Mechanism Design (Auctions)

Tuesday, September 13, 2022 10:57 AM

- A seller/ auctioneer.
- selling "single item".
- N = set of agents/ bidders / players.

agent $i \in N$ values the item at say $\underset{\sim}{v_i}$
$\downarrow$
private info./type of agent i.

☆ Sealed Bid Auctions:

① Auctioneer solicits "bids" from the agents in a sealed envelope

agent i bids b_i in a sealed envelope.

↳ b_i need not be v_i

② Auctioneer opens all envelopes, looks at the b_i's & decides

Auctioneer's Goal: max s.w.
$\Updownarrow$
give the item to agent of max v_i

$$\text{winner} = \underset{i}{\operatorname{argmax}}\ v_i$$

$$= i^*$$

payment = P

$$u_i(b_1, \ldots, b_n) = v_i - P \quad \text{if } i = i^* \text{ (winner)}$$
$$= 0 \quad \text{o.w.}$$

---

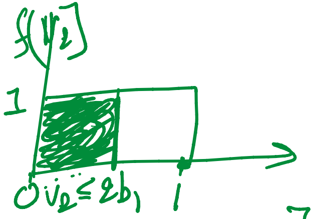
winner = Highest bidder

	1	2	3
v_i	[illegible]	[illegible]	[illegible]

		1	2	3
	v_i	1500	800	200
payment = pay your bid	b_i	1499	600	180
payment = pay second highest bid.	b_i	1600	800	2000

☆ First price Auction: Highest bidder wins, pays the bid.

Example : $N = \{1, 2\}$, $V_1, V_2 \sim U[0, 1]$



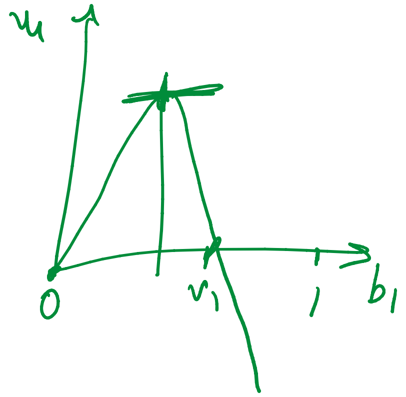
Suppose : $b_2 = \frac{V_2}{2}$

$$u_1(b_1, b_2) = (V_1 - b_1) \, Pr[b_1 \geq b_2] + 0 \, Pr[b_1 < b_2]$$

$$= (V_1 - b_1) \, Pr[2b_1 \geq V_2]$$

$$= (V_1 - b_1)(2b_1)$$

$$b_1^* = \underset{b_1}{\arg\max} \; u_1(b_1, b_2) = \underset{b_1}{\arg\max} \; \underline{2V_1 b_1 - 2b_1^2}$$



$$\frac{d}{db_1}(2V_1 b_1 - 2b_1^2) = 0$$

$$\Rightarrow 2V_1 - 4b_1 = 0$$

$$\Rightarrow b_1 = \frac{V_1}{2}$$

Similarly, if $b_1 = \frac{V_1}{2}$ then best bid for agent 2 is $b_2 = \frac{V_2}{2}$

2 is $b_2 = \frac{v_2}{2}$

$$\Downarrow$$

$\left(\frac{v_1}{2}, \frac{v_2}{2}\right)$ is a BNE.

Generalize, $N = \{1, \dots, n\}$ $\quad v_i \sim U[0,1]$

$b_i = \frac{(n-1)}{n} v_i \quad \forall i$ is a NE.

① what if v_i's have a complex distribution?

② " " " " different " " ?

③ What if agents are not fully rational?

④ what if there are other NE & agents could not coordinate on what to play?

★ Second Price : Highest bidder wins, pays the second highest bid.

winner $= \underset{i}{\arg\max}\, b_i = i^*$

$B_i =$ critical bid of agent i.

payment $p_i = \boxed{\max_{k \neq i} b_k}$ $\quad$ if $i = i^*$

$= 0 \quad$ o.w.

(Vickrey '61): Under second price, for each i, Thm (Vickery '61): Under second price, for each i, $b_i = V_i$ is an optimal bid no matter what others are bidding

|||

$\forall i, \quad b_i = V_i$ is a DSE.

|||

$\forall i, \ \forall b_{-i}, \quad u_i(V_i, b_{-i}) \geq u_i(b_i, b_{-i}), \ \forall b_i$

Pf: Fix an agent i, fix b_{-i} (arbitrarily)

agent i bids b_i

$V_i = \arg\max_{b_i} u_i(b_i, b_{-i})$

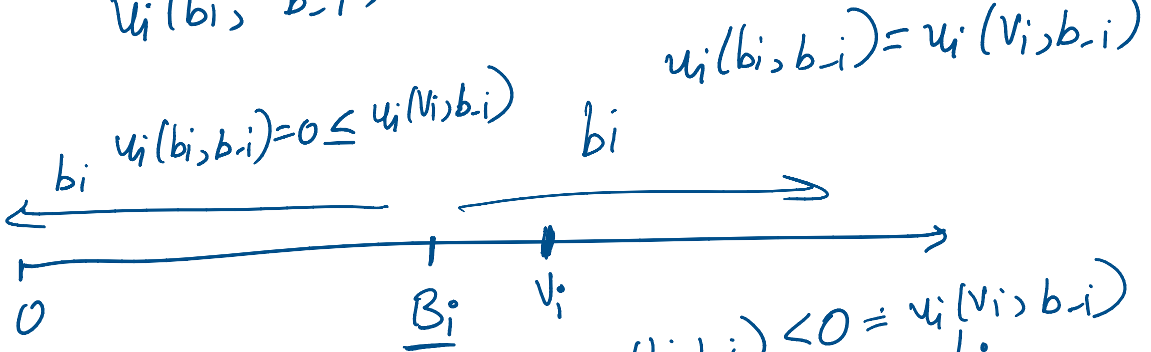
Case I: i wins.

$u_i(b_i, b_{-i}) = \underline{V_i - B_i}$ $\quad \nearrow \max_{k \neq i} b_k$ $\quad (b_i \geq B_i)$

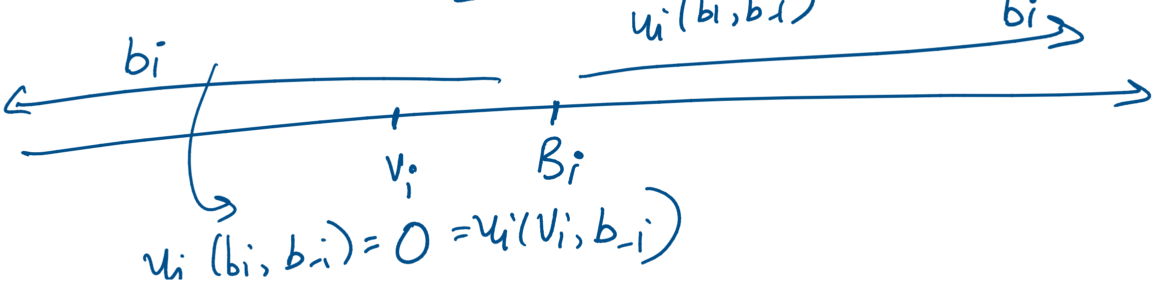
Case II: i looses.

$u_i(b_i, b_{-i}) = 0$.

case I



case II



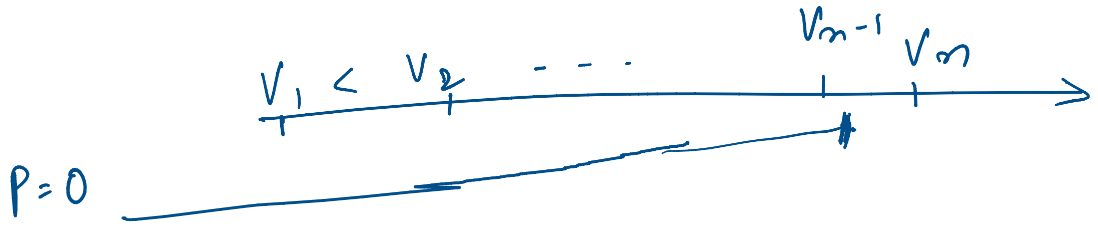
$u_i(b_i, b_{-i}) = 0 = u_i(v_i, b_{-i})$

Dominant stategy Incentive Compatible
(DSIC)
Truthful Auction.

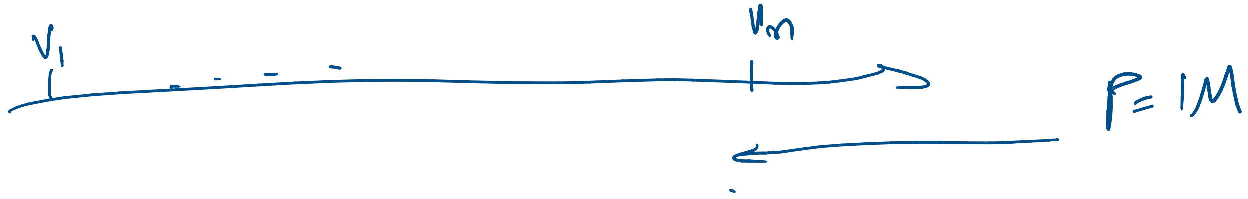
---

* Ebay = second price.

English Auction.: Increasing price.



* Dutch Auction : Decreasing price



|||.

First price.