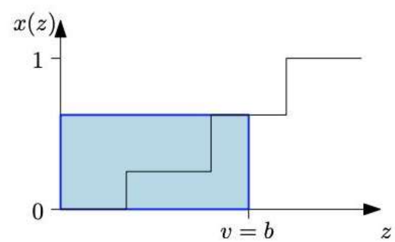
UNIVERSITY OF ILLINOIS URBANA-CHAMPAIGN

Optimal Bidding in Large Scale Multi-Item Auctions

Presented by:
Ishas Kekre and Tom Herschberg

Single vs Multi-Item

- Single Item
  - n bidders with valuations v_i
  - Second Price Auction -> $b_i = v_i$ maximizes utility and social welfare
- Multi-Item Auction
  - Collection of m items to be distributed to n bidders
  - V_{ij} for some allocation $j = \{x_1, x_2, \ldots\}$ is agent i's valuation of allocation j
  - Introduces lots of additional complexity



(a) $v \cdot x(v)$

(Roughgarden 2013)

Background

- Different types of multi-item auctions
  - Are items identical?
  - Are items divisible?
  - Are items complements/substitutes?
    - $V(A) + V(B) = V(\{A,B\})$?
  - Are items auctioned simultaneously?
  - Sealed or opened?
- Discussion
  - Spectrum Auctions
  - Sponsored Search Auctions

Spectrum Auctions

- Auction system used to sell and assign spectrum resources to various bidders.
- Why Auction?
  - Administrative process, lottery, first come first served etc.
  - Transparent & Fair
  - Maximize Revenues and Utility
  - Economic (McMillan 1995)
- Not identical
- Not divisible
- Simultaneous and Open Bidding
- Not monotone
  - Package Bidding

Simultaneous Ascending Auction

- Introduced by FCC in 1994

Bidder 1	Bidder 2	Bidder 3
(0,0,**1**)	(0,**1**,0)	(**1**,0,0)
(**2**,0,**1**)	(0,**1**,0)	(1,0,0)
(2,0,**1**)	(0,**1**,0)	(**3**,0,0)
(2,0,**1**)	(0,**1**,0	(**3**,0,0)

Package Bidding

- More efficient and revenue generating format
- Rise of free rider problem for individual license bids
    - V1(A) = 4, V2(B) = 4, V3(AB) = 3

	B1	B2	B3
A	1	.	.
B	.	1	.
AB	.	.	2

	p2	1-p2
p1	8-4 = 4	8 - 3 = 5
1-p1	8 - 3 = 5	3-2 = 1

Collusion

- Bidders working together to reduce auction prices
- Bid Signaling
  - Retaliation
  - Withdrawal
  - Market Division
- Bid Restricting
- Bidder Identities
  - Transparency
  - Competition for higher revenue
  - Signaling
- Withdrawal Limit
  - FCC introduced two round limit
- Reserve Prices
- Market Closing

Reserve Prices Example

- Two identical items, two bidders A and B
- V_a(one item) = 20, V_a(both) = 50, V_b(one) = 17
- Bidder B will put one truthful bid of 17
- No Reserve Price
  - A puts one bid at 20 -> A and B win an item for 0, A's utility is **20**
  - A puts one bid at 20 and one at 30 -> A wins both items for 17, utility is 50 - 2(17) = 16
  - Demand Reduction
- Reserve Price = 5
  - A puts one bid at 20 -> A and B win an item for 5, A's utility is 20-5 = 15
  - A puts one bid at 20 and one at 30 -> A wins both items for 17, utility is 50 - 2(17) = **16**

Price Forecasting + Budgeting

- For licenses that are substitutes, where should you bid to get the best licenses at the best price? - Price Forecasting (Stanford 2009)
- Exposure
  - Sum of all a bidder’s bids in a given round
  - Total Price * Demand/Supply
  - Equilibrium where Demand = Supply, exposure meets budget
  - Useful for revenue prediction < 10% error

(Bulow, Levin, Milgrom 2009)

Managing Exposure

- Agent bidding on two licenses, $v_{12} > v_1 + v_2$
- Unknown final price c_i from distribution F_i
- If opponent on license *i* exits, expected utility:

$$\pi_i(p_1, p_2) = v_i - p_i + Q_j(p_1, p_2). \tag{1}$$

$$Q_j(p_1, p_2) = \int_{p_j}^{\max\{b-p_i, p_j\}} \max\{0, (v_{12} - v_i - c_j)\} dF_j(c_j | c_j \geq p_j). \tag{2}$$

$$\pi_1(p_1, p_2) = 0 \qquad \text{and} \qquad \pi_2(p_1, p_2) = 0. \tag{3}$$

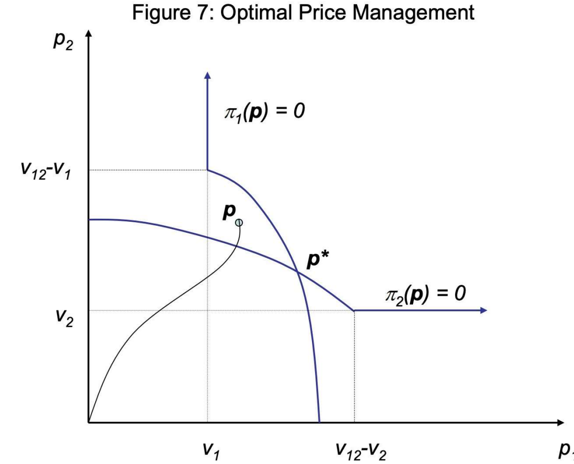
(Bulow, Levin, Milgrom 2009)

Managing Exposure cont.

- Prop1 - Optimal strategy involves raising prices along any path to the unique price pair (p_{1*}, p_{2*}) that will solve for (3), drop out at this point if both competitors are still active.
- Prop2 - If the individual bidder for for license *i* drops out first, continue bidding on license *j* until $p_j = v_{12} - \max\{p_i, v_i\}$

(Bulow, Levin, Milgrom 2009)

Figure 7: Optimal Price Management

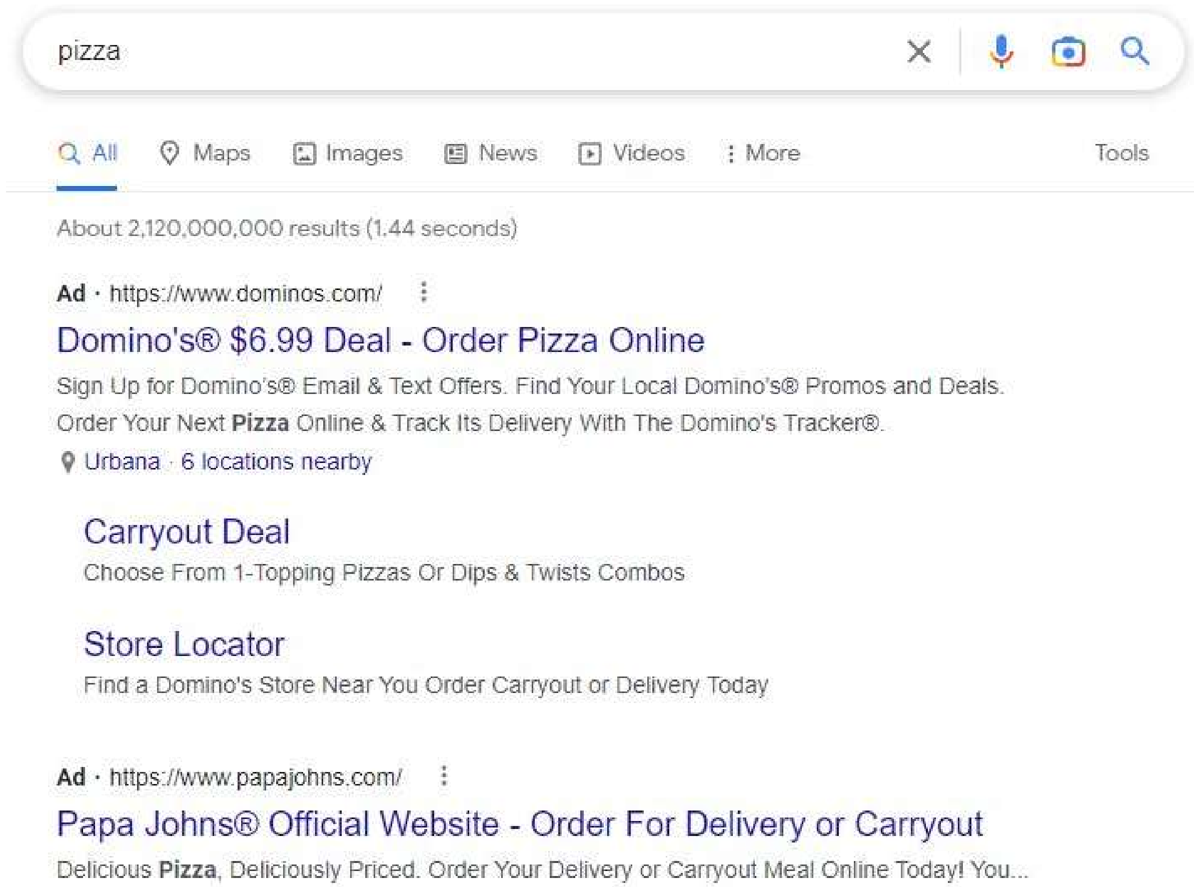


Managing Price Increase

- Holding Back Demand
- Parking
- Jump Bidding
  - Ineffective if done early
  - Risk of overpaying if done late
- SpectrumCo
  - Forecasted final prices
  - Alter rate of price increase
  - Massive market share with a billion dollar discount vs competitor’s prices

Sponsored Search Auctions (SSAs)

- Advertisers bid to be shown alongside results of search engine query
- Multiple advertising slots are available for each search
  - Slots higher on the page are considered more valuable
- Highest bidder gets the highest slot, second-highest bidder gets second slot, etc.
- Auction is run every time a query is made
- Most use generalized second price (GSP) auctions
- For this discussion, assume unlimited budget and full rationality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Sign Up for Domino's® Email & Text Offers. Find Your Local Domino's® Promos and Deals.
Order Your Next Pizza Online & Track Its Delivery With The Domino's Tracker®.
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Carryout Deal
Choose From 1-Topping Pizzas Or Dips & Twists Combos
Store Locator
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Delicious Pizza, Deliciously Priced. Order Your Delivery or Carryout Meal Online Today! You...

Generalized Second Price Auctions

- n bidders compete for k ad slots
  - $n > k$
  - Slot 1 is displayed first, slot k is displayed last
  - $CTR_1 \geq CTR_2 \geq \ldots \geq CTR_k$
- Each bidder *i* has a private value v_i
- Each bidder submits a bid $b_i \leq v_i$
- Bidders are numbered in the descending order of their bids
- Bidder *i* wins slot *i* and pays b_{i+1} each time their ad is clicked
- Truth-telling is ***not a dominant strategy***

(Varian 2007)

Truth-Telling in GSPs

- 3 bidders with values per click {$10, $4, $2}
- 2 ad slots with CTRs {200, 199}
- If bidder 1 bids truthfully:
  - Payoff = $(v_i - b_{k+1}) * CTR_k = (10 - 4) * 200 =$ $1200
- If bidder 1 shades bid to $3:
  - Payoff = $(v_i - b_{k+1}) * CTR_k = (10 - 2) * 199 =$ $1592 > $1200
- Bidder 1 increases payout by bidding less than their true value

(Edelman, Ostrovsky, and Schwarz 2005)

Optimal Bidding in SSAs

- Because GSPs are not incentive-compatible, bidders must choose a bidding strategy
- This results in a continuum of possible Nash equilibria
- VCG Equilibrium:
  - Possible Nash equilibrium of GSP
  - Player payments are identical to those made if VCG auction was used
  - Cheapest envy-free equilibrium for bidders
- How can we converge to VCG equilibrium over repeated GSPs?

Best-Response Bidding

- Bidder i chooses a bid for the next round that maximizes their utility assuming all other bidders b_{-i} behave the same
  - Utility $u_i = \text{CTR}_s(v_i - p_s)$
- i will make a bid $b' \in (p_{s^*}(i), p_{s^*-1}(i))$ to win slot s^*
  - $p_{s^*}(i)$ = price that i pays to win slot s^*
- Where to bid within this range?
  - Too high: Bidder $i + 1$ will undercut i and force them into a slot with lower utility
  - Too low: Bidder $i - 1$ will out bid
  - Want competitors to pay as much as possible

(Nisan et. al. 2011)

The Balanced Bidding (BB) Strategy

- Bidder *i* targets the slot s^*_i which maximizes their utility given b_{-i}
  - $s^*_i = \text{argmax}_s\{CTR_s(v_i - p_s(i))\}$
- Bidder *i* makes the max bid *b'* that wouldn't lower utility if they were undercut
  - $CTR_{s^*i}(v_i - p_{s^*i}(i)) = CTR_{s^*i-1}(v_i - b')$
- The BB Strategy converges to VCG equilibrium if exactly one random bidder updates their bid after every round
  - Known as the *asynchronous bidding* model
- What if all bidders simultaneously submit new bids after each round?
  - Requires slightly different strategy called Restricted Balanced Bidding (RBB)

(Cary et. al. 2014)

The Restricted Balanced Bidding (RBB) Strategy

- Bidder *i* targets the slot $s^*_i \leq \underline{s_i}$ which maximizes their utility given b_{-i}
  - $s^*_i = \mathrm{argmax}_s\{CTR_s(v_i - p_s(i))\}$
  - s^*_i must have a CTR no higher than the CTR of s_i
- Bidder *i* makes the max bid *b'* that wouldn't lower utility if they were undercut
  - $CTR_{s^*i}(v_i - p_{s^*i}(i)) = CTR_{s^*i-1}(v_i - b')$
- The RBB Strategy converges to the VCG equilibrium if all bidders change bids after each round
  - Known as the *synchronous model*
- Both BB and RBB were later proven to be incentive compatible
  - If all bidders use best-response bidding, there is no benefit to deviating

(Cary et. al. 2014)

Collusion in SSAs

- Advertisers have started delegating bidding campaigns to digital marketing agencies (DMAs)
- DMAs often bid on the behalf of multiple advertisers in the same auction
- Coordinating prices between advertisers causes payments to decrease
- Introduces completely different bidding strategies

Collusion in SSAs

- Reminder: Bidder *i* chooses a bid such that getting underbid won't affect utility
  - $b_i = v_i - (\mathrm{CTR}_i / \mathrm{CTR}_{i-1}) * (v_i - b_{i+1})$
- Consider a scenario with 8 bidders and 7 slots
  - Valuations v = (12, 10.5, 10.4, 10.3, 10.2, 10.1, 10, 1)
  - CTRs = (50, 40, 30.1, 20, 10, 2, 1, 0)
  - DMA D contains bidders 5 and 6, i.e. D = {5, 6}

(Decarolis et. al. 2020)

Bidder	VCG Equilibrium Bid	Bid After Collusion
1	X	X
2	10.11	9.91
3	10.01	9.76
4	9.88	9.12
5	9.67	9.5
6	9.18	7.94
7	5.5	5.5
8	1	1

<- Lowered by DMA to benefit bidder 5

Revenue: 55.35   Revenue: 52.73

Collusion in SSAs

- Reminder: Utility $u_i = \mathrm{CTR}_s(v_i - p_s)$
- Bidder 4 undercuts bidder 5 to increase their utility after the DMA lowered the bid of bidder 6
  - Utility of slot 4 = 20(10.3 - 9.5) = 16
  - Utility of slot 5 = 10(10.3 - 7.94) = 23.6
- Bidder 5 then moved up to slot 4, which increased their utility *relative to the VCG eq.*
  - Utility from VCG eq. = 10(10.2 - 9.67) = 5.3
  - Utility from collusion eq. = 20(10.2 - 9.12) = **21.6**
- Resulting allocation is inefficient

Conclusion

- Optimal bidding strategies became difficult to ascertain when these auctions were scaled up
- Both types of auction became susceptible to collusion at large scale
- A clear trade-off emerged between security and efficiency
  - Spectrum auctions are complex and holistic due to the importance and cost of the items being auctioned
  - SSAs are much simpler and intuitive to allow for quick completion and low barrier of entry, making them more susceptible to collusion and inefficiencies

Questions?