Mechanisms for matching markets on ride-sharing

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Uber

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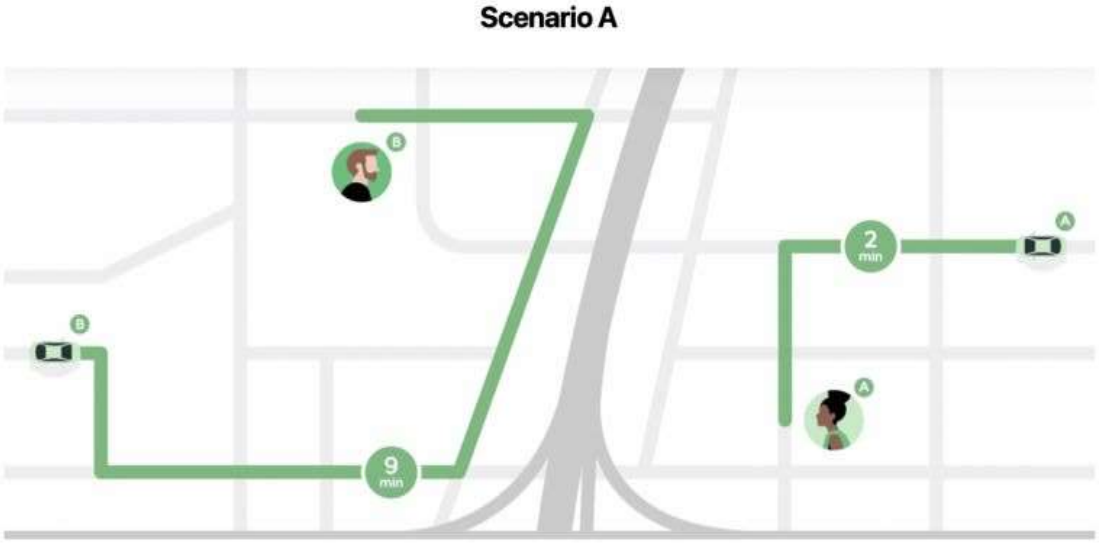

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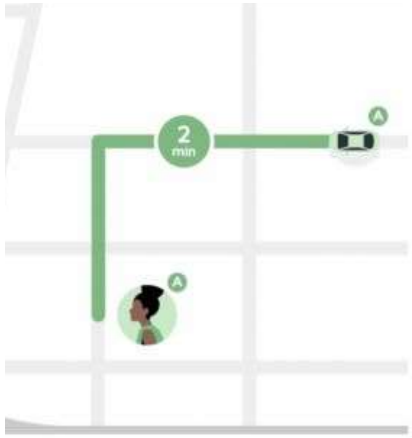

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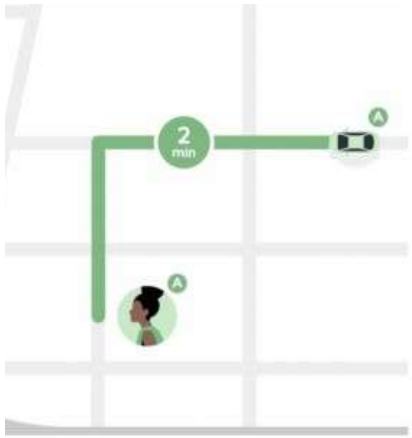

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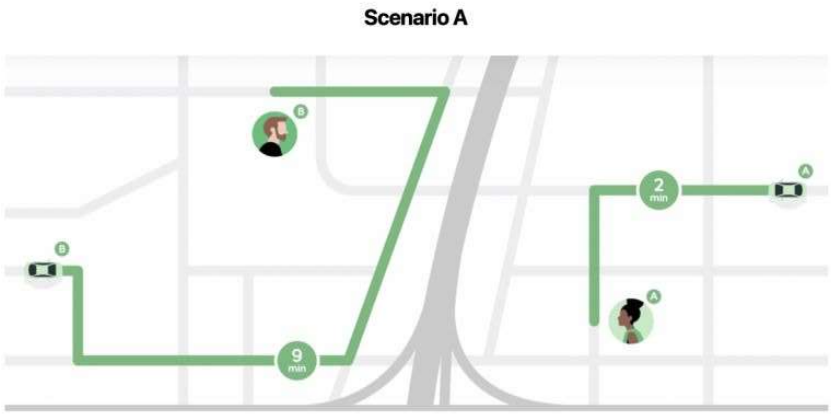

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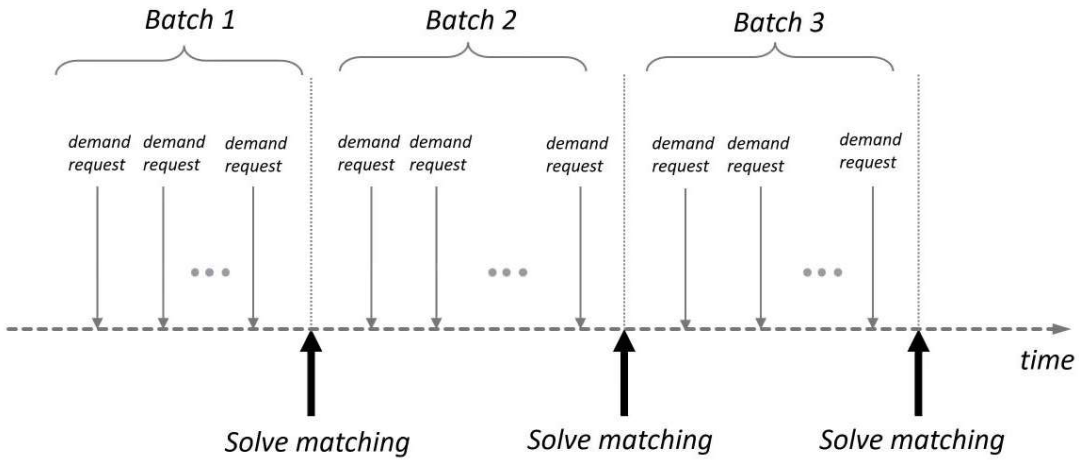

Image Source: Yiding Feng

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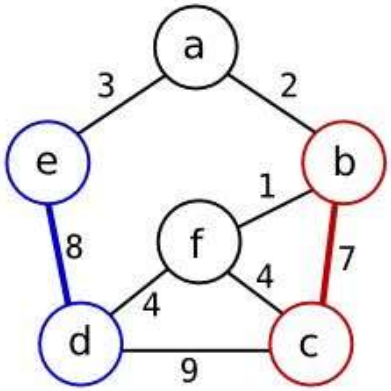

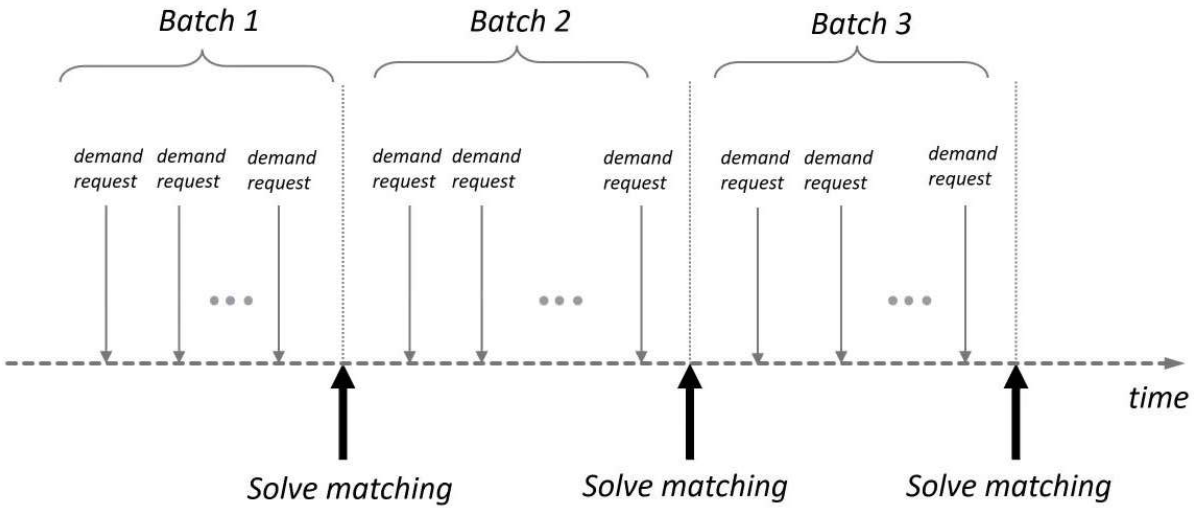

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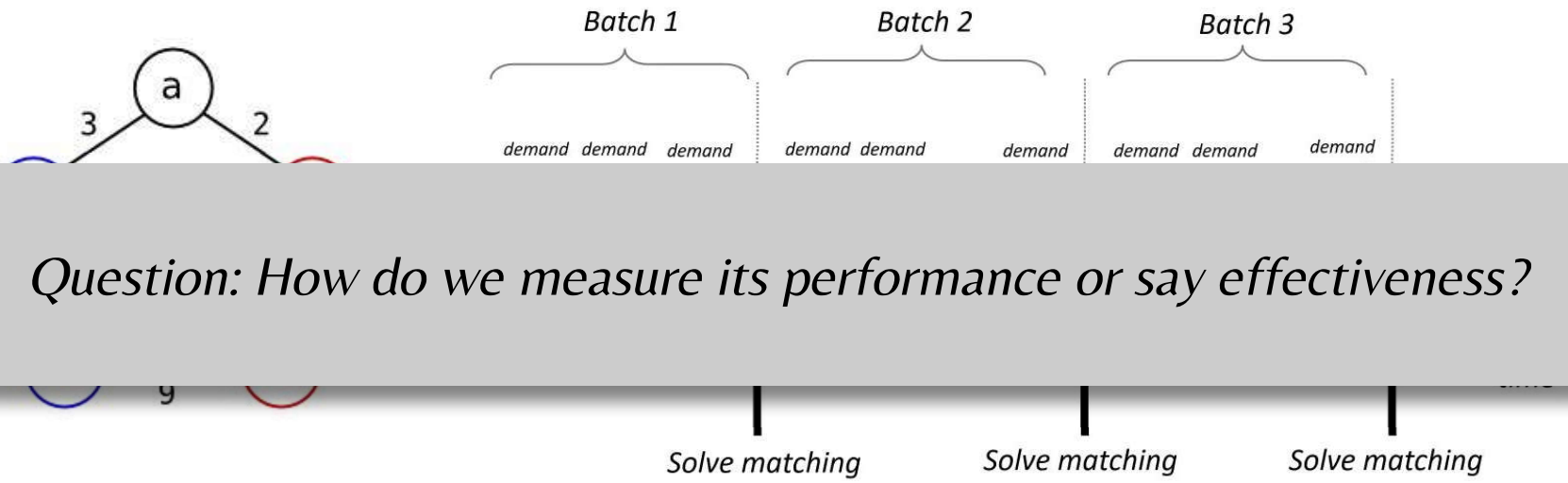

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Competitive ratio: The worst case ratio between the performance of these two.

Namely, worst performance of our online algorithm/ performance of optimal offline algorithm

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\text{subject to} \quad & \sum_{j\in[n]:\ i<j} x_{ij} + \sum_{j\in[n]:\ j<i} x_{ji} \le 1 \qquad \forall i \in [n], \qquad \text{(Offline Matching)} \\
& x_{ij} \in \{0,1\} \ \forall i<j,\ i,j\in[n].
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Thm 2 (*random arrival*)**:** *No algorithm is more than 1/2-competitive.*
Proof sketch: If it is (½+ε)-competitive, consider a graph with three vertices {1, 2, 3}, with $v_{12} = \alpha$ *and* $v_{23} = v_{13} = \varepsilon'\alpha$*, d = 1, then choosing* $\varepsilon' < \frac{2}{3}\varepsilon$ *gives us the desired contradiction.*

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➢ Secretary Matching (Ezra et al. 2020) - 5/12 competitive ratio

ALGORITHM 1: 5/12-matching secretary for vertex arrival (for $k = \lfloor \frac{n}{2} \rfloor$)

Let $v_1, \ldots, v_n$ be the vertices in arrival order
$A = V$ // A is the set of available vertices
$\mu = \varnothing$ // μ is the returned matching
for $t \in \{k+1, ..., n\}$ **do**
  Let $V_t = \{v_1, \ldots, v_t\}$ // V_t is the set of vertices arrived up to time t
  if t is odd **then**
    Select $r_t \in \{1, \ldots t-1\}$ uniformly at random
    Set $V'_t = V_t \backslash \{v_{r_t}\}$ // delete a random vertex from $v_1, \ldots, v_{t-1}$
  else
    Set $V'_t = V_t$
  end if
  Let μ_t be the maximum weighted matching in $G(V'_t)$
  if $\mu_t(v_t) \in V_t \cap A$ **then**
    Add $e_t \stackrel{\text{def}}{=} v_t \mu_t(v_t)$ to μ // add the chosen edge to the matching
    Remove v_t and $\mu_t(v_t)$ from A
  end if
end for
Return matching μ

Notice: Vertices will not depart in this algorithm, besides the bound here is proven to be tight. The corresponding offline algorithm is also different.

Source: Secretary Matching with General Arrivals, Nov 2020

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1. Vertices can be viewed as passengers who request to share a ride.
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Could we do the Batching with some stochastic information in the future?

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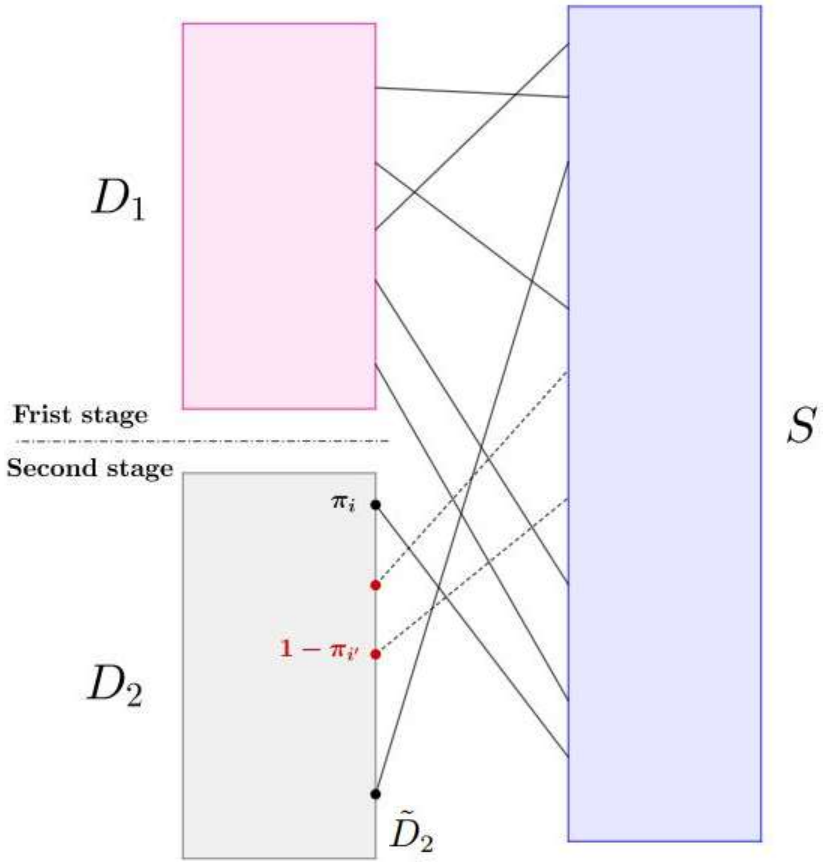

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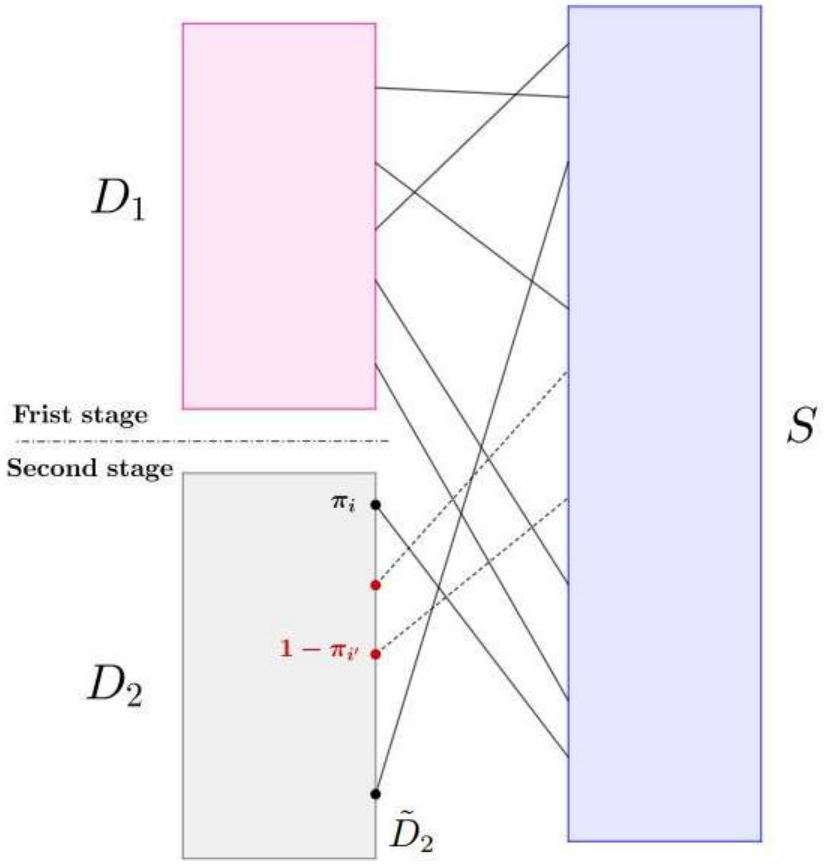

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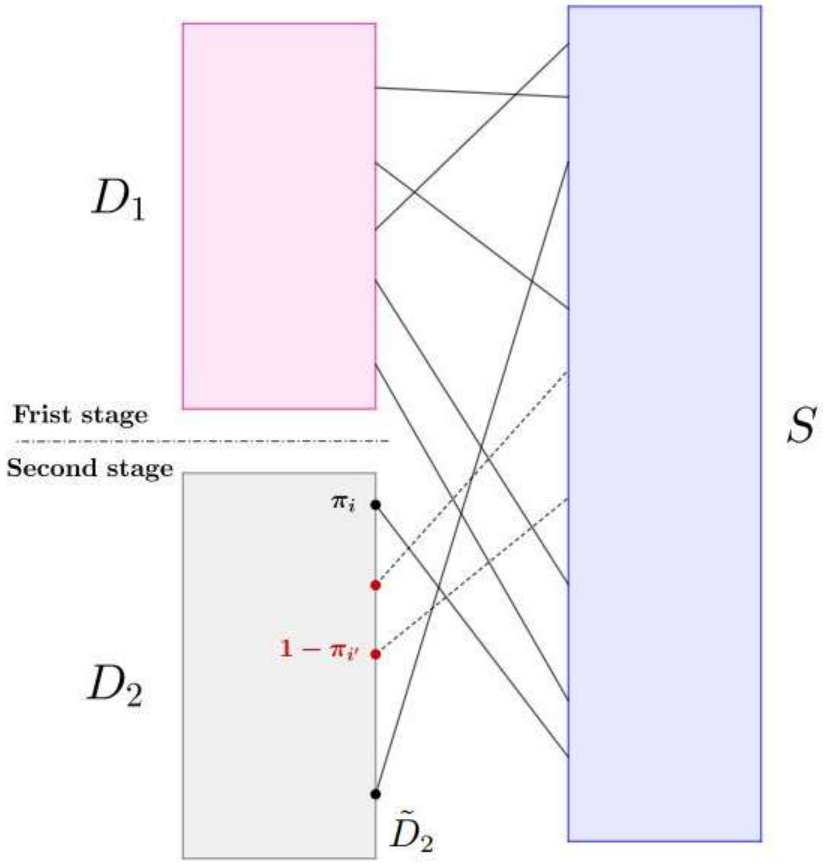

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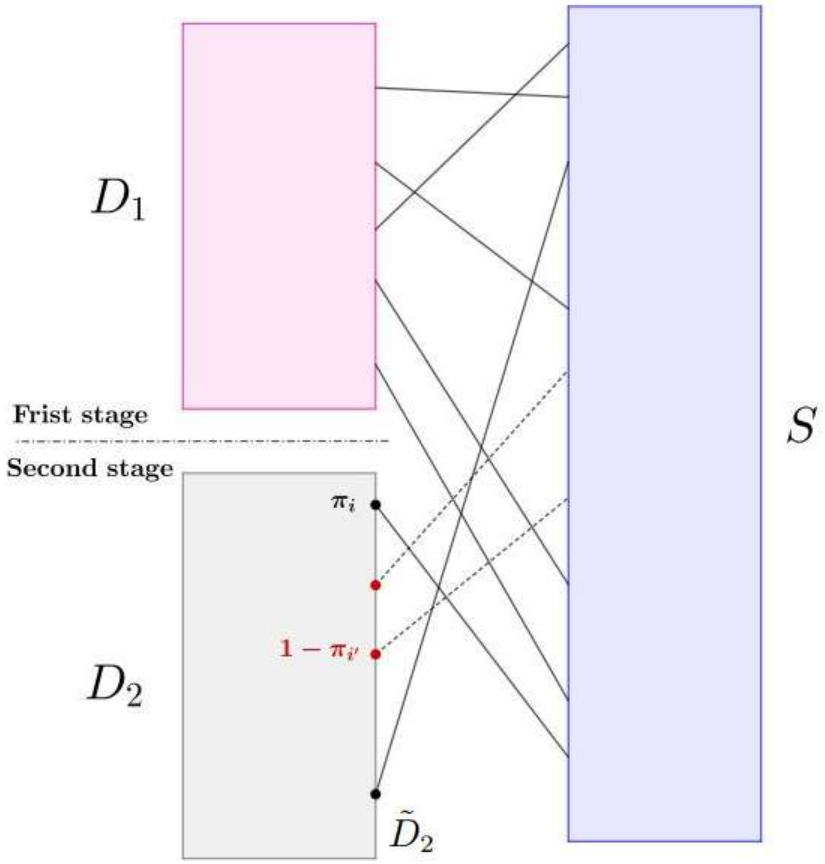

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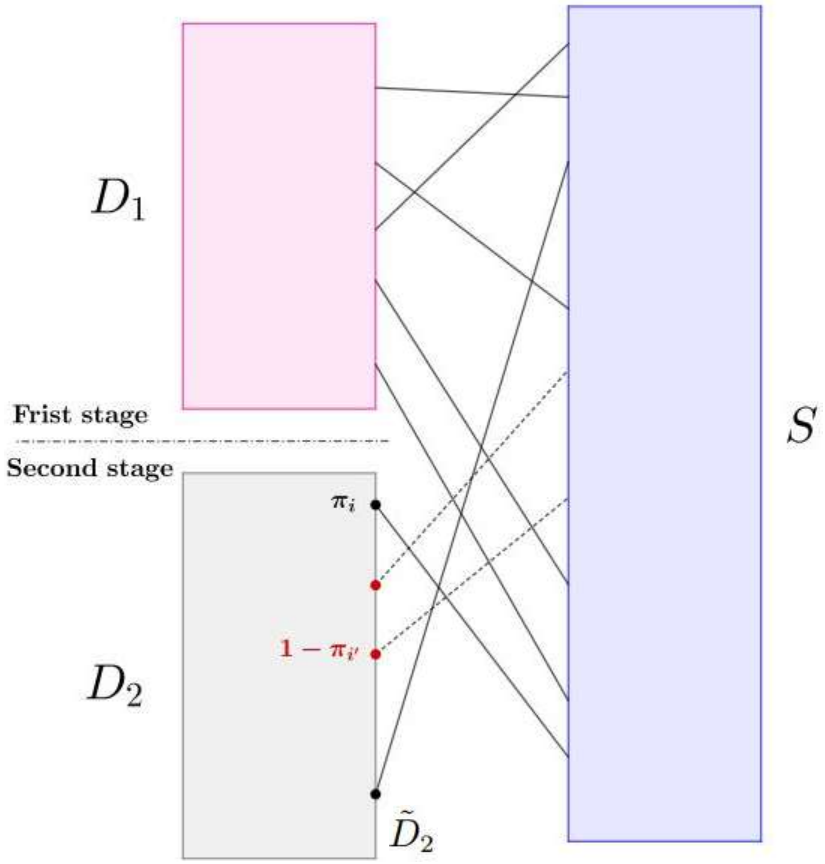

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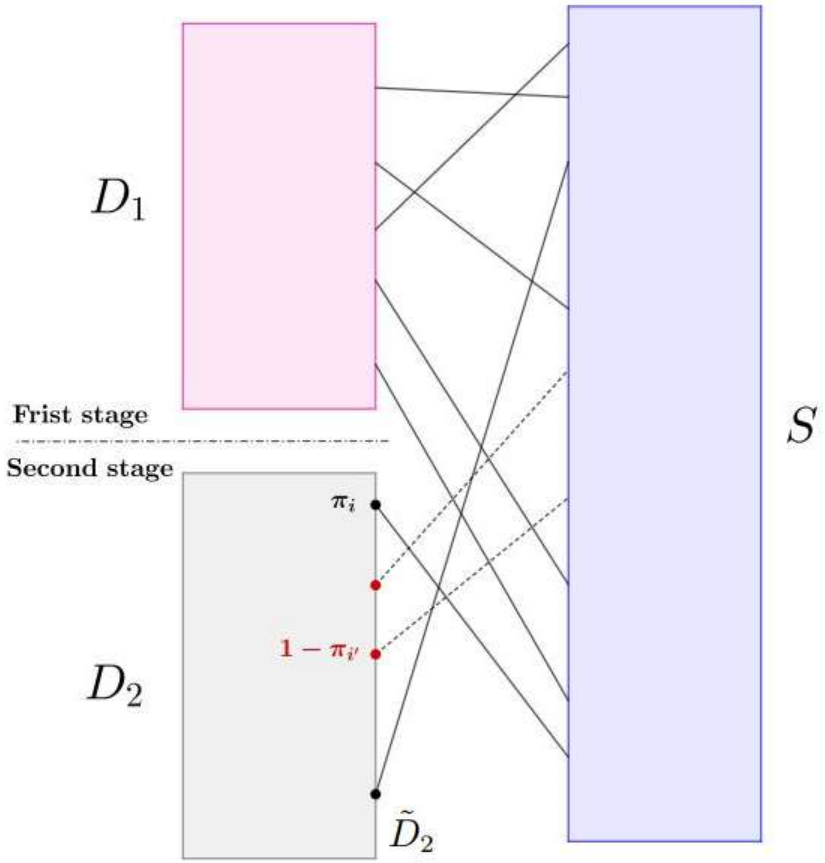

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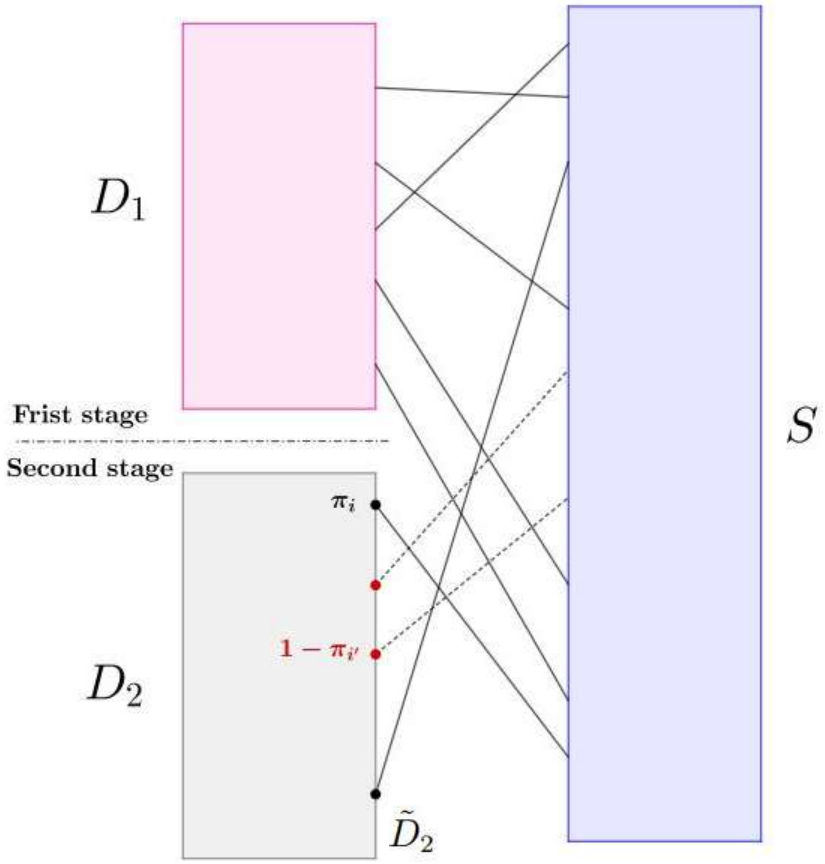

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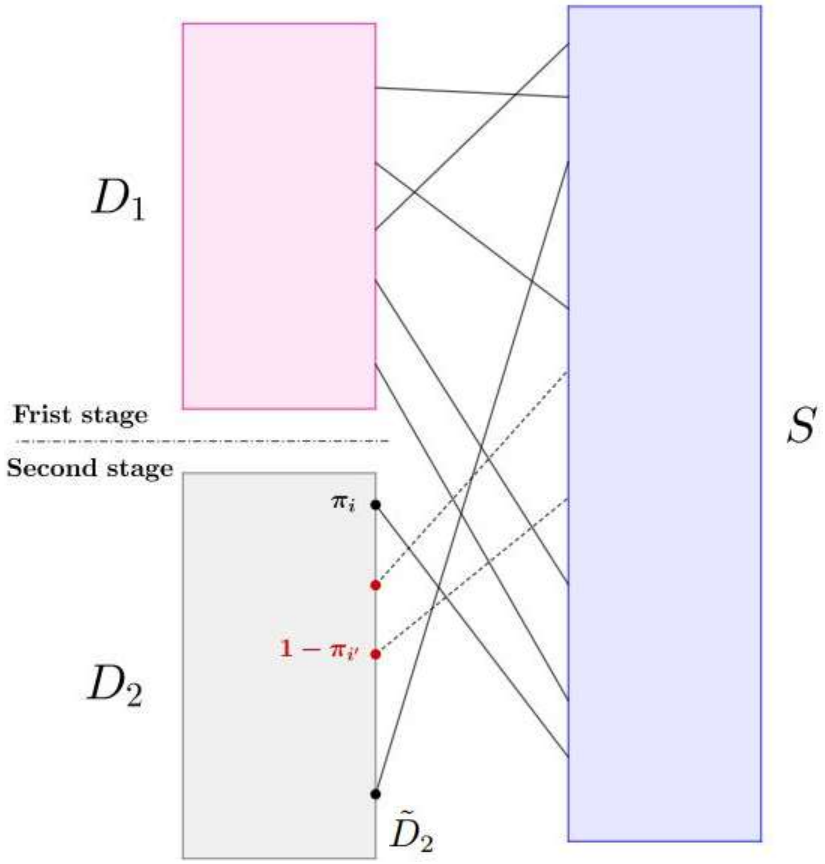

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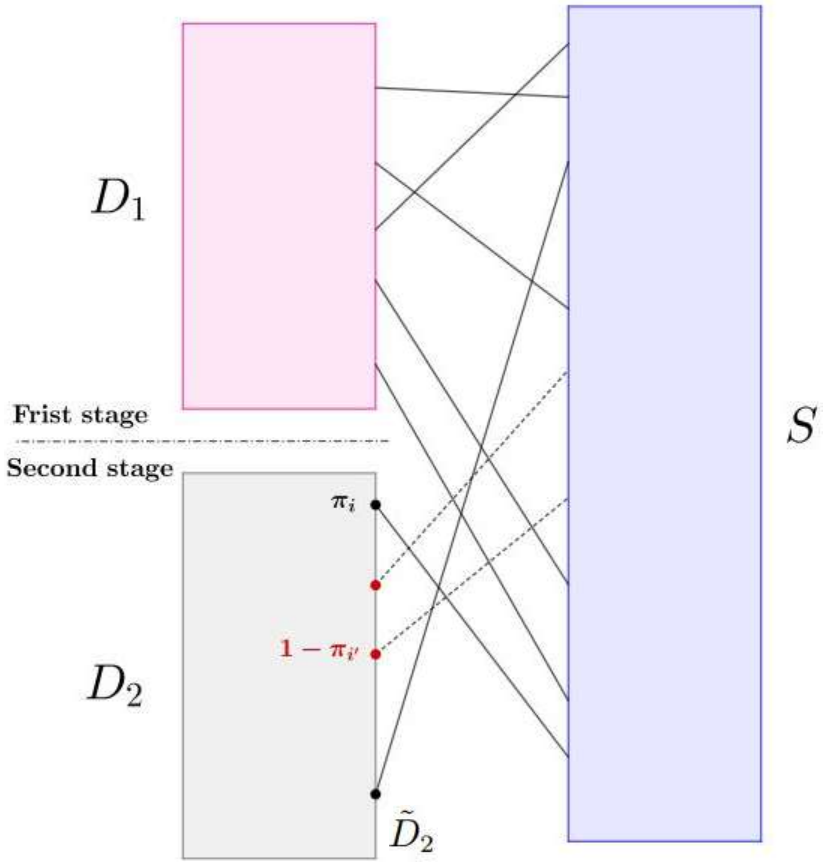

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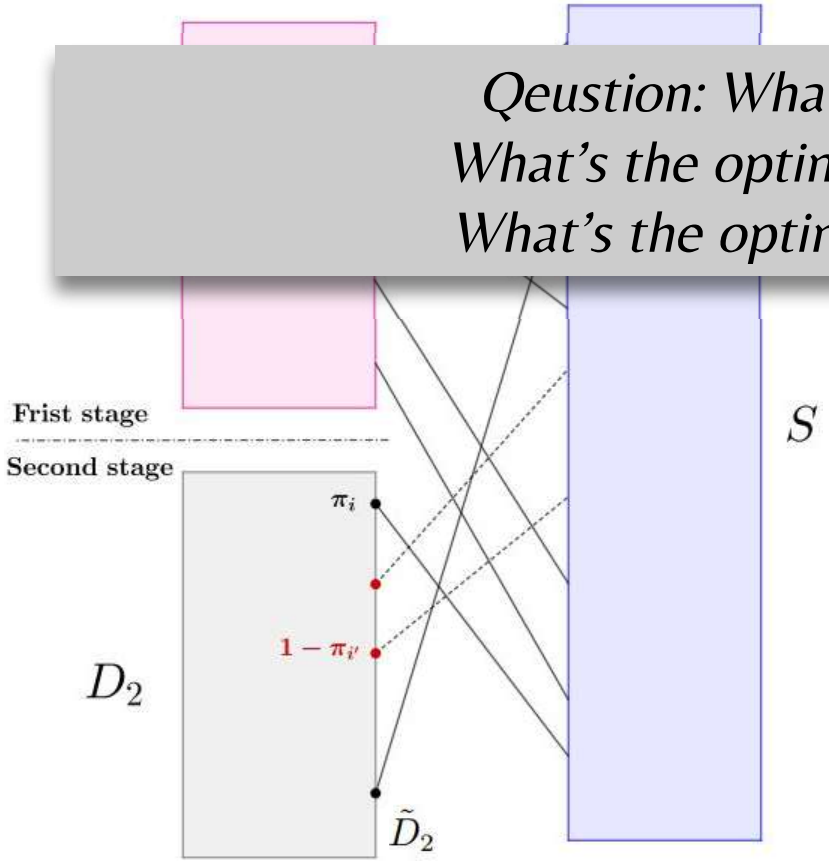
  - ❖ D_1 represents the set of riders who have made a request for a ride
  - ❖ D_2 represents the set of riders who are entering their destinations in the application and may or may not make a request.

❑ Two-stage Matching in a batch (Feng, Niazadeh, Saberi, 2020)



Qeustion: What's the randomness here?
What's the optimal offline algorithm here?
What's the optimal online algorithm here?

- Bipartite Graph $G = (D, S, E)$
  - ❖ rider set D, driver set S, edge set E
  - ❖ D is partitioned into D_1 and D_2
- Two stage
  - ❖ first stage: pick matching between D_1 and S
  - ❖ second stage: each rider in D_2 arrives independently at random with probability π_i, then the platfofrm picks the matching between the arrived rider in D_2 and the unmatched driver in S
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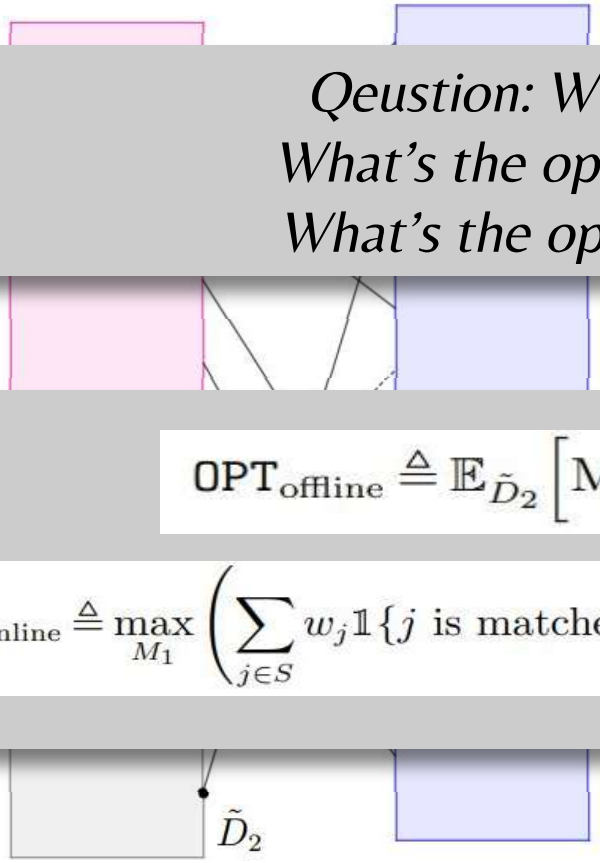

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  - ❖ first stage: pick matching between D_1 and S

$$\texttt{OPT}_{\text{offline}} \triangleq \mathbb{E}_{\tilde{D}_2}\left[\textsc{Max-WeightMatch}^{\mathbf{w}}(D_1 \cup \tilde{D}_2, S)\right]$$

$$\texttt{OPT}_{\text{online}} \triangleq \max_{M_1}\left(\sum_{j\in S} w_j \mathbb{1}\{j \text{ is matched in } M_1\} + \mathbb{E}_{\tilde{D}_2}\left[\textsc{Max-WeightMatch}^{\mathbf{w}}(\tilde{D}_2, S\setminus S_1)\right]\right)$$

request for a ride

  - ❖ D_2 represents the set of riders who are entering their destinations in the application and may or may not make a request.

❑ Case 1: general weight *w(i,j)*

Algorithm 5 Greedy with Random Discard

1: **input:** bipartite graph $G = (D, S, E)$, non-negative edge weights $\{w_e\}_{e \in E}$.

2: **output:** bipartite matching M_1 in the graph $G[D_1, S]$.

3: Let M_1 be the maximum weighted matching in the graph $G[D_1, S]$.

4: Discard each edge $e \in M_1$ from matching M_1 with probability $1/2$.

5: Return M_1.

Thm 3 : *It is ½-competitive against the optimum offline policy.*

Thm 4 : *In the two-stage matching problem with weighted demand vertices, no policy can obtain a competitive ratio better than ½ against the optimum offline policy.*

❑ Case 1: general weight *w(i,j)*

Thm 3 : *It is ½-competitive against the optimum offline policy.*

Proof.

- At the first stage, the algorithm will first do the maximum weighted matching and then randomly discards each edge in the matching with probability 1/2, its final matching M1 is at least 1/2-approximation to the total weights of edges matched in the optimum offline policy at the first stage.

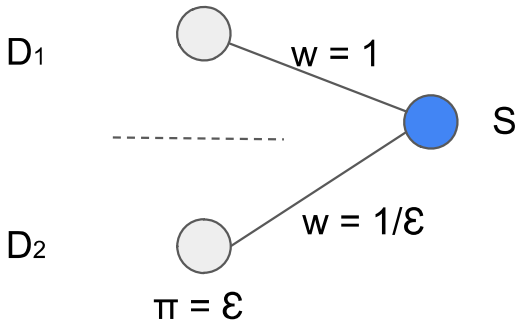
- At the second stage, notice each supply vertex in S is matched at the first stage with probability at most 1/2, then the maximum weighted matching with all remaining supply vertices and new demand vertices in D_2 at the second stage is at least 1/2-approximation to the total weights of edges matched in the optimum offline policy at the second stage.

Case 1: general weight *w(i,j)*

Thm 4 : *In the two-stage matching problem with weighted demand vertices, no policy can obtain a competitive ratio better than ½ against the optimum offline policy.*

Proof.

- Consider the following instance of two-stage matching with weighted demand vertices: There is one supply vertex and two demand vertices. Demand vertex 1 is at the first stage with weight 1. Demand vertex 2 with weight 1/Ɛ appears at the second stage with probability Ɛ. Both demand vertices can be matched to the supply vertex.

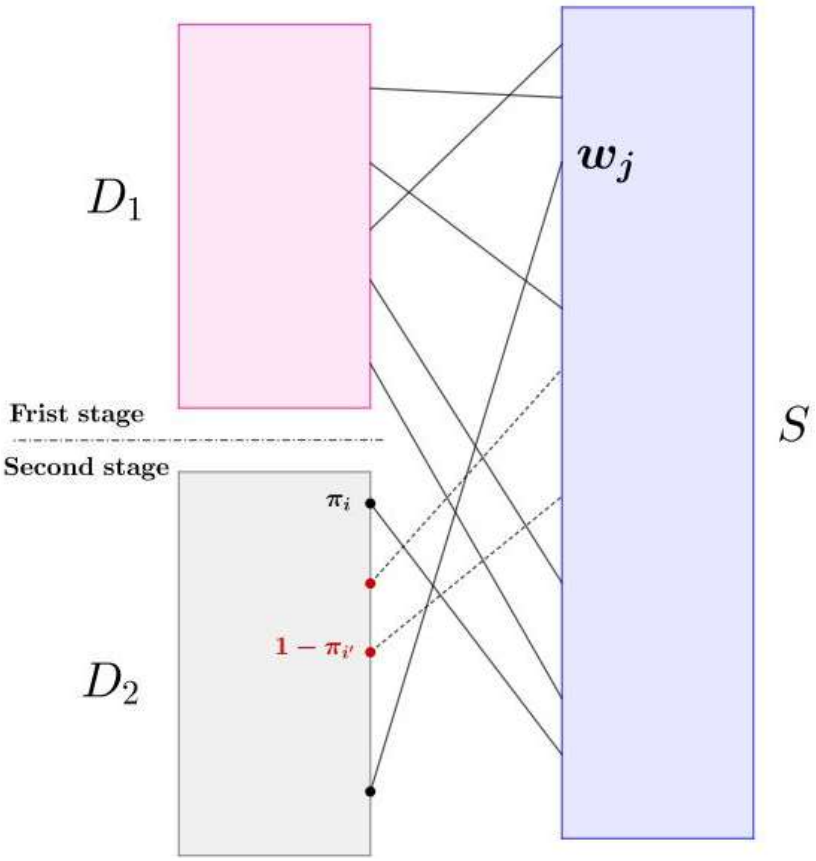


The expected total weight for any online policy is 1

The expected total weight for optimum offline policy is

$$Ɛ * 1 / Ɛ + (1 - Ɛ) * 1 = 2 - Ɛ$$

Case 2: with only driver weight *w(j)*



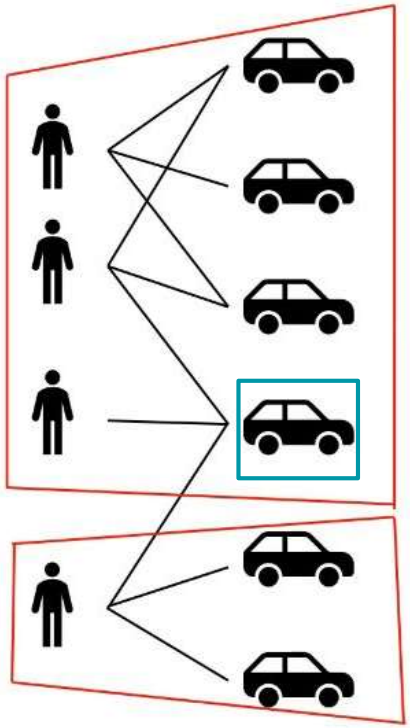
- HEDGE: ¾ competitive against optimal offline

Intuition: Consider the unweighted case first (treat all w_j as 1)

- Suppose we do not know anything about the second stage. How would we pick the matching for D_1 and S
- If it is a *complete* bipartite graph in the first stage, then we should output the matching uniformly at random, namely, each driver in S will be matched with c = min (1, $|D_1|/S$), which is the demand per driver.
- What if it is not a *complete* bipartite graph?
  => Demand-balanced (DB) Decomposition

  Goal: decompose it to several small complete graph, and then do the matching uniformly at random in each sub graph.

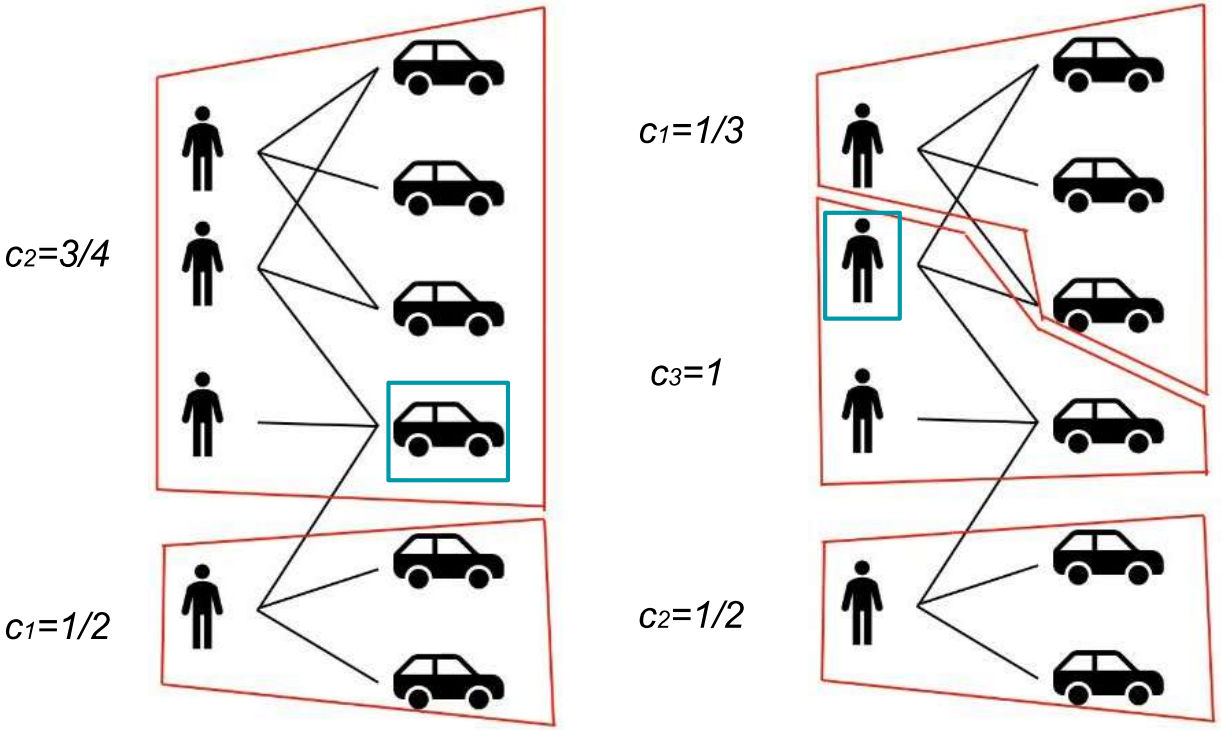
Demand-balanced (DB) Decomposition



Uniformity I: the driver in each sub graph will be matched with the same probability $c = \min(1, |D_{sub}|/S_{sub})$

Source: Two-stage Matching and Pricing with Applications to Ride Hailing, Jul 2020

Demand-balanced (DB) Decomposition



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Uniformity II: no edge rom riders in high demand pair to drivers in low demand pair

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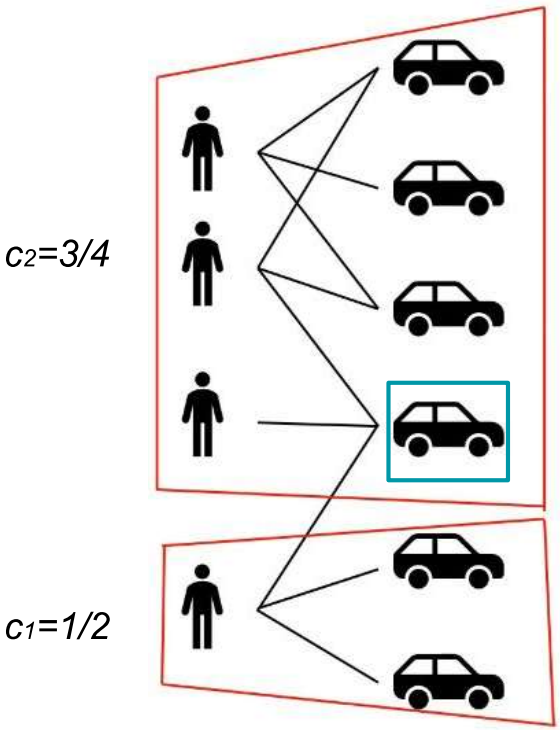
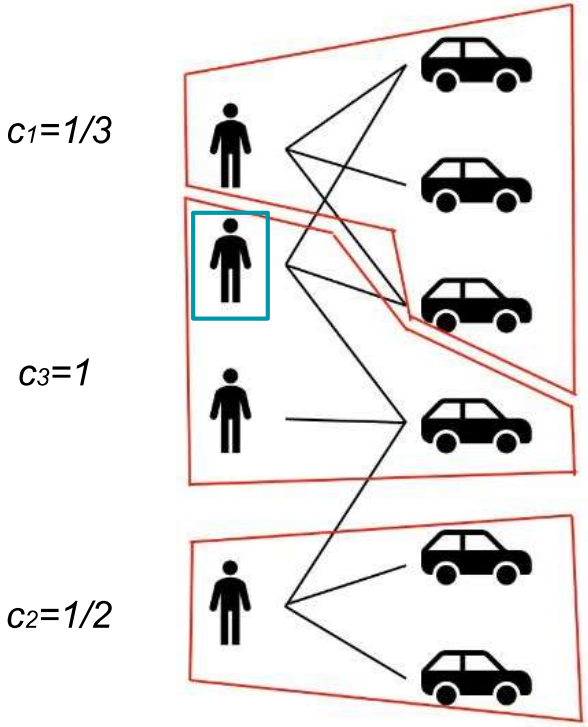
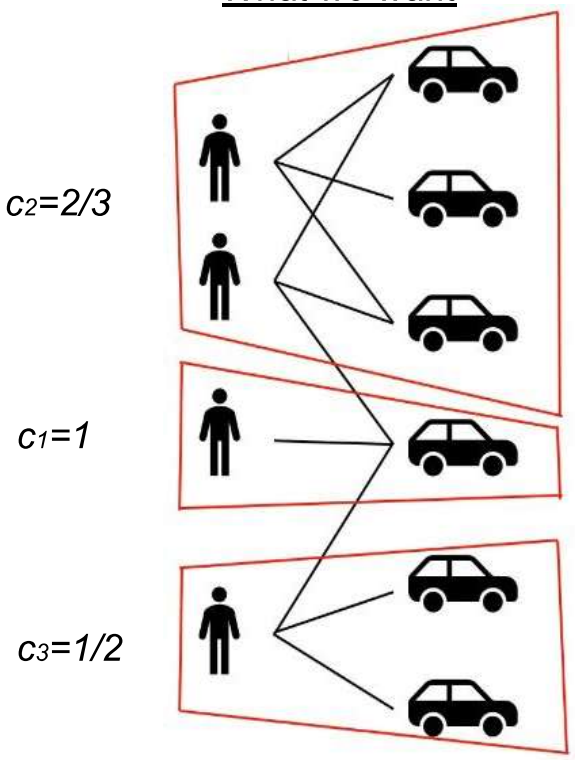




What we want



Uniformity I: the driver in each sub graph will be matched with the same probability $c = \min(1, |D_{sub}|/S_{sub})$

Uniformity II: no edge rom riders in high demand pair to drivers in low demand pair

Monotonicity: the c can be strictly sorted.

Source: Two-stage Matching and Pricing with Applications to Ride Hailing, Jul 2020

❑ Demand-balanced (DB) Decomposition

- Convex programming for balanced supply utilization

$$\{x_{ij}^*\} \in \arg\min \sum_{j \in S} g\left(w_j\left(1 - \sum_{i \in N(j)} x_{ij}\right)\right) \text{ s.t.}$$

$$\sum_{j \in N(i)} x_{ij} \leq 1 \qquad i \in D_1 \ ,$$

$$\sum_{i \in N(j)} x_{ij} \leq 1 \qquad j \in S \ ,$$

$$x_{ij} \geq 0 \qquad i \in D_1, j \in S, (i,j) \in E$$

Unweighted version

Source: Two-stage Matching and Pricing with Applications to Ride Hailing, Jul 2020

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Unweighted version

$$\{x_{ij}^*\} \in \arg\min \sum_{j\in S} \frac{1}{w_j} g\left(w_j\left(1-\sum_{i\in N(j)} x_{ij}\right)\right) \text{ s.t.}$$
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Weighted version

Remark

1. g(.) : R → R can be any differentiable, increasing, and strictly convex function.
2. By picking proper g, the objective here can be Rawlsian social welfar or Nash social welfare
3. All vertices in sub-graph are fully matched, namely, the sum of x^*_{ij} for any j is 1.
4. The x^*_{ij} is the probability induced by the random matching satisfying the *Uniformity I*.

Source: Two-stage Matching and Pricing with Applications to Ride Hailing, Jul 2020

Demand-balanced (DB) Decomposition

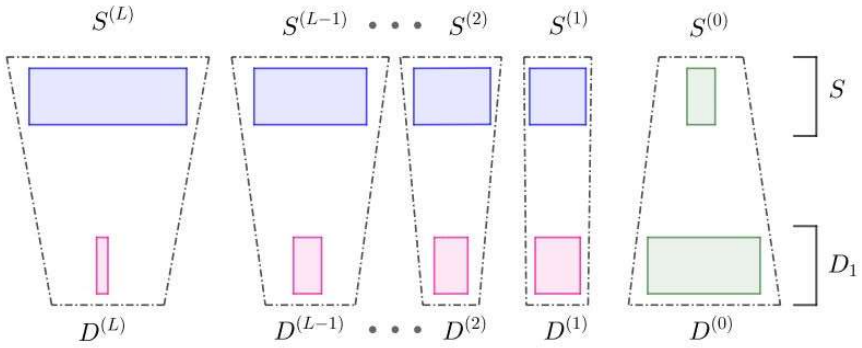


Figure 2 Structural decomposition in Lemma 1; the pairs $(D^{(l)}, S^{(l)})$ are sorted such that $0 = c^{(0)} < c^{(1)} < \ldots < c^{(L)}$. Red dashed-lines are forbidden *non-existent* edges, i.e., any edge between $D^{(l)}$ and $S^{(l')}$ when $l < l'$.

Algorithm 1 WEIGHTED-BALANCED-UTILIZATION

input: bipartite graph $G = (D, S, E)$, non-negative weights $\{w_j\}_{j \in S}$, convex function $g(\cdot)$.

output: bipartite matching M_1 in $G[D_1, S]$, bipartite matching M_2 in $G[\tilde{D}_2, S]$.

Solve convex program $\mathscr{P}^g$ to obtain $\{x^*_{ij}\}$.

Sample matching M_1 with edge marginal probabilities $\{x^*_{ij}\}$.

In the second stage, return the maximum supply-weighted matching M_2 between $\tilde{D}_2$ and the remaining vertices of S.

❑ Demand-balanced (DB) Decomposition

Thm 5 : *It is ¾-competitive against the optimum offline policy.*

Thm 6 : *In the two-stage stochastic matching for maximizing supply efficiency, no policy obtains a competitive ratio better than ¾ against OPT offline, even when all the weights are equal.*

Other Results

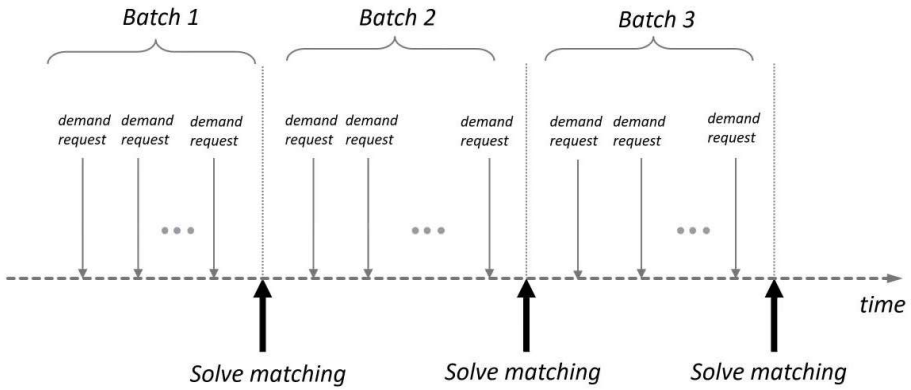
			Lower-bound (previous work)	Upper-bound (this paper)	Lower-bound (this paper)
Two-stage stochastic matching (Section 3)	Optimum offline	Weighted	$(1-\frac{1}{e})$ (*)	$\frac{3}{4}$	$\frac{3}{4}$
		Unweighted	$\frac{2}{3}$ (**)	$\frac{3}{4}$	$\frac{3}{4}$
	Optimum online	Weighted	$(1-\frac{1}{e})$ (*)	FPTAS-hard	$\left(1-\frac{1}{e}+\frac{1}{e^2}\right) \approx 0.767$
		Unweighted	$\frac{2}{3}$ (**)	FPTAS-hard	0.7613

Two-stage stochastic joint matching/pricing for market efficiency (Section 5)	-	$\frac{1}{2}$	$\frac{1}{2}$
Single-stage stochastic joint matching/pricing for demand efficiency (Appendix EC.7)	-	$1-\frac{1}{e}$ (ex-ante)	$1-\frac{1}{e}$
Two-stage stochastic matching with general edge weights (Appendix EC.8)	-	$\frac{1}{2}$	$\frac{1}{2}$

FPTAS - Fully Polynomial Time Approximation Scheme

Discussion

Key in this pre:



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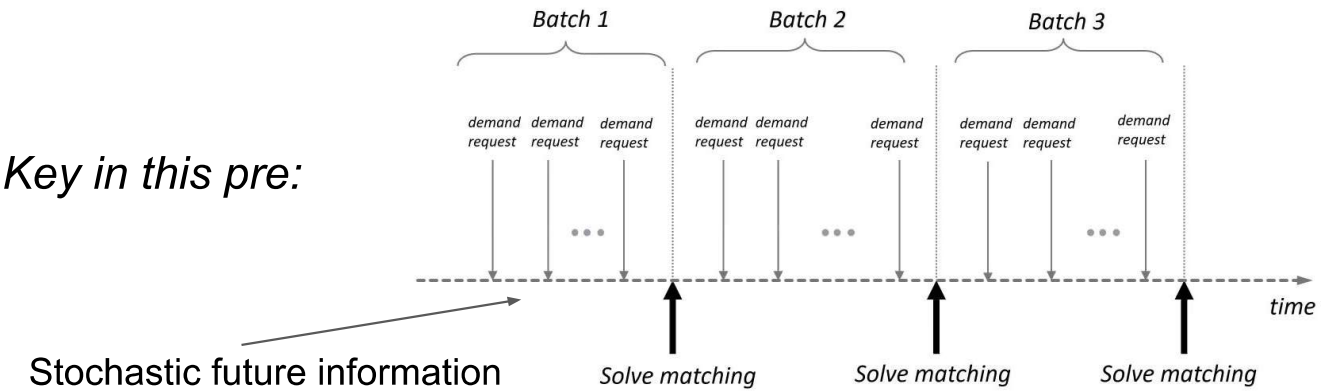

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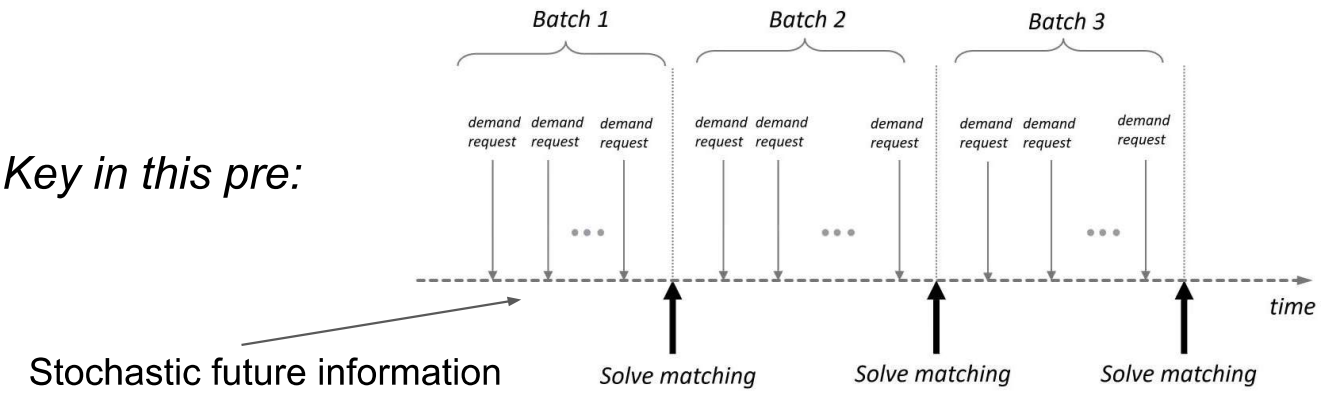

1. Most of the algorithms do not take any batch correlation into the account.
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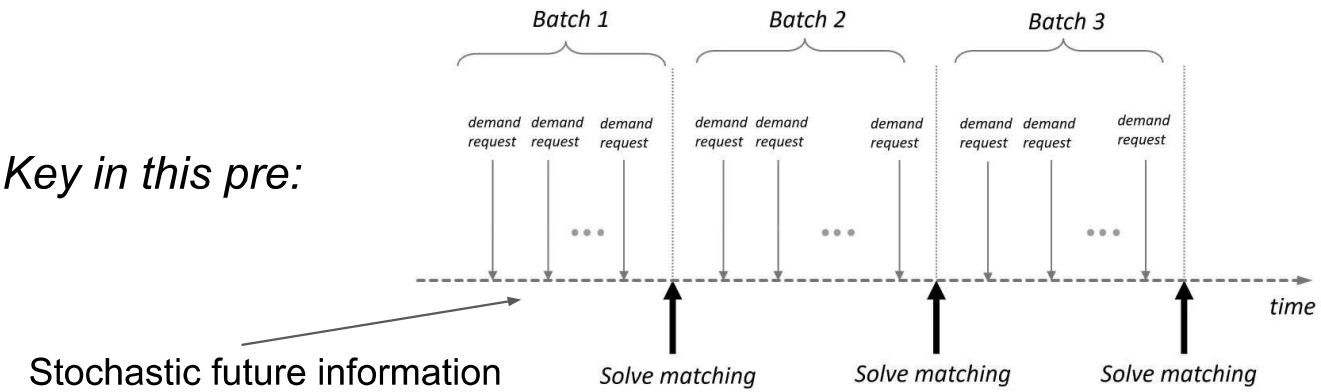

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   a. Currently, we do the matching after a specific time interval T. Then the unmatched vertices in the first batch have wait as least another time interval T to be matched with a driver. Then, could we leverage a dynamic shifting window?

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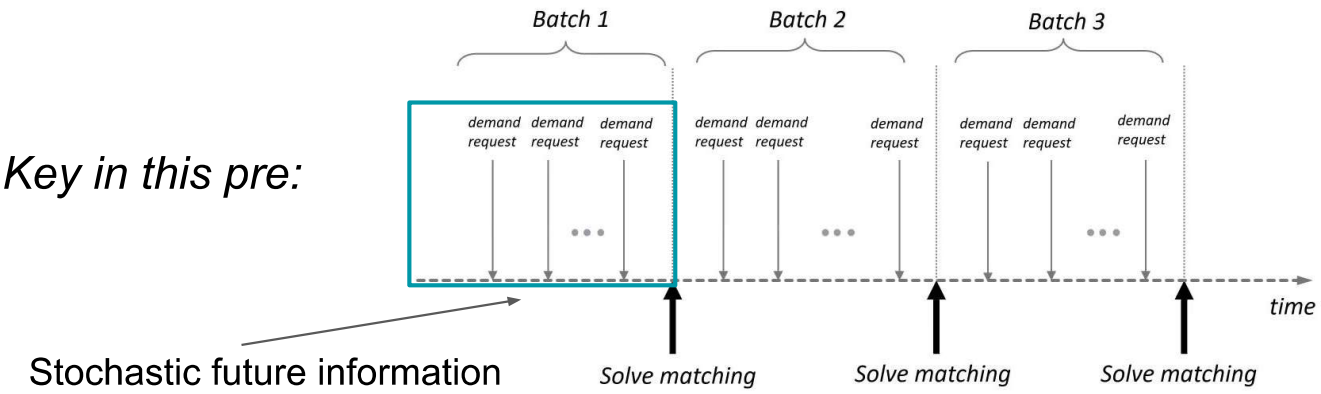

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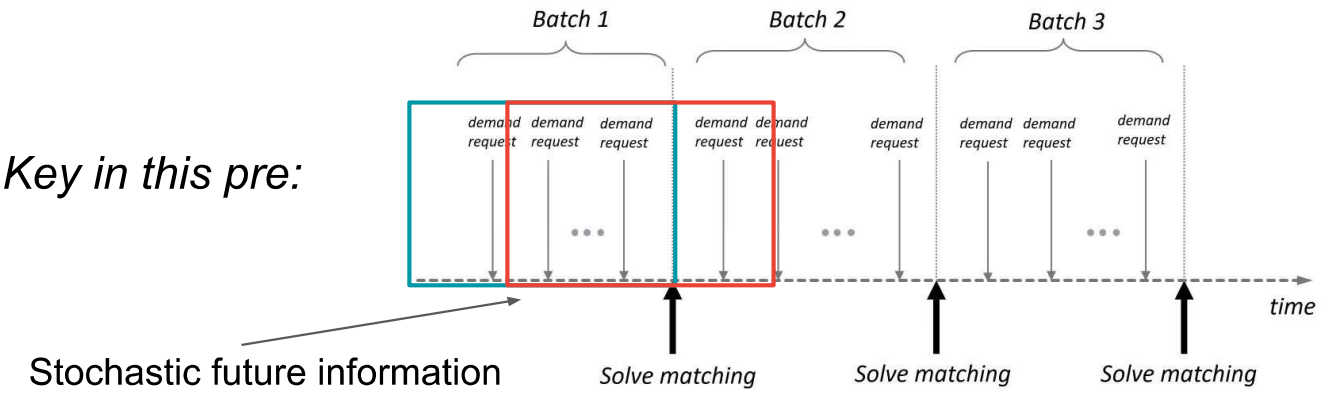

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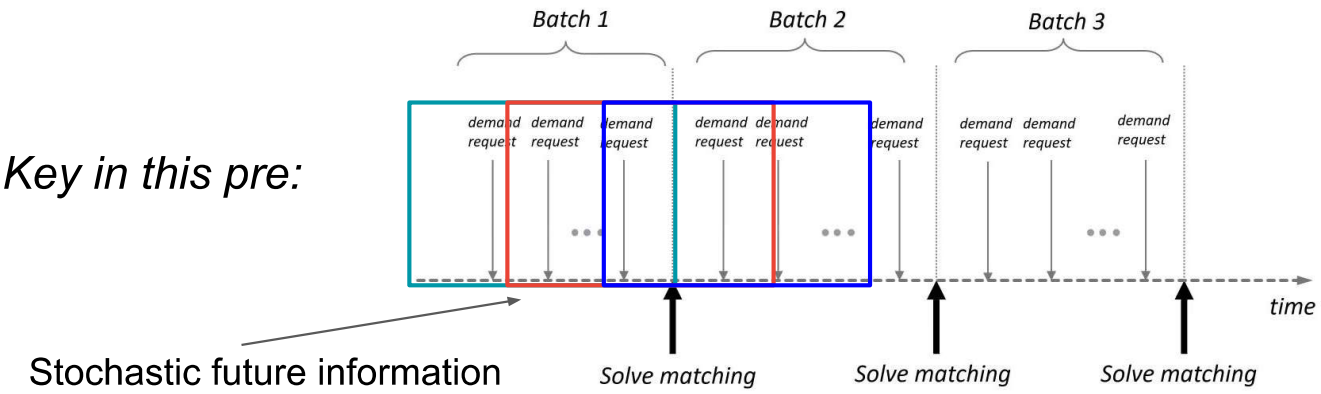

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  - Unmatched demand and supply will incur waiting or holding costs, and will be carried over to the next period with abandonment (Hu, 2021).
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Any question?