Total Function Complexity

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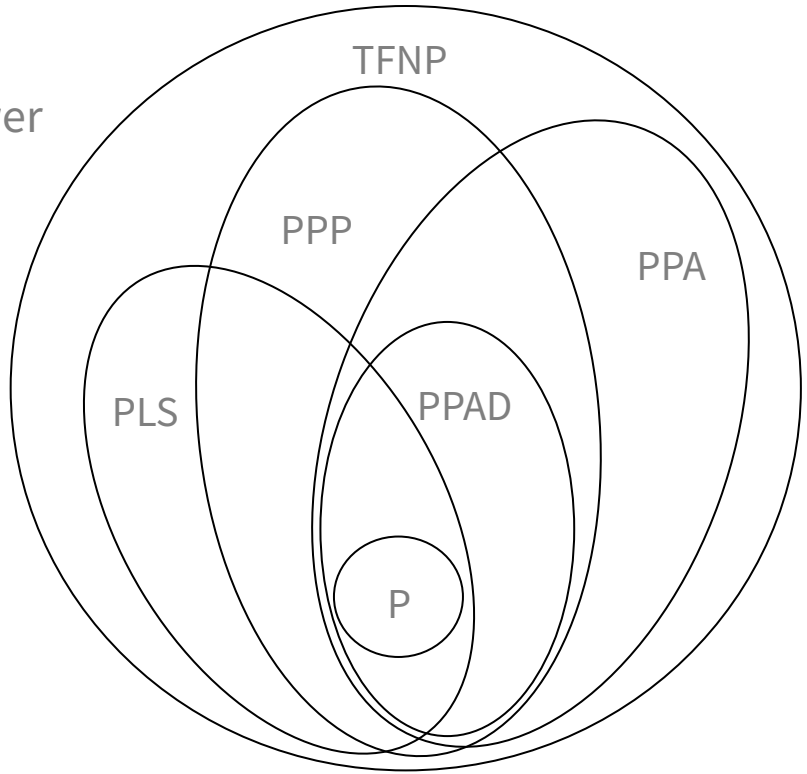
Function Problems Review

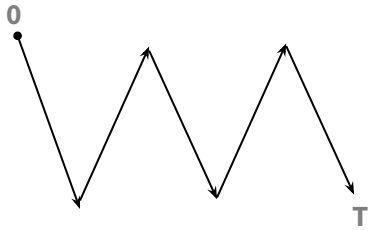
- NP asks decision problems with Yes/No answer
  - Usually tied to existence or solution checking
- FNP problems find the solution
- TFNP problems are guaranteed to have a solution
- TFNP problems are likely not NP-hard

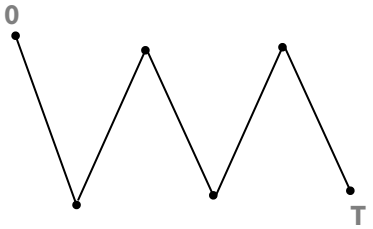


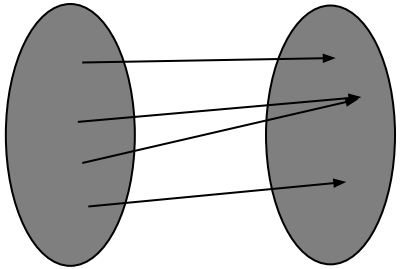
Paper Overview

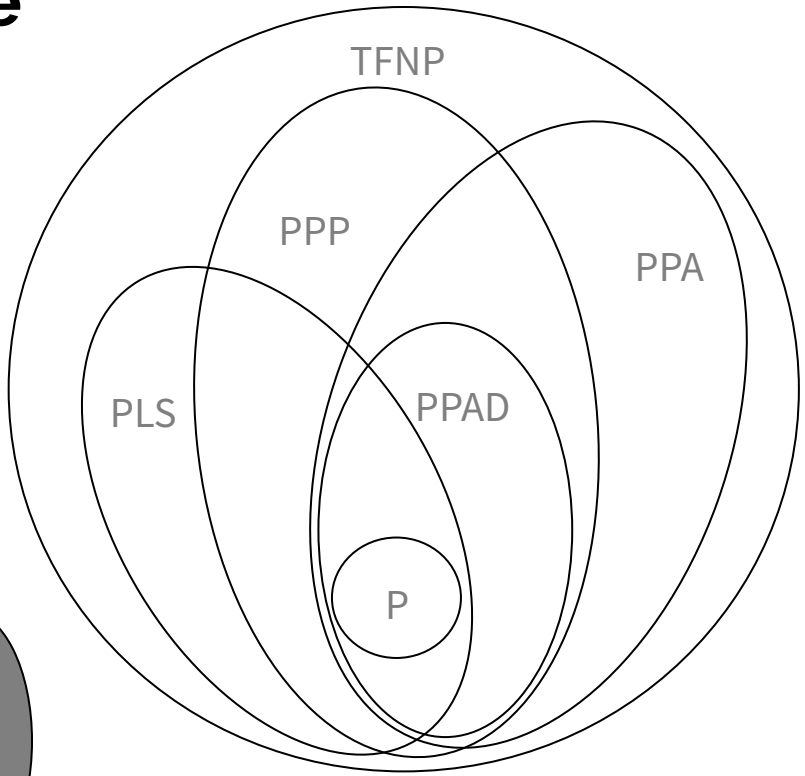
Inefficient proofs of existence

- Why are these new complexity classes useful?
  - Class-Complete problems



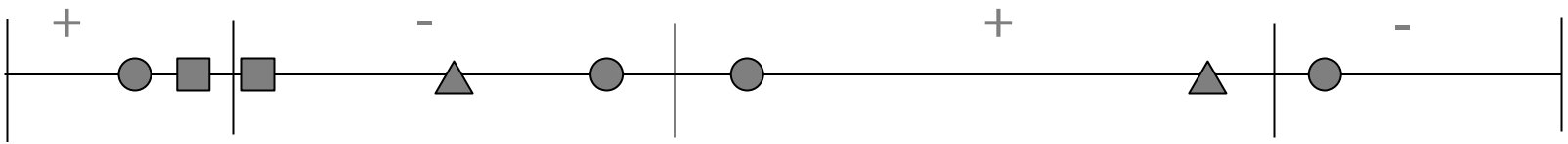






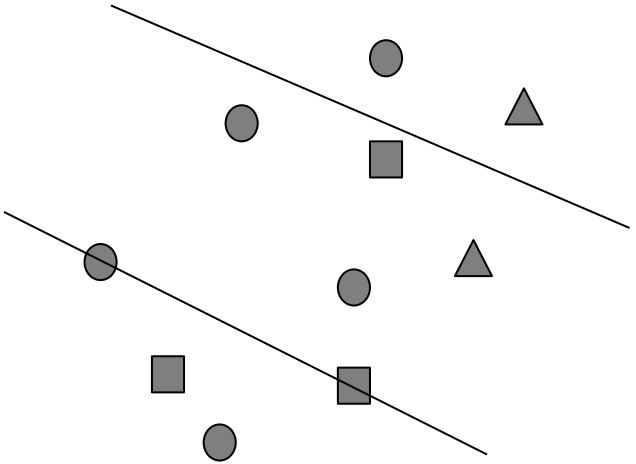
Hardness Results for Consensus Halving

- A dictator must split resources into two groups so that every agent believes those two groups to be equal
- Solution will always exist with n cuts if we are satisfying n agents
- Valuations are on a interval [0,1]



Do approximations make it easier?

- ε-approximate consensus halving problem
  - 2 Block Uniform
    - PPA-Complete
  - Inverse Exponential ε
    - PPA-Complete
  - Constant $\varepsilon < 0.2$
    - PPA-Complete
- ε-approximate necklace splitting problem
  - PPA-Hard, but complete for $\varepsilon < 0.2$
- ε-approximate pizza cutting problem
  - PPA-Hard, but complete for $\varepsilon < 0.2$



Hardness results of Various equilibria

- High-payoff 2-player NE is NP-Complete
- 2-player NE is PPAD-Complete
- NE for Congestion games is PPAD∩PLS-Complete
    - Leads to a corollary that finding a fixed point of a gradient descent problem for a smooth gradient is also PPAD∩PLS-Complete

Discussion

Have these new complexity classes helped?

- Complexity classes presented here were only the most useful
  - CLS
  - CCLS
  - EOPL
  - UEOPL
- Some classes still do not have “natural” complete problems
  - If the complexity class’s existence is defined by a graph with a circuit, and all complete problems also need to use a circuit, isn’t the class almost trivial?

Have these new complexity classes helped?

- Classes are still being populated by new research into problems
- Many problems are related to game theory or social welfare
- PPA had a non-circuit based problem (Consensus Halving) defined in 2018
- Provides insight into how closely related many social welfare problems are

Algorithmic Progress

- Algorithms to solve NE have existed since before TFNP
- Algorithms to solve many social welfare problems have not been efficiently created yet, and the aforementioned algorithms could link into that process
- The collapsing of CLS, CCLS, etc. also links many problems with gradient descent algorithms