auction]

Pf: Fix an agt i, say others are bidding b_{-i} arbitrarily.

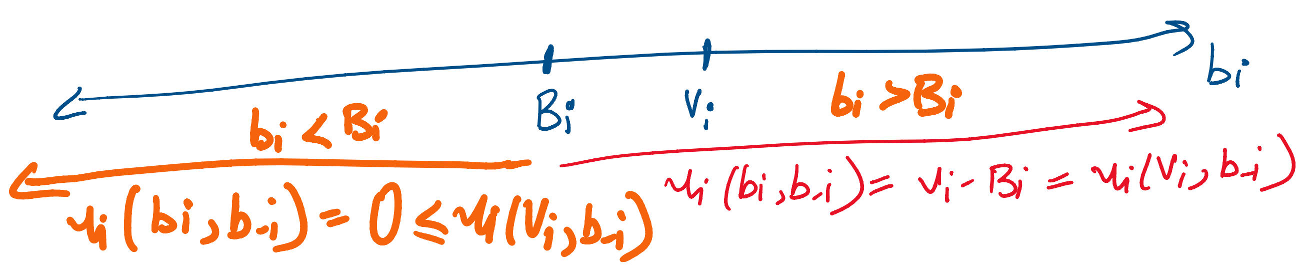
suppose agt i bids $\boxed{b_i = v_i}$

Q: Can i deviate to b_i & improve?

$\exists b_i$ s.t. $u_i(b_i, b_{-i}) > u_i(v_i, b_{-i})$

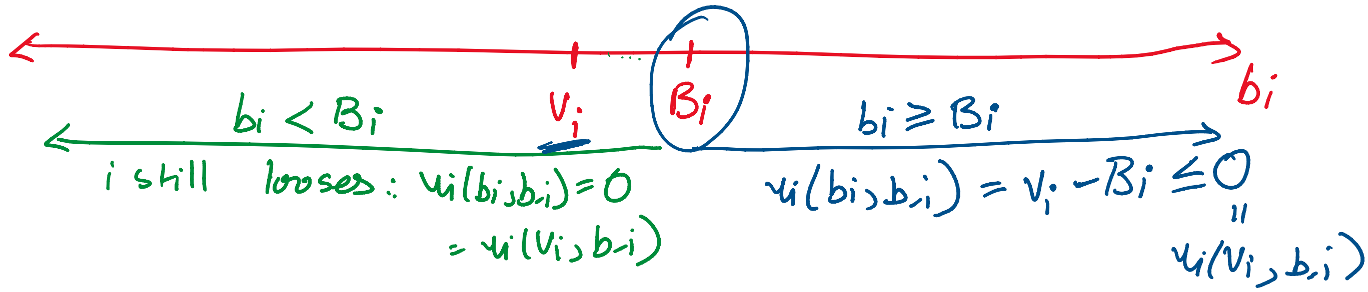
Case I: i wins when $b_i = v_i \geq \max_{k \neq i} b_k = B_i = p_i$

$u_i(v_i, b_{-i}) = v_i - B_i \geq 0$



Case II: i looses when $b_i = v_i$

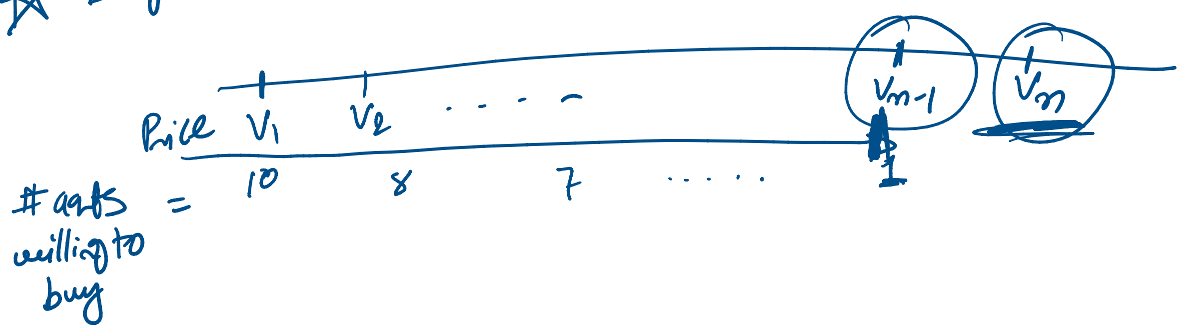
$u_i(v_i, b_{-i}) = 0$



EBay Auction: essentially S.P.

★ English Auction

Second Price



★ Dutch Auction

First Price

Q: what if pay third highest bid?

still DSIC?

If not then counter eg.!