Chess Strategy Survey

Flora Zheng
Instructor: Professor Ruta Mehta
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History of chess programs

Chess is a two-player zero-sum game on

a 8x8 board.

1989: Chess world champion Garry Kasparov

won against Deep Thought

1997: State-of-the-art chess program Deep Blue

used alpha-beta search to defeat Kasparov

Problem Introduction

Problem: Chess is an extensive-form game. Given the current situation of a chess board or node in the game tree, determine the best move to make next. The goal is to find a winning strategy.

Motivation: The game tree for chess has a large number of search states

Literature

Claude Shannon presented chess playing with an evaluation function and quiescence search in 1949.

John McCarthy devised alpha-beta search in 1956.

Papers surveyed

Shannon, C.: Programming a Computer for Playing Chess. Journal **41**(314), 1-18 (1949)

Description: An evaluation function can be used to compute the value of a node in the game tree. Shannon presents quiescence search.

Gérard M. Baudet. 1978. An analysis of the full alpha-beta pruning algorithm. In Proceedings of the tenth annual ACM symposium on Theory of computing (STOC '78). Association for Computing Machinery, New York, NY, USA, 296–313. https://doi.org/10.1145/800133.804359

Description: Minimax search runs in $O(n^d)$. Minimax search with alpha-beta pruning and optimal ordering of nodes in the game tree reduces the amount of computation to $O(n^{d/2})$.

C. B. Browne *et al.*, "A Survey of Monte Carlo Tree Search Methods," in *IEEE Transactions on Computational Intelligence and AI in Games*, vol. 4, no. 1, pp. 1-43, March 2012, doi: 10.1109/TCIAIG.2012.2186810.

Description: Stochastic search provides another technique. Find a leaf node to search. Evaluate to the desired depth as determined by how much computation is affordable. Then average the utility values over all simulations.

Paper

Shannon, C.: Programming a Computer for Playing Chess. Journal **41**(314), 1-18 (1949)

Infeasible to evaluate all possible moves

Large number of search states

Over 10^{40} nodes in the game tree

The horizon effect is that the game tree is searched to a depth d due to limited computational resources. There could be a detrimental move beyond the horizon at depth d+1 such as the capture of a queen. The impact of the horizon effect can be mitigated by searching more game states past depth+1. In quiescence search, continue searching unstable moves such as moves that capture a piece. In stochastic search, continue searching a small subset of possible paths.

Representation of a chess position

Shannon stated that a chess position can include

1) The positions of all pieces on the board
2) White or black to represent which player's turn to move
3) Whether the king and rooks have moved to know whether castling is legal
4) The last move to determine whether en passant capture is legal

Evaluation function

Calculate the value of the chess board at a node of the game tree

Seek low computational complexity so a linear evaluation function is frequently used.

Let the features be $f_1(s)$, $f_2(s)$, …. Let the weights be w_1, $w_{2, ...}$

Value of a position P $f(P)=w_1 f_1(s)+w_2 f_2(s)+...$

One simple evaluation function is to assign points to remaining pieces.

Queen: 9, Rook: 5, Bishop: 3, Knight: 3, Pawn: 1

f(P) = Score of white - Score of black

The evaluation function can also take the positions of the pieces into account.

For the next move, take the move with the maximum score over all possible next moves. The chess program could not win against human chess grandmasters.

Paper

Gérard M. Baudet. 1978. An analysis of the full alpha-beta pruning algorithm. In Proceedings of the tenth annual ACM symposium on Theory of computing (STOC '78). Association for Computing Machinery, New York, NY, USA, 296–313. https://doi.org/10.1145/800133.804359

Minimax search

Minimax(node) =

$\min_{action}$ Minimax(Successor(node, action)) if the player is MIN

$\max_{action}$ Minimax(Successor(node, action)) if the player is MAX

utility value of the node if the node is a leaf node

Minimax search

Define n as the number of moves and d as the depth of the game tree.

Computational complexity of minimax is $O(n^d)$

Minimax search with alpha-beta pruning

Let α be the highest score that MAX is able to force MIN into receiving. Let β be the lowest score MIN is able to force MAX into receiving.

The pruning thresholds are α and β. If a node in the game tree has $\alpha \geq \beta$, then the subsequent children of the node can be pruned.

At the root node of the game tree, $\alpha=-\infty$ and $\beta=\infty$.

Minimax search with alpha-beta pruning

Alpha-beta pruning seeks to decrease the computational complexity of minimax search from $O(n^d)$ to $O(n^{d/2})$. The following properties have to hold to attain $O(n^{d/2})$ computational complexity. The children of a MIN node are computed beginning with the lowest-value child and then in ascending order. The children of a MAX node are computed beginning with the highest-value child and then in decreasing order.

The difference between minimax search and alpha-beta search is analogous to the difference between a chess beginner and a grandmaster.

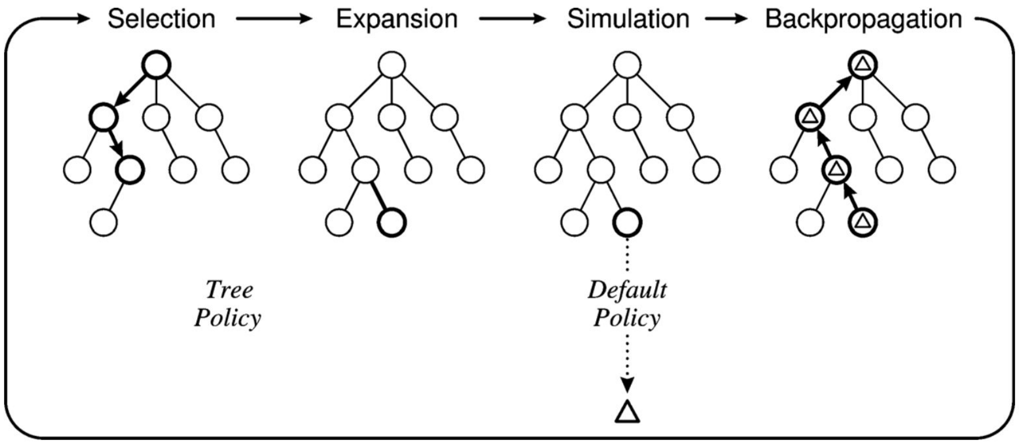
Paper

C. B. Browne *et al.*, "A Survey of Monte Carlo Tree Search Methods," in *IEEE Transactions on Computational Intelligence and AI in Games*, vol. 4, no. 1, pp. 1-43, March 2012, doi: 10.1109/TCIAIG.2012.2186810.

Stochastic search

Stochastic search can be used to calculate the value of a state in the game tree.

There is a list of possible next moves. For e
the values from the p paths to calculate th
over all possible next moves.



Results

Define n as the number of moves and d as the depth of the search.

Computational complexity of minimax search is $O(n^d)$

Computational complexity of minimax search with alpha-beta pruning and optimal ordering of nodes in the game tree is $O(n^{d/2})$

Define n as the number of initial moves and p as the number of different paths to evaluate for each initial move. Computational complexity of stochastic search is O(npd).

Discussion

The search with alpha-beta pruning performs faster than the minimax search because some of the leaf nodes in the game tree are no longer calculated. The stochastic strategy likely performs worse than the minimax search strategy and alpha-beta search strategy due to not evaluating over all possible states.

Conclusions

A game tree models a zero-sum game between two players. An evaluation function can be used to calculate the value of a game state. Limited-horizon search is needed but provides an estimate.

References

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Russell, Stuart and Norvig, Peter: Artificial Intelligence: A Modern Approach. 4th edn. Pearson, 221 River Street, Hoboken, NJ 07030 (2021). Image for alpha-beta pruning from https://aima.cs.berkeley.edu/figures.pdf

Appendix

Alpha-beta Pruning

Max node Upwards triangle

Min node Downwards triangle