Lecture 12
Stackelberg Eq. & Nash Bargaining

CS580

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Commitment
(Stackelberg strategies)

Commitment

Unique Nash equilibrium
(iterated strict dominance
solution)

1, 1	3, 0
0, 0	2, 1

von Stackelberg

- Suppose the game is played as follows:
  - Alice **commits** to playing one of the rows,
  - Bob observes the commitment and then chooses a column
- Optimal strategy for Alice: commit to Down

Computing the optimal mixed strategy to commit to

[Conitzer & Sandholm EC'06]

- Player 1 (Alice) is a leader.
- Separate LP for every column $j^* \in S_2$:

$\arg\max_{j^*} LP(j^*)$

maximize $\sum_i x_i A_{ij^*}$ — Alice's utility when Bob plays j^*

subject to $\forall j,\ \left(x^T B\right)_{j^*} \geq \left(x^T B\right)_j$ — Playing j^* is best for Bob

$x \geq 0,\ \sum_i x_i = 1$ — x is a probability distribution

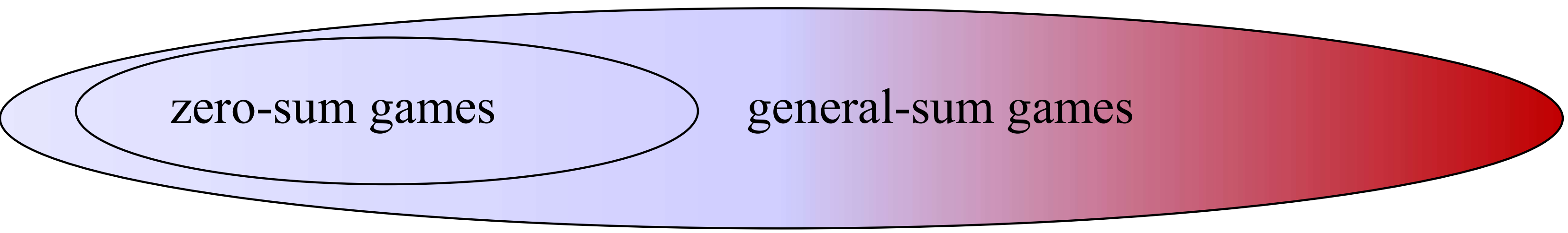
Among soln. of all the LPs,
pick the one that gives max utility.

Generalizing beyond zero-sum games

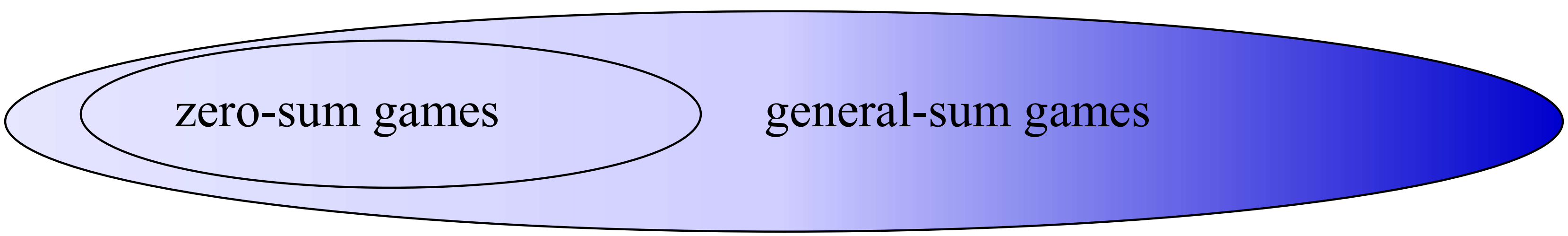
Minimax, Nash, Stackelberg all agree in zero-sum games

0, 0	-1, 1
-1, 1	0, 0

minimax strategies



Nash equilibrium



Stackelberg mixed strategies

Other nice properties of commitment to mixed strategies

- No equilibrium selection problem

0, 0	-1, 1
1, -1	-5, -5

- Leader's payoff at least as good as any Nash eq. or even correlated eq.

(von Stengel & Zamir [GEB '10])

≥

$\max_{x \in \Delta_m} \left(\arg\max_{y:} x^T B y \right) x^T A y.$

Nash Bargaining

Nash Bargaining: Dividing Utilities

Two agents: 1, 2
Outside option utilities: c_1, c_2
Feasible set of utilities: $U \subseteq R^2$ (convex),

$$(c_1, c_2) \in U$$

Goal: define a bargaining function $\mathrm{f}(c_1, c_2, U) \in U$ satisfying certain good properties

Nash Bargaining: Axioms

Two agents: 1, 2
Outside option with utilities: c_1, c_2
Feasible set of Utilities: $U \subseteq R^2$ (convex), $(c_1, c_2) \in U$

Goal: $\mathrm{f}(c_1, c_2, U) \in U$ that is

1. Scale free

2. Symmetric

3. Pareto Optimal

4. Independent of Irrelevant Alternatives (IIA)

5. Individually Rational

Nash Bargaining: Theorem

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Goal: $f(c_1, c_2, U) \in U$ that is

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Theorem (Nash'50). f satisfies the 5 axioms if and only if, $f(c_1, c_2, U)$ is

$$argmax\ (u_1 - c_1)(u_2 - c_2)$$
$$\text{s.t.} \quad (u_1, u_2) \in U$$
$$u_1 \geq c_1, u_2 \geq c_2$$

Nash Bargaining: Theorem

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Proof. ($\Leftarrow$)

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2. Symmetric

3. Pareto Optimal

4. Independent of Irrelevant Alternatives (IIA)

5. Individually Rational