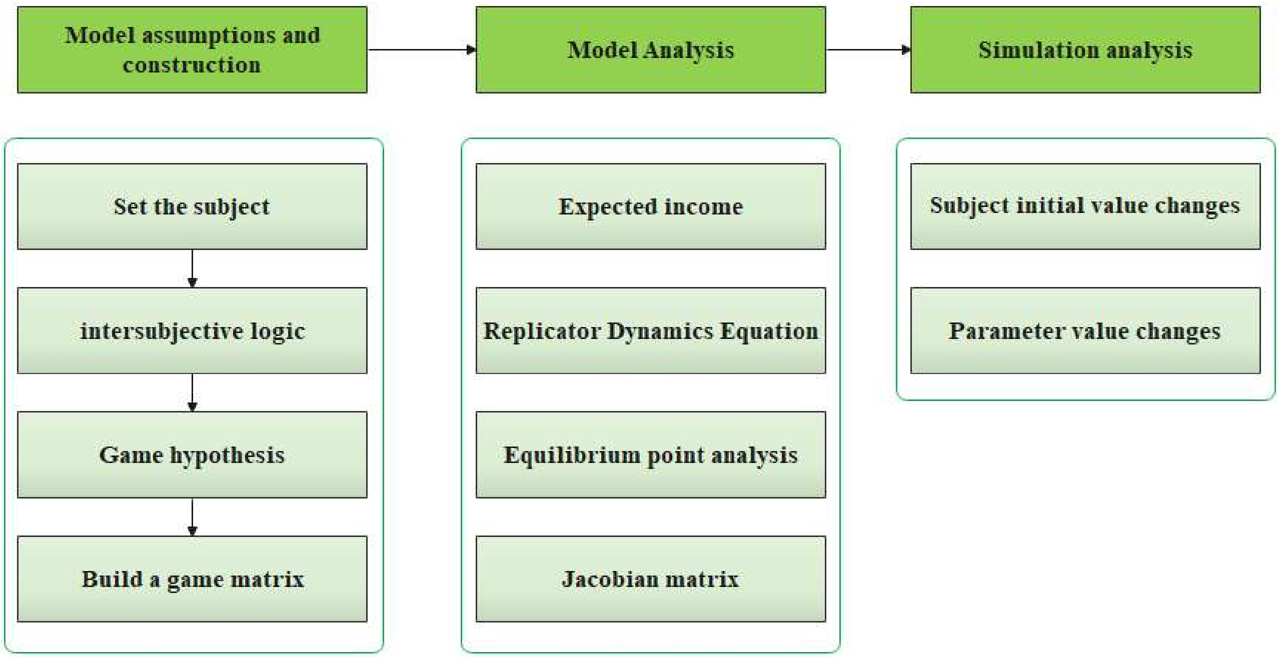
Three-Party Evolutionary Game Analysis of NGO Participation in Corporate Carbon Emissions

By Zhenyu Liao

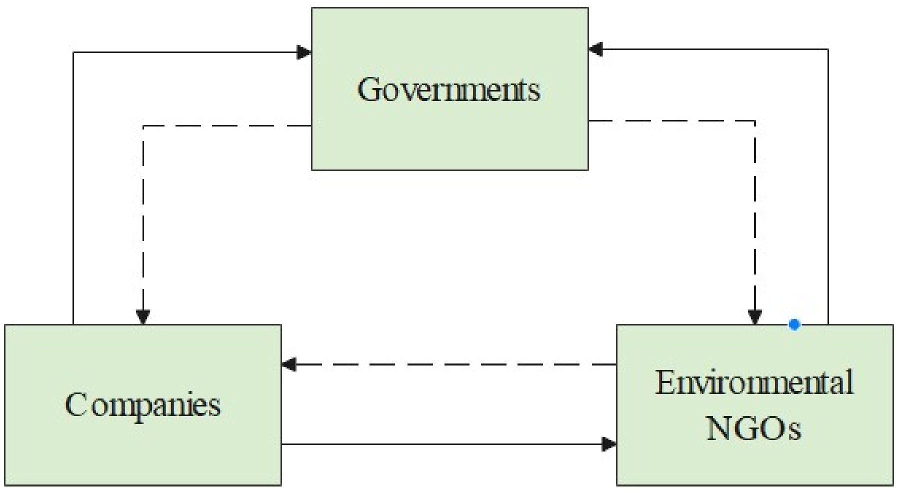
What is an evolutionary game

- Evolutionary Stable Strategy
  - Due to the limited rationality of the game player, it is impossible for the game player to find the optimal strategy and the optimal equilibrium point from the beginning. After a period of imitation and error correction, all game parties will tend to a certain stable strategy.
- Replication Equation
  - A dynamic differential equation describing the frequency or frequency of a particular strategy adopted by a group.

The Structure of the Three-Party Evolutionary Game



Game Subject



- Companies
  - Maximize profits and favor high-carbon production strategies to save the company's environmental costs
- Governments
  - Public interest as goal
- Environmental NGOs
  - Pursue comprehensive benefits such as ecological and environmental benefits as the goal

Game Subject Behavior Strategy

- Companies behavior strategy set
  - S1={K1, K2}
- Government behavior strategy set
  - S2={M1, M2}
- Environmental NGOs behavior strategy set
  - S3={N1, N2}

The Probability of Behavioral Strategy

- The probability that the enterprise chooses the “K1" strategy is x, and the probability of choosing the “K2" strategy is 1-x;
- The probability that the government chooses the “M1" strategy is y, and the probability of choosing the “M2" strategy is 1-y;
- The proportion of environmental NGOs choosing the “N1” strategy is z, and the proportion choosing the “N2” strategy is 1-z.
- Among them, $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.
- A total of eight strategy combinations are available, (K,M,N)

Relevant Parameter Assumptions

Companies	Governments	Environmental NGOs
benefits from adopting low-carbon production E1	Potential benefits from low-carbon production E3	Regulatory costs C5
benefits from adopting traditional production E2	The cost of government regulation C3	Government funding S2
The cost of low-carbon production C1	Environmental pollution control costs C4	Public funding S3
The cost of traditional production C2		
Low carbon production subsidies S1		
Fines for traditional production form Governments G1		
Fines for traditional production form NGOs G2		

Strategy portfolio and income matrix

Strategy protfolio	Companies	Governments	NGOs
(K_1, M_1, N_1)	$E_1 - C_1 + S_1$	$E_3 - C_3 - S_1 - S_2$	$-C_5 + S_2 + S_3$
(K_1, M_1, N_2)	$E_1 - C_1 + S_1$	$E_3 - C_3 - S_1$	0
(K_1, M_2, N_1)	$E_1 - C_1$	$E_3 - S_2$	$-C_5 + S_2 + S_3$
(K_1, M_2, N_2)	$E_1 - C_1$	E_3	0
(K_2, M_1, N_1)	$E_2 - C_2 - G_1 - G_2$	$-C_3 - C_4 + G_1 - S_2$	$-C_5 + S_2 + S_3$
(K_2, M_1, N_2)	$E_2 - C_2 - G_1$	$-C_3 - C_4 + G_1$	0
(K_2, M_2, N_1)	$E_2 - C_2 - G_2$	$-C_4 - S_2$	$-C_5 + S_2 + S_3$
(K_2, M_2, N_2)	$E_2 - C_2$	$-C_4$	0

Replicator Dynamics Equation

- Companies
  - Adopt low-carbon production V_{11}

$$V_{11} = yz(E_1 - C_1 + S_1) + y(1-z)(E_1 - C_1 + S_1) + (1-y)z(E_1 - C_1) + (1-y)(1-z)(E_1 - C_1)$$

  - Adopt traditional production V_{12}

$$V_{12} = yz(E_2 - C_2 - G_1 - G_2) + y(1-z)(E_2 - C_2 - G_1) + (1-y)z(E_2 - C_2 - G_2)$$

  - Average expectation V_1

$$V_1 = xV_{11} + (1-x)V_{12}$$

  - Replicator Dynamics Equation

$$F(x) = \mathrm{d}x/\mathrm{d}t = x(V_{11} - V_1)$$
$$= x(1-x)[E_1 - C_1 + y(C_2 + G_1 + S_1 - E_2 + zE_2 - zC_2) + z(C_2 + G_2 - E_2)]$$

Replicator Dynamics Equation

- Governments
  - Replicator Dynamics Equation

$$F(y) = y(1-y) \quad [x(zE_3 - zE_2 - G_1 - S_1) - C_3 + G_1]$$

- Environmental NGOs
  - Replicator Dynamics Equation

$$F(z) = z(1-z)(S_2 + S_3 - C_5)$$

Equilibrium point stability analysis

- Jacobian Matrix of Three-Party Evolutionary Game

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \\ J_4 & J_5 & J_6 \\ J_7 & J_8 & J_9 \end{bmatrix} = \begin{bmatrix} \partial F(x)/\partial x & \partial F(x)/\partial y & \partial F(x)/\partial z \\ \partial F(y)/\partial x & \partial F(y)/\partial y & \partial F(y)/\partial z \\ \partial F(z)/\partial x & \partial F(z)/\partial y & \partial F(z)/\partial z \end{bmatrix}$$

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[- x*(C2 - C1 + E1 - E2 + G1*y + G2*z + S1*y) - (x - 1)*(C2 - C1 + E1 - E2 + G1*y + G2*z + S1*y),
[                                                     y*(y - 1)*(G1 + S1 + E2*z - E3*z),
[                                                                                  0,
                                                                                        -x*(x - 1)*(G1 + S1),
                            y*(C3 - G1 + G1*x + S1*x + E2*x*z - E3*x*z) + (y - 1)*(C3 - G1 + G1*x + S1*x + E2*x*z - E3*x*z),
                                                                                                          0,
                                                                                                          -G2*x*(x - 1)]
                                                                                                 y*(E2*x - E3*x)*(y - 1)]
                                                                            - z*(S2 - C5 + S3) - (z - 1)*(S2 - C5 + S3)]
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Equilibrium point stability analysis

- Equilibrium Point
  - Let F(x) = 0, F(y)= 0, F(z) = 0

$D_1(0,0,0)$
$D_2(0,0,1)$
$D_3(0,1,0)$
$D_4(1,0,0)$
$D_5(0,1,1)$
$D_6(1,1,0)$
$D_7(1,0,1)$
$D_8(1,1,1),$
$D_9(-(C_3-G_1)/(G_1+S_1),(C_1-E_1)/(C_2-E_2+G_1+S_1),0)$
$D_{10}(-(C_3-G_1)/(E_2-E_3+G_1+S_1),-(C_2-C_1+E_1-E_2+G_2)/(G_1+S_1),1)$

Equilibrium point stability analysis

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$D_1(0,0,0)$
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$D_4(1,0,0)$
$D_5(0,1,1)$
$D_6(1,1,0)$
$D_7(1,0,1)$
$D_8(1,1,1)$,

Pure strategy

$D_9(-(C_3-G_1)/(G_1+S_1),(C_1-E_1)/(C_2-E_2+G_1+S_1),0)$
$D_{10}(-(C_3-G_1)/(E_2-E_3+G_1+S_1),-(C_2-C_1+E_1-E_2+G_2)/(G_1+S_1),1)$

Abandon mixed strategy

- Next, compute eigenvalues by substituting the equilibrium point into the Jacobian matrix.

Equilibrium point stability analysis

- Next, compute eigenvalues by substituting the equilibrium point into the Jacobian matrix

Nash Equilibrium	Jacobian Matrix Eigenvalues
$D_1(0,0,0)$	$E_1 - C_1, G_1 - C_3, S_2 - C_5 + S_3$
$D_2(1,0,0)$	$C_1 - E_1, - C_3 - S_1, S_2 - C_5 + S_3$
$D_3(0,1,0)$	$C_2 - C_1 + E_1 - E_2 + G_1 + S_1, C_3 - G_1, S_2 - C_5 + S_3$
$D_4(0,0,1)$	$C_2 - C_1 + E_1 - E_2 + G_2, G_1 - C_3, C_5 - S_2 - S_3$
$D_5(1,1,0)$	$C_1 - C_2 - E_1 + E_2 - G_1 - S_1, C_3 + S_1, S_2 - C_5 + S_3$
$D_6(1,0,1)$	$C_1 - C_2 - E_1 + E_2 - G_2, E_3 - E_2 - C_3 - S_1, C_5 - S_2 - S_3$
$D_7(0,1,1)$	$C_2 - C_1 + E_1 - E_2 + G_1 + G_2 + S_1, C_3 - G_1, C_5 - S_2 - S_3$
$D_8(1,1,1)$	$C_1 - C_2 - E_1 + E_2 - G_1 - G_2 - S_1, C_3 + E_2 - E_3 + S_1, C_5 - S_2 - S_3$

- Apply Liapunov's First Method to determine the stability of the equilibrium point
  - The equilibrium point is stable when the real parts of all eigenvalues are negative.
  - The equilibrium point is unstable when at least one eigenvalue has a positive real part.
  - The equilibrium point is semi-stable when there are some eigenvalues with negative or zero real parts, but no positive real part eigenvalues.

Nash Equilibrium	Jacobian Matrix Eigenvalues	Real Parts	Stability Conclusion
$D_1(0,0,0)$	$E_1 - C_1, G_1 - C_3, S_2 - C_5 + S_3$	(−, −, ×)	uncertain
$D_2(1,0,0)$	$C_1 - E_1, - C_3 - S_1, S_2 - C_5 + S_3$	(−, −, ×)	uncertain
$D_3(0,1,0)$	$C_2 - C_1 + E_1 - E_2 + G_1 + S_1, C_3 - G_1, S_2 - C_5 + S_3$	(+, ×, ×)	unstable
$D_4(0,0,1)$	$C_2 - C_1 + E_1 - E_2 + G_2, G_1 - C_3, C_5 - S_2 - S_3$	(×, +, ×)	unstable
$D_5(1,1,0)$	$C_1 - C_2 - E_1 + E_2 - G_1 - S_1, C_3 + S_1, S_2 - C_5 + S_3$	(−, ×, ×)	uncertain
$D_6(1,0,1)$	$C_1 - C_2 - E_1 + E_2 - G_2, E_3 - E_2 - C_3 - S_1, C_5 - S_2 - S_3$	(×, +, −)	unstable
$D_7(0,1,1)$	$C_2 - C_1 + E_1 - E_2 + G_1 + G_2 + S_1, C_3 - G_1, C_5 - S_2 - S_3$	(+, −, −)	unstable
$D_8(1,1,1)$	$C_1 - C_2 - E_1 + E_2 - G_1 - G_2 - S_1, C_3 + E_2 - E_3 + S_1, C_5 - S_2 - S_3$	(−, −, −)	ESS

- Analyze $D_8(1,1,1)$:
  - When the sum of subsidies and penalties exceeds the profit difference compared to traditional production, government regulations and environmental NGO oversight will eventually compel the company to adopt low-carbon production methods.

Nash Equilibrium	Jacobian Matrix Eigenvalues	Real Parts	Stability Conclusion
$D_1(0,0,0)$	$E_1 - C_1$, $G_1 - C_3$, $S_2 - C_5 + S_3$	(−,−,×)	uncertain
$D_2(1,0,0)$	$C_1 - E_1$, $- C_3 - S_1$, $S_2 - C_5 + S_3$	(−,−,×)	uncertain
$D_3(0,1,0)$	$C_2 - C_1 + E_1 - E_2 + G_1 + S_1$, $C_3 - G_1$, $S_2 - C_5 + S_3$	(+, ×, ×)	unstable
$D_4(0,0,1)$	$C_2 - C_1 + E_1 - E_2 + G_2$, $G_1 - C_3$, $C_5 - S_2 - S_3$	(×,+, ×)	unstable
$D_5(1,1,0)$	$C_1 - C_2 - E_1 + E_2 - G_1 - S_1$, $C_3 + S_1$, $S_2 - C_5 + S_3$	(−,×, ×)	uncertain
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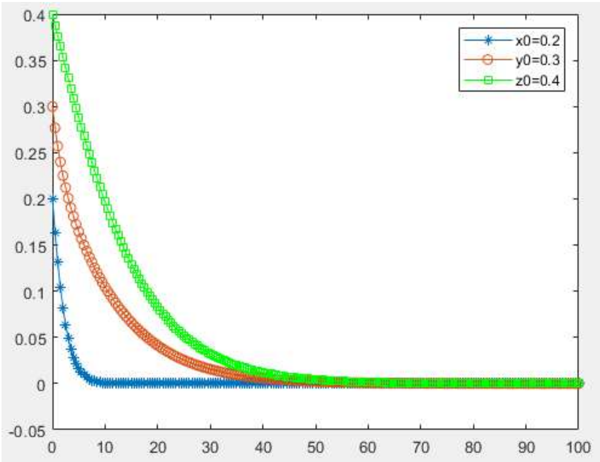
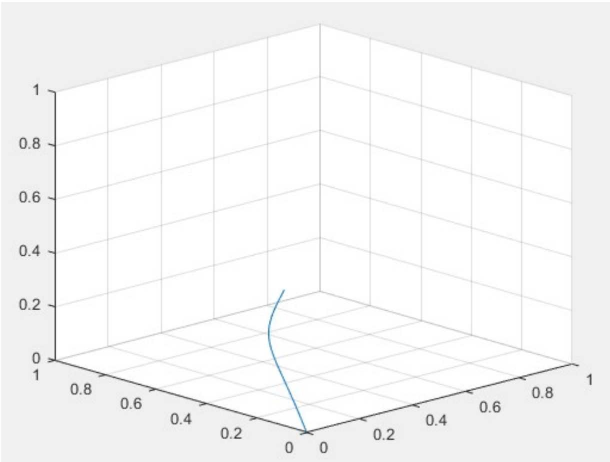
$D_8(1,1,1)$	$C_1 - C_2 - E_1 + E_2 - G_1 - G_2 - S_1$, $C_3 + E_2 - E_3 + S_1$, $C_5 - S_2 - S_3$	(−,−,−)	ESS

Simulation Analysis

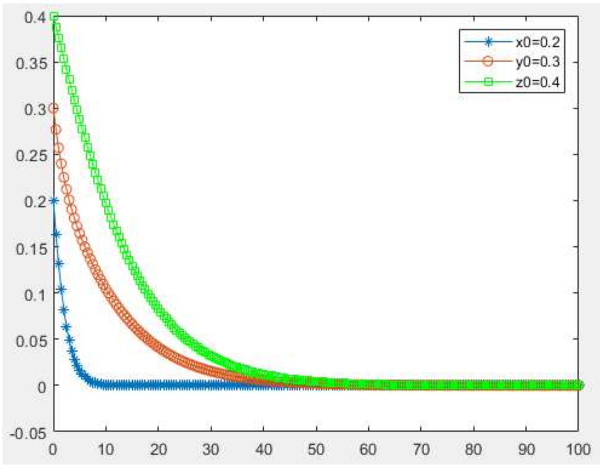
- Let x0, y0 and z0 represent the initial proportions of companies choosing the "adopting" strategy, the government choosing the "regulation" strategy, and environmental NGOs choosing the "supervision" strategy respectively.
- The initial time is 0, end time is 100, and the initial state is (0. 2, 0. 3, 0. 4).
- The parameter values are E1 =0. 2, E2 = 0. 5, E3 = 0. 9, C1 = 0. 9, C2 = 0. 3, C3 =0. 9, C5 = 0. 4, S1 = 0. 05, S2 = 0. 2, S3 = 0. 1, G1 =0. 8, G2 = 0. 2.

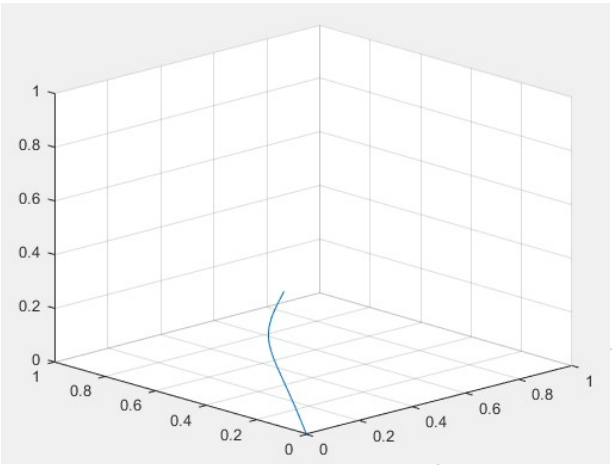




Simulation Analysis

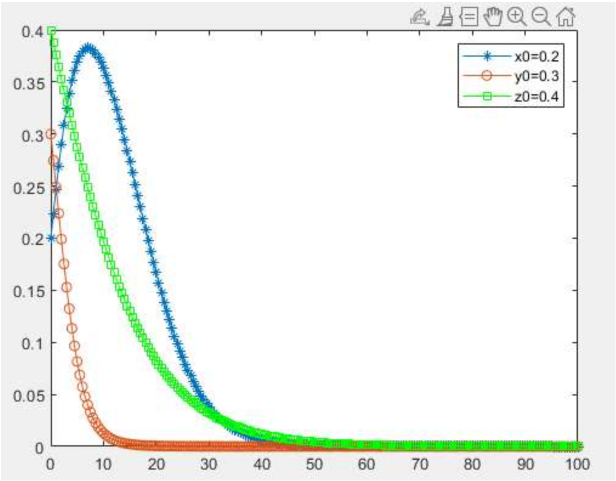
- E1 =0. 2, E2 = 0. 5, E3 = 0. 9, C1 = 0. 9, C2 = 0. 3, C3 =0. 9, C5 = 0. 4, S1 = 0. 05, S2 = 0. 2, S3 = 0. 1, G1 =0. 8, G2 = 0. 2.
- When the cost of adopting low-carbon production methods is high, and the subsidies and benefits gained by the company cannot offset the expenses incurred, the company is unlikely to adopt such methods. In such cases, both government and NGOs may prove ineffective.

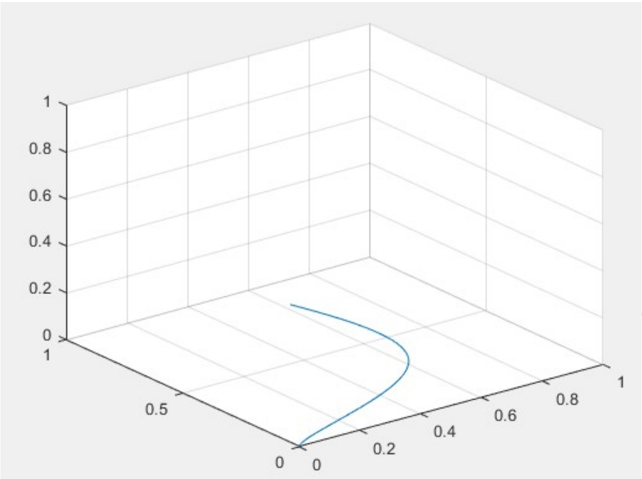




Simulation Analysis

- E1 =0. 2, E2 = 0. 5, E3 = 0. 9, C1 = 0. 9, C2 = 0. 3, C3 =0. 9, C5 = 0. 4, S1 = 0. 05, S2 = 0. 2, S3 = 0. 1, G1 =0. 8, G2 = 0. 6.
- Increasing the intensity of environmental NGO supervision will only temporarily increase the rate at which companies use low-carbon production. In the long run, the system will still collapse.





Thanks