Fair and Pareto-Optimal Allocation of Indivisible Chores

CS 580 Final Project

Zhuangjin (Jane) Du

University of Illinois Urbana-Champaign

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1. Introduction

2. Some Existing Subproblems
   - EF1 with mixed chores + goods
   - EF1 + PO using fPO

3. Other fair allocations of chores
   - MMS, EFX

Recap from Class

- **EF1**:

$$v_i(A_i) \geq v_i(A_j) - \max_{g \in A_j} v_i(g) \quad \forall i, j \in N$$

- **PROP1**

$$v_i(A_i) \geq v_i(M)/n - \max_{g \in M \setminus A_i} v_i(g) \quad \forall i \in N$$

- **MMS** For $k \in \mathbb{N}$, let $\mathcal{P}^k(M)$ be the set of all partitions of $\mathcal{M}$ into k bundles.

$$\mathrm{MMS}_i^k = \max_{(S_1, \ldots, S_k) \in \mathcal{P}^k(M)} \min_{t \in [k]} v_i(S_t) \quad i \in N$$

- **PO**: x is PO if it is not dominated by some integral allocation x', i.e.

$$v_i(\mathbf{x}_i) \leq v_i(\mathbf{x}_i')$$

Recap from Class

- Round Robin for EF1 goods for weakly additive utilities
- Envy-cycle-elimination for EF1 goods with monotone in creasing utilities
- EF1 + PO for goods exists for *additive* valuations. Computation in pseudopolynomial time.

EF1 for Chores

An example of an allocation of mixed chores and goods

	$\mathbf{g_1}$	$\mathbf{g_2}$	$\mathbf{g_3}$	$\mathbf{g_4}$
$\mathbf{a_1}$	3	-4	-4	-4
$\mathbf{a_2}$	3	-4	-4	-4

Definitions in Chore Allocation

Envy-freeness up to one item (EF1)

An integral allocation x is said to be envy-free up to one item (EF1) if for every pair of agents $i, j \in N$ such that $x_i \cup x_j \neq \emptyset$, there exists an item $r \in x_i \cup x_j$ such that $v_i\left(x_i \setminus \{r\}\right) \geq v_i\left(x_j \setminus \{r\}\right)$

Pain-per-buck

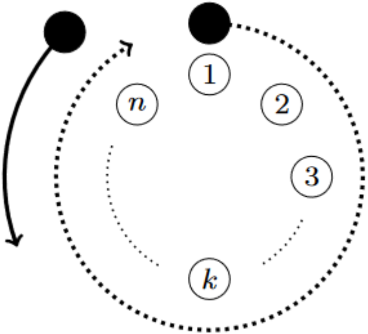
We can look at minimum payments where each agent receives p_j per unit of chore. Each agent then seeks to minimize her total cost subject to payment constraint (receiving a total payment above a threshold)

Double Round Robin

Partition the set of objects O into two sets O^+, O^-:

$$O^+ = \{o \in O \mid \exists i \in N \text{ s.t. } u_i(o) > 0\}$$

$$O^- = \{o \in O \mid \forall i \in N, u_i(o) \leq 0\}$$



Runtime: $O(\max m \log m, mn)$[1]

[1] Haris Aziz et al. *Fair allocation of combinations of indivisible goods and chores*. Mar. 17, 2021. arXiv: 1807.10684[cs]. URL: http://arxiv.org/abs/1807.10684 (visited on 09/21/2022).

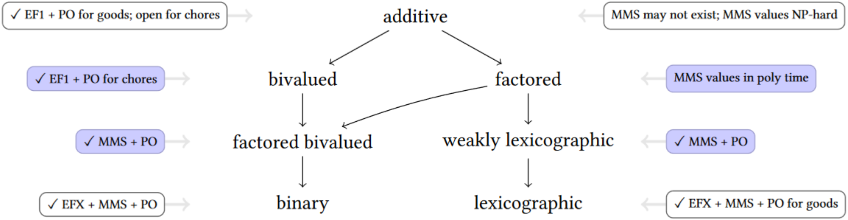
Bivalued and other subproblems

- **binary**: all valuations are in $\{0, -1\}$. Trivial: first apply 0's, then assign -1's evenly if possible.
- **factored bivalued**: all valuations lie in $\{a, b\}$ for integer b/a. Polynomial-time if b/a is an integer, NP-hard if a, b are coprime [1]
- **general bivalued**: easy and difficult chores: 1 for difficult chores, and $p > 1$ for easy chores

Weakly Lexicographic

Fractional Pareto Optimal (fPO)

For a price vector p such that $p(c) > 0 \forall c \in M$, an allocation is **fractionally Pareto optimal** if it is not Pareto dominated by any fractional allocation. Computing: pseudopolynomial time for goods.



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[2] Soroush Ebadian, Dominik Peters, and Nisarg Shah. *How to Fairly Allocate Easy and Difficult Chores.*

Bivalued chore algorithm

How do we achieve Pareto Optimality?

In bivalued chores, we know that fPO and PO are equivalent.[3]

Price envy-free up to one chore (pEF1)

For all $i, j \in N$ there is a chore $c \in x_i$ such that $P(x_i \setminus c) \leq (x_j)$. i is said to pEF1-envy j if this inequality is not true [5].

Bivalued chore algorithm[3]

1. *Initialization*: Partition agents into groups $N_1 \ldots N_R$, who are $pEF1$ within the group.
2. *Chore Reallocation* The biggest spender (BS b) envies the least spender (LS ℓ). If MBB edge exists, transfer from b to ℓ.
3. *Price Adjust* If edge doesn't exist, we may raise prices of chores in b. Groups are raised exactly once and in order $N_1 \ldots N_R$
4. *Chore Reallocation* Once LS enters a raised group, no more price-rise. Transfer from BS to LS via alternating paths.

[3] Jugal Garg, Aniket Murhekar, and John Qin. "Fair and Efficient Allocations of Chores under Bivalued Preferences". In: *Proceedings of the AAAI Conference on Artificial Intelligence* 36.5 (June 28, 2022), pp. 5043–5050. ISSN: 2374-3468, 2159-5399. DOI: 10.1609/aaai.v36i5.20436. URL: https://ojs.aaai.org/index.php/AAAI/article/view/20436 (visited on 10/13/2022).

MMS with chores

Weakly lexicographic:

Ask each agent to rank items, allowing for ties.

- Computing MMS values is NP-hard (for both chores and goods) under general additive valuations, but is polynomial-time for factored utility functions.
- Weakly lexicographic or factored bivalued utilities have an MMS+PO allocation computable in polynomial time [4].
- Work has been done for approximately-EFX chore assignment as well, including poly-time 5-approximation of EFX for 3 agents [6].

References

[1] Hannaneh Akrami et al. *Maximizing Nash Social Welfare in 2-Value Instances*. Oct. 1, 2021. arXiv: 2107.08965[cs]. URL: http://arxiv.org/abs/2107.08965 (visited on 12/01/2022).

[2] Haris Aziz et al. *Fair allocation of combinations of indivisible goods and chores*. Mar. 17, 2021. arXiv: 1807.10684[cs]. URL: http://arxiv.org/abs/1807.10684 (visited on 09/21/2022).

[3] Siddharth Barman, Sanath Kumar Krishnamurthy, and Rohit Vaish. "Finding Fair and Efficient Allocations". In: *Proceedings of the 2018 ACM Conference on Economics and Computation*. EC '18: ACM Conference on Economics and Computation. Ithaca NY USA: ACM, June 11, 2018, pp. 557–574. ISBN: 978-1-4503-5829-3. DOI: 10.1145/3219166.3219176. URL: https://dl.acm.org/doi/10.1145/3219166.3219176 (visited on 12/01/2022).

[4] Soroush Ebadian, Dominik Peters, and Nisarg Shah. *How to Fairly Allocate Easy and Difficult Chores*.

[5] Jugal Garg, Aniket Murhekar, and John Qin. "Fair and Efficient Allocations of Chores under Bivalued Preferences". In: *Proceedings of the AAAI Conference on Artificial Intelligence* 36.5 (June 28, 2022), pp. 5043–5050. ISSN: 2374-3468, 2159-5399. DOI: 10.1609/aaai.v36i5.20436. URL: https://ojs.aaai.org/index.php/AAAI/article/view/20436 (visited on 10/13/2022).

[6] Shengwei Zhou and Xiaowei Wu. "Approximately EFX Allocations for Indivisible Chores".