Multi-Item Auctions

Krzysztof Dutka

Quick Refresh of Auctions

- Selling items of unknown value
- Agents put forth bids to buy items
- With single item, best is second-price/Vickrey auctions

Image from: https://oko.uk/blog/first-price-vs-second-price-auctions

Challenges of multi-item

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One bidder, two items $v_1, v_2 \in \{4, 7\}$

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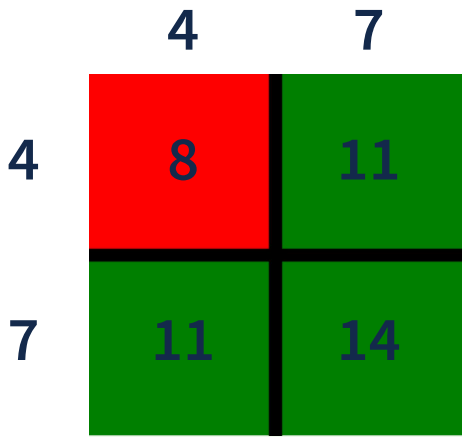
Better strategy is to bundle with price at 11

$$\mathbb{E}[Revenue] = 8.25$$



Simultaneous Multi-Round Auction

Simultaneous Multi-Round Auction

Each Round

- If agent has highest bid on an item:
  - Doesn't need to do anything
- Else:
  - Place a higher bid on an item
  - Or drop out

Auction ends when no new bids

CBRS Auction	Winning Bid ($ million)
Verizon	$1,893
Wetterhorn Wireless LLC (Dish)	$913
Spectrum Wireless Holdings, LLC (Charter)	$464
XF Wireless Investment, LLC (Comcast)	$459
Cox Communications, Inc.	$212

C-band Auction	Winning Bid ($ million)
Verizon	$45.45
AT&T	$23.41
T-Mobile	$9.34
US Cellular	$1.28
New Level II, LP	$1.28

Image from: https://www.counterpointresearch.com/us-october-spectrum-auction/

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Goals:

- **Reveals information**
- Gives bidders ability to respond

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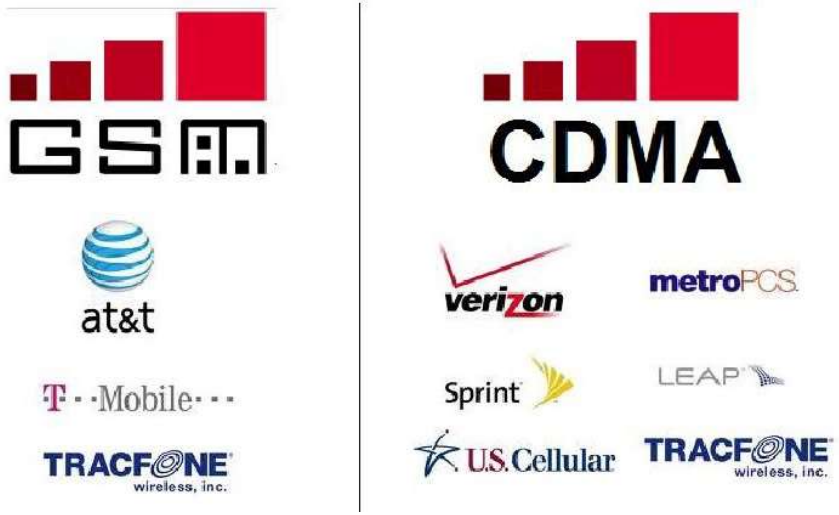

*Notable problems particularly with spectrum auctions, may not occur with other markets, some also applicable to auctions in general
Image from: https://www.neway.mobi/news/what-is-the-difference-between-gsm-and-cdma-and-which-one-is-better.html

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Image from: https://freebiesupply.com/logos/pagenet-logo/

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Next Wave Communications

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Image from: https://www.nwtelco.com/

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  - Bid shading/demand reduction
- **Bundles are hard** to form

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Next Wave Communications

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Combinatorial Clock Auction

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- **Anonymous** Bids
- Round reports include price and demand, no winners

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- Agents report demand for product at price point
- **Additional final round** of closed bids for bundles

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- **Easy to auction identical/divisible goods**
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- Bid Sniping
- Computationally **Complex**

Combinatorial Clock Auction

Payment: Vickrey-nearest-core pricing

- **Vickrey pricing** introduces truthful bids

$$b_1\{A\} = 28$$
$$b_2\{B\} = 20$$
$$b_3\{AB\} = 32$$
$$b_4\{A\} = 14$$
$$b_5\{B\} = 12$$

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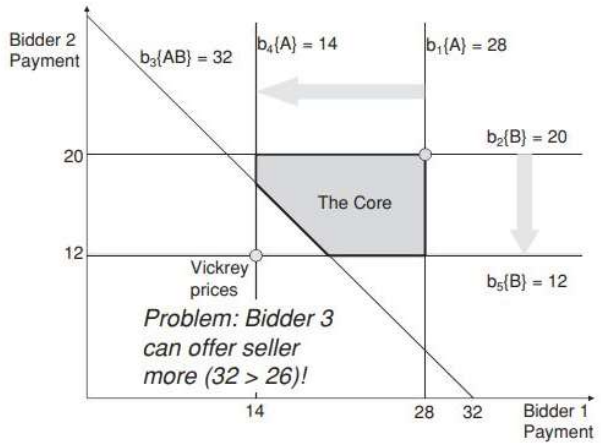
Payment: Vickrey-nearest-core pricing

- Vickrey pricing introduces truthful bids
- **Core**
  - Set of payment by the winning coalition
  - No other coalition offered to pay more



Combinatorial Clock Auction

Payment: Vickrey-nearest-core pricing

- Vickrey pricing introduces truthful bids
- Core
  - Set of payment by the winning coalition
  - No other coalition offered to pay more
- Find closest core prices to Vickrey prices
  - Is **unique**

$$b_1\{A\} = 28$$
$$b_2\{B\} = 20$$
$$b_3\{AB\} = 32$$
$$b_4\{A\} = 14$$
$$b_5\{B\} = 12$$

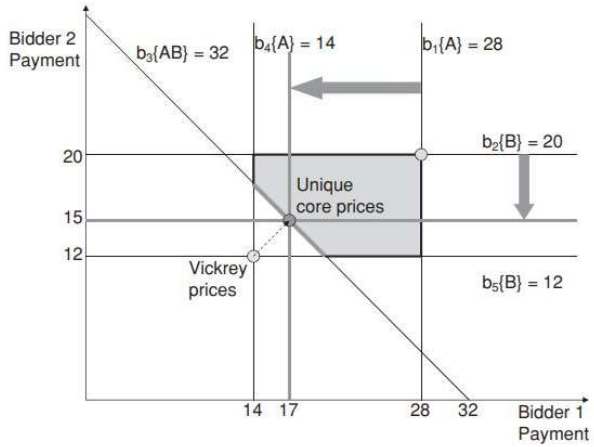


Figure 6. Core point closest to Vickrey prices.

Ausubel Auction

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- Auctioneer records demand for each agent at each price

Ausubel Auction

- Auctioneer records demand for each agent at each price
- Price is cost where rival demand falls below supply
  - Calculated per item and demand and supply is adjusted accordingly

$	*Alice*	*Bob*	*Carl*
0	4	4	4
5	4	4	3
10	4	3	2
15	4	2	1
20	3	1	1
25	3	1	0

$$s = 4$$

Ausubel Auction

- Auctioneer records demand for each agent at each price
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  - Calculated per item and demand and supply is adjusted accordingly

$$Alice : 15$$

$$s = 3$$

$	*Alice*	*Bob*	*Carl*
0	4	4	4
5	4	4	3
10	4	3	2
15	3	2	1
20	2	1	1
25	2	1	0

Ausubel Auction

- Auctioneer records demand for each agent at each price
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$$Alice : 15, 20$$

$$s = 2$$

$	*Alice*	*Bob*	*Carl*
0	4	4	4
5	4	4	3
10	4	3	2
15	3	2	1
20	1	1	1
25	1	1	0

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$$Alice : 15, 20, 25$$
$$s = 1$$

$	*Alice*	*Bob*	*Carl*
0	4	4	4
5	4	4	3
10	4	3	2
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25	0	1	0

Ausubel Auction

- Auctioneer records demand for each agent at each price
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$$Alice : 15, 20, 25$$
$$Bob : 25$$
$$s = 0$$

$	*Alice*	*Bob*	*Carl*
0	4	4	4
5	4	4	3
10	4	3	2
15	3	2	1
20	1	1	1
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Generalized Second-Price Auction

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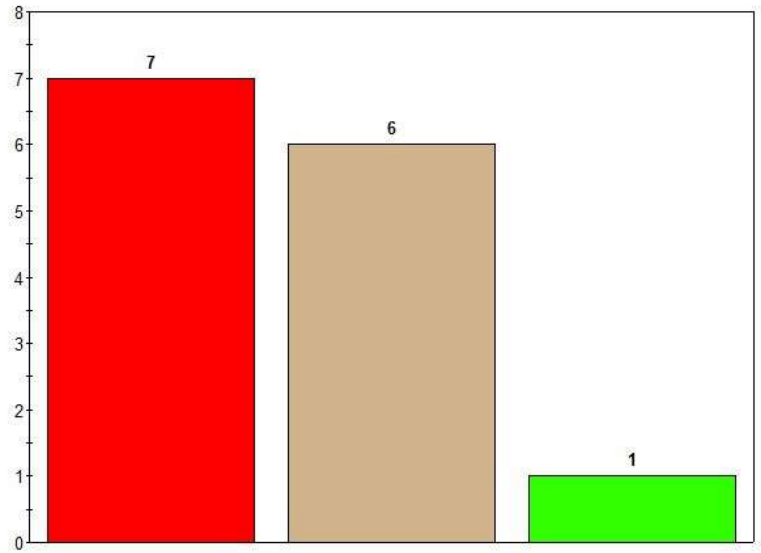
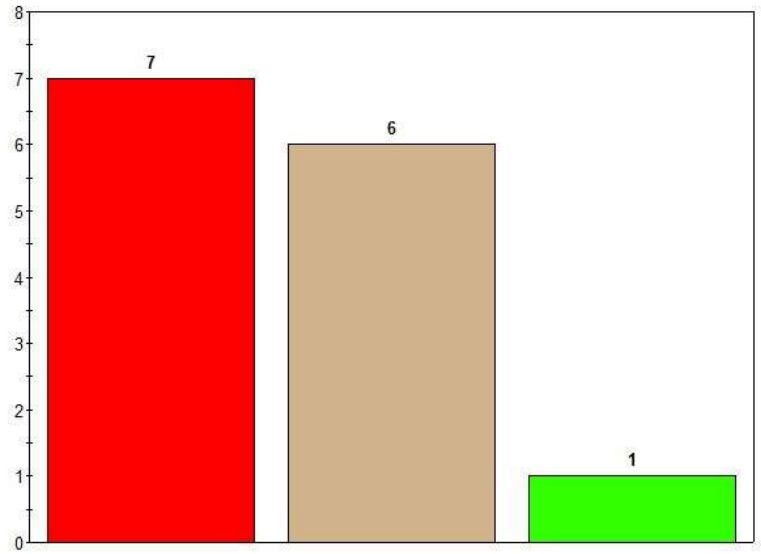
- Used in **Internet Advertising**



All Graphs created with: https://nces.ed.gov/nceskids/graphing/classic/bar_pie_data.asp?ChartType=bar

Generalized Second-Price Auction

- Used in Internet Advertising
- **Single** Round Auction

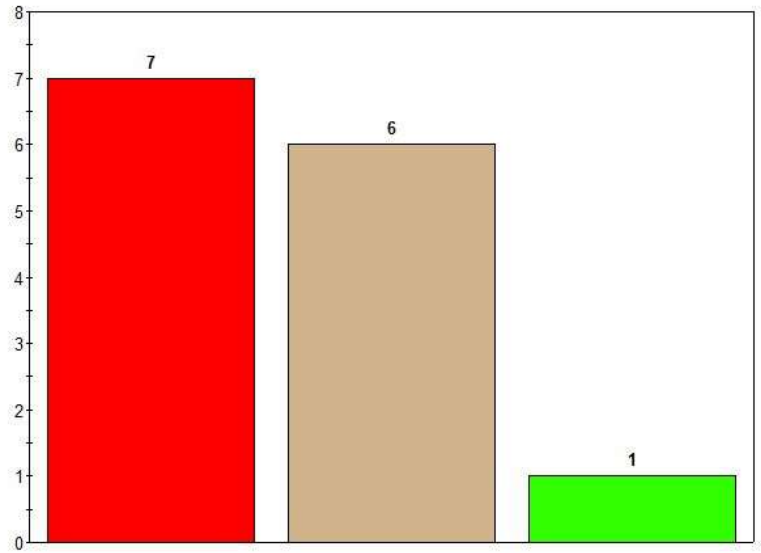


Generalized Second-Price Auction

- Used in Internet Advertising
- Single Round Auction
- Price paid is bid of **next lower bid**

$$100 \cdot 6 = 600$$

$$40 \cdot 1 = 40$$



Generalized Second-Price Auction

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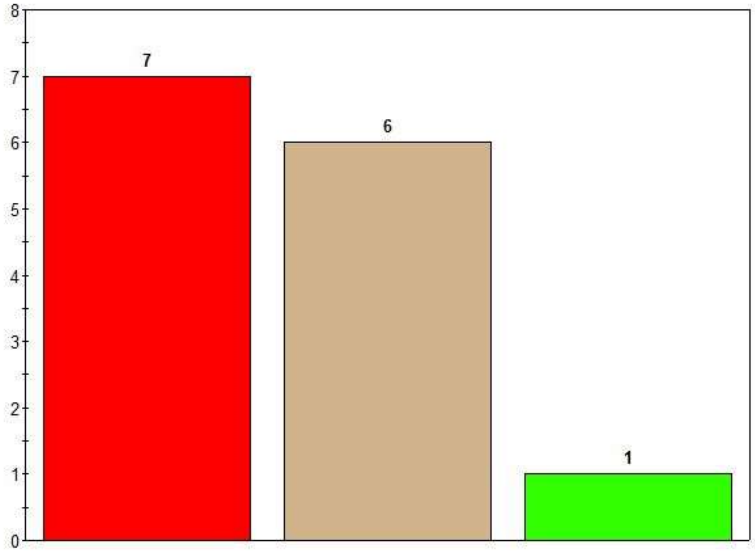
- Price paid is bid of next lower bid
- **Untruthful**



$$\alpha_1 = 100 \qquad 100\ (7 - 6) = 100$$

$$\alpha_2 = 40$$

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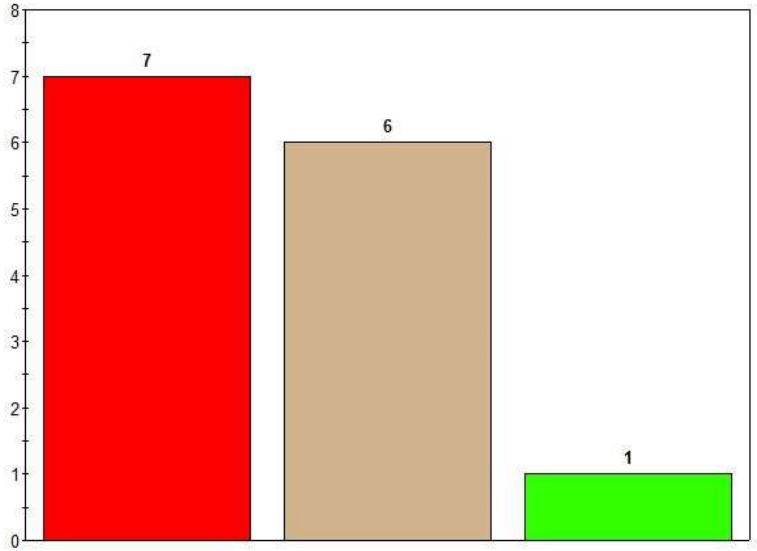

$$\alpha_1 = 100 \qquad 100\ (7-6) = 100$$

$$\alpha_2 = 40 \qquad 40\ (7-1) = 240$$

Generalized Second-Price Auction

- Used in Internet Advertising
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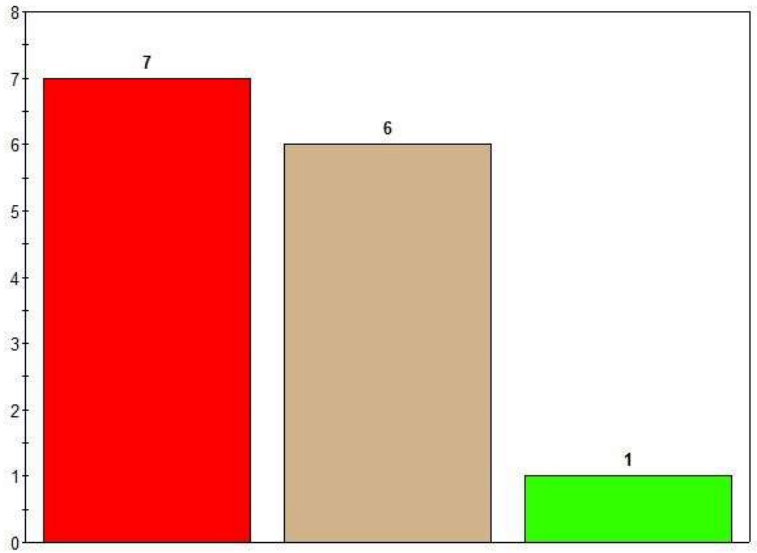
- Price paid is bid of next lower bid
- Untruthful
- **Price of Anarchy**
  - 2.927 in partial information
  - 1.282 in full information in Nash Eq.



Vickrey-Clarke Groves Auction

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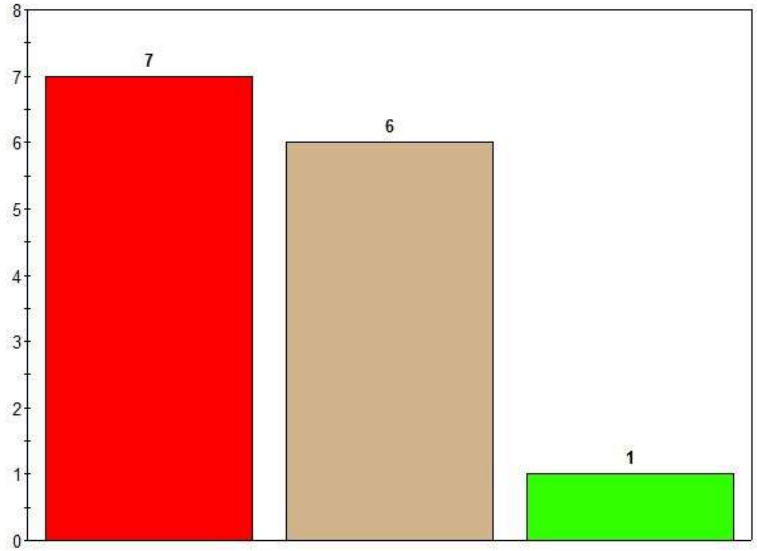
- Single Round Auction
- Price paid is **cost to rest of bidders due to participation**

$$V^{M}_{N\setminus\{b_i\}} - V^{M\setminus\{t_i\}}_{N\setminus\{b_i\}}$$



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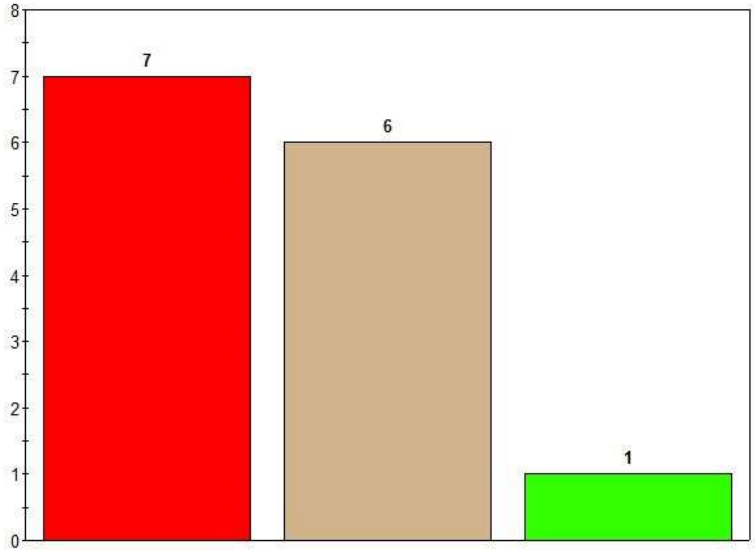
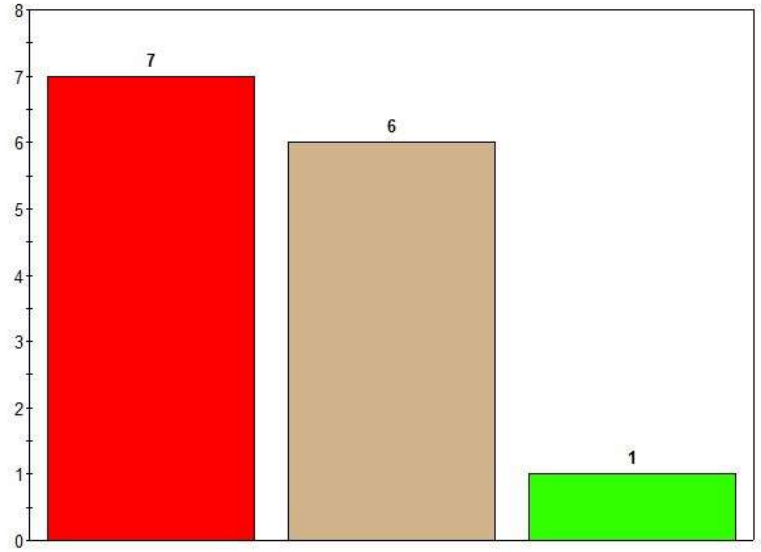
$$\alpha_2 = 40$$



$$(100 \cdot 6 + 40 \cdot 1) - (40 \cdot 6 + 0 \cdot 1) = 400$$

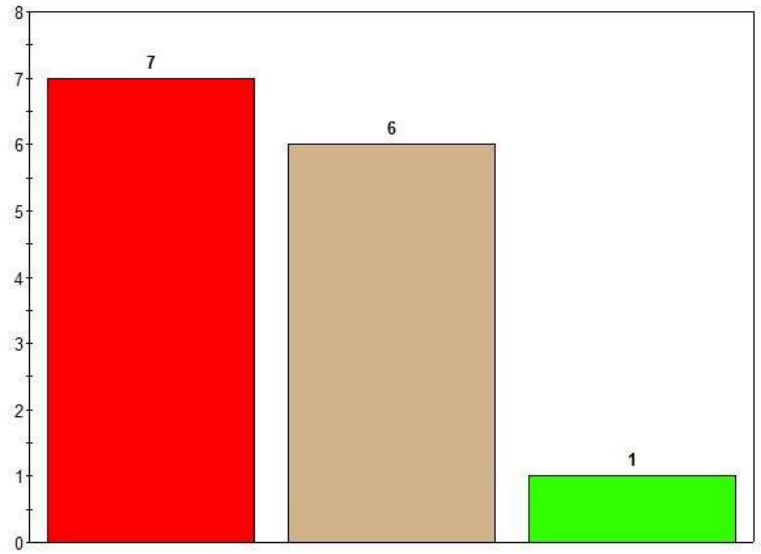
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- **Truthful**



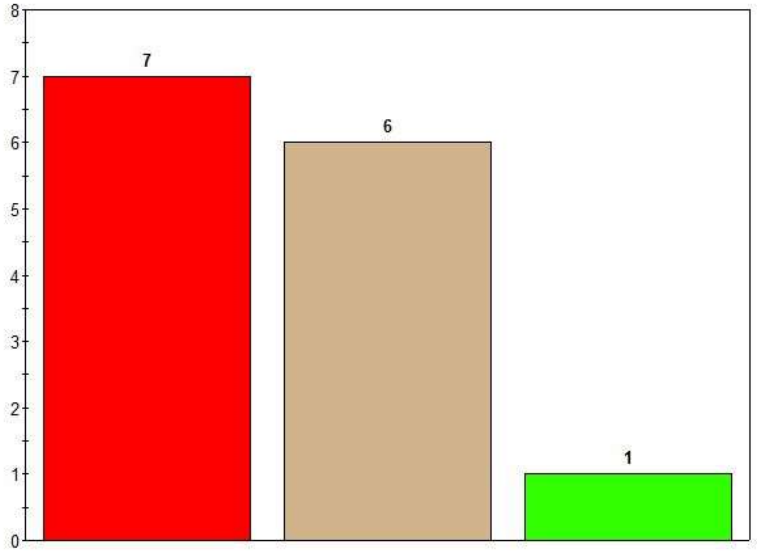
Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- Truthful
- **Complex** and **unintuitive**



Vickrey-Clarke-Groves Auction

- Single Round Auction
- Price paid is cost to rest of bidders due to participation
- Truthful
- Complex and unintuitive
- **Lower revenue** than GSP under truthful scenarios



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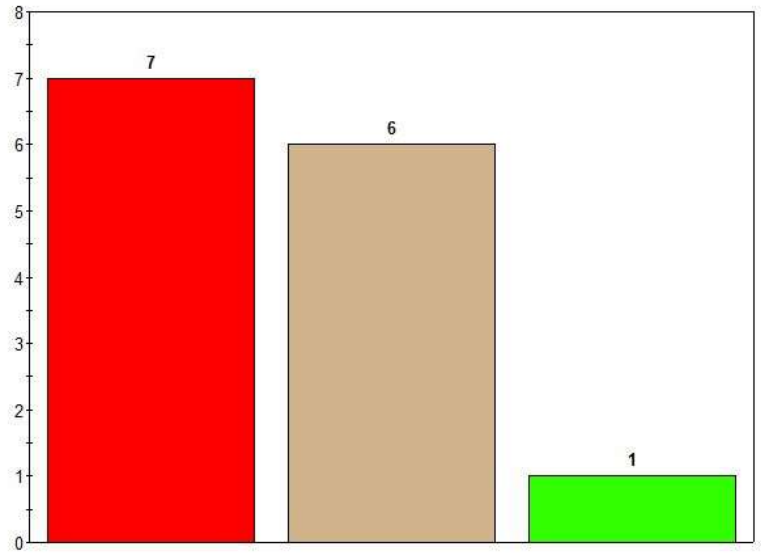
- Truthful
- Complex and unintuitive
- Lower revenue than GSP under truthful scenarios and **in experiments**



Efficiency vs. Equity

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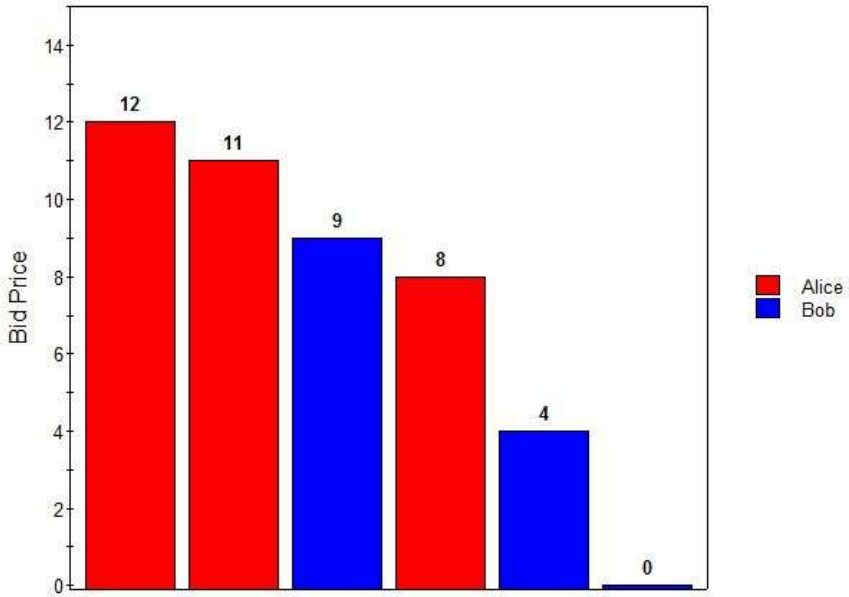
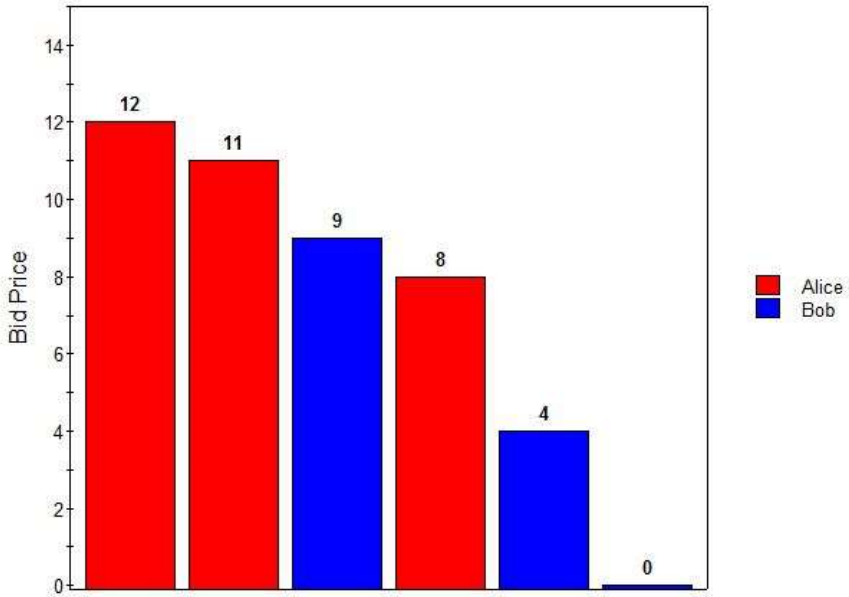
- With **uniform pricing**, price is highest losing bid



Efficiency vs. Equity

- With **uniform pricing**, price is highest losing bid
  - May belong to one of the winners



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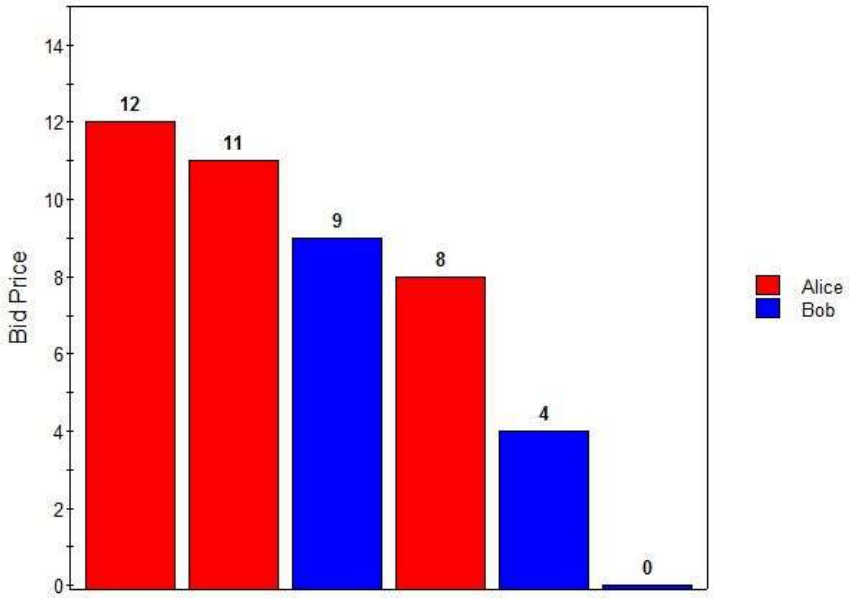
  - Encourages bid shading/**demand reduction**



Efficiency vs. Equity

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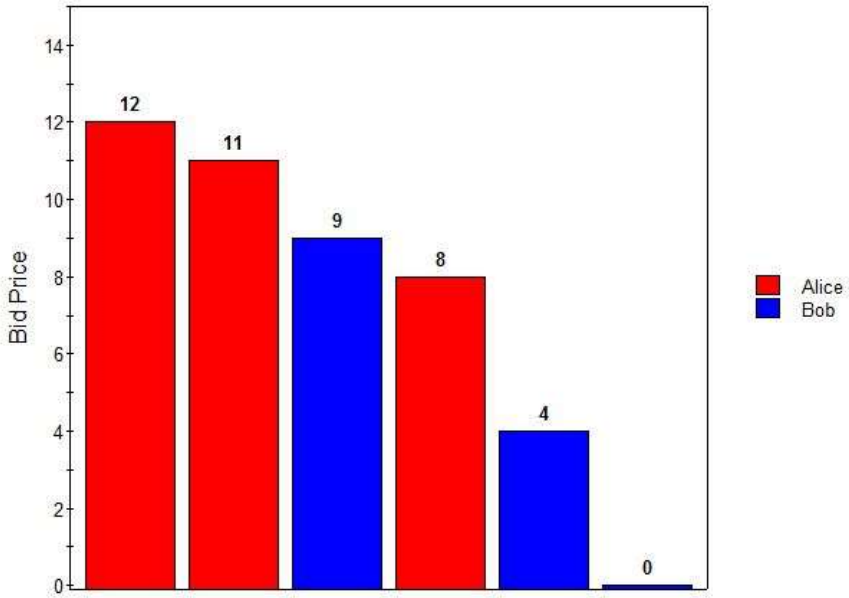

Efficiency vs. Equity

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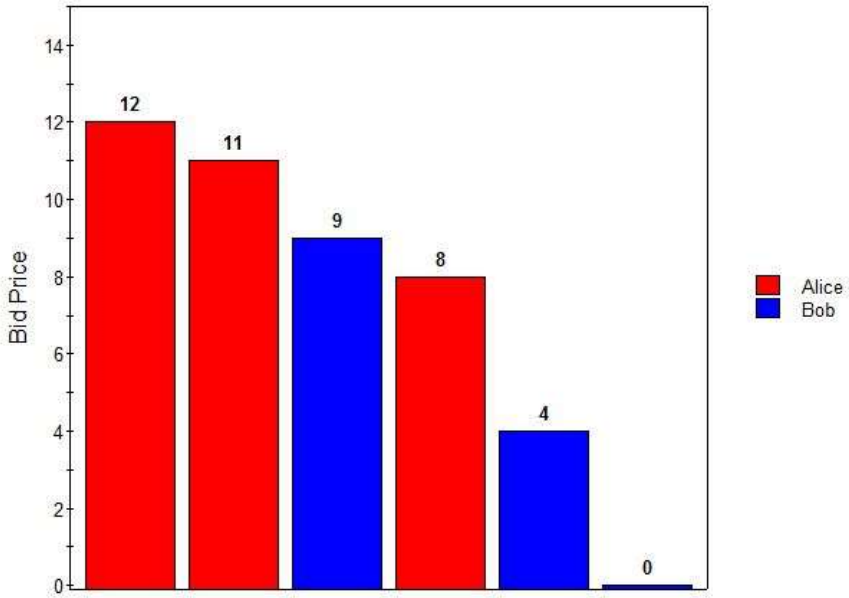
  - May belong to one of the winners
  - Encourages bid shading/**demand reduction**
- Uniform Price doesn't **extract the full value**
- Bid shading also happens with discriminatory auctions



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