cs579: Computational Complexity	Assigned: *Thu., Jan. 31, 2019*
Problem Set #2	
Prof. Michael A. Forbes	Due: *Thu., Feb. 14, 2019 (3:30pm)*

Some details from the syllabus.

- **Submission Policy:** Problem sets will be posted by 3:30pm on the day of release, and are due 3:30pm on the due date. An electronic (pdf) copy must be submitted by email to the course staff (`miforbes` and `awk2`). Each problem should be on a separate page. The subject line of the email should be "[cs579] ps$N$ submission" (N = pset number) and the filename must be "*NETID*_ps$N$.pdf" (N = pset number, *NETID* = your netid).

- **Collaboration Policy:** Students are forbidden from directly searching for solutions on the internet, but may consult the exercise-hints in the textbooks for the course. That said, students are highly encouraged to collaborate in small groups. However, this must be a two-way collaboration. Students should not dictate complete solutions to other students, either verbally or written. Solutions must be written independently regardless of collaboration, and psets must list the collaborators you worked with.

- **Late Policy:** Students are highly encouraged to turn-in assignments on-time to avoid falling behind on the material, and to incentivize this any late homework will automatically lose 10%. However, to be flexible, students have a total of 6 late days (24 hours each, rounded up to an integral number of days), no more than 3 of which can be used for any particular homework. Problem sets will not be accepted for credit once the late days are exhausted.

- **Solutions:** Hard-copy sample solutions will be distributed to students when the problem sets are returned (please keep the internet free of easily-found solutions). These sample solutions will be selected from student submissions (with names omitted). Please inform the course staff if you wish to *opt-out* of ever being selected.

Each problem is worth 10 points.

1. (Sipser #7.21) Let $\mathsf{DOUBLESAT} = \{\langle\varphi\rangle : \varphi$ has at least two satisfying assignments$\}$. Show that $\mathsf{DOUBLESAT}$ is NP-complete.

2. (Sipser #7.39) In the proof of the Cook-Levin theorem, a window is a 2×3 rectangle of cells. Show why the proof would have failed if we had used 2×2 windows instead.

3. (Sipser #7.17) Show that, if $\mathsf{P} = \mathsf{NP}$, then every language $A \in \mathsf{P}$, except $A = \emptyset$ and $A = \Sigma^*$, is NP-complete.

4. (Sipser #8.12) Show that TQBF restricted to formulas where the part following the quantifiers is in conjunctive normal form is still PSPACE-complete.

5. (Sipser #8.22)

   (a) Let $\mathsf{ADD} = \{\langle x, y, z\rangle : x, y, z > 0$ are binary integers and $x + y = z\}$. Show that $\mathsf{ADD} \in \mathsf{L}$.

(b) Let $\mathsf{PALADD} = \{\langle x, y\rangle : x, y > 0$ are binary integers where $x + y$ is an integer whose binary representation is a palindrome$\}$. (Note that the binary representation of the sum is assumed not to have leading zeros. A palindrome is a string that equals its reverse.) Show that $\mathsf{PALADD} \in \mathsf{L}$.

6. (Sipser #8.27) Recall that a directed graph is strongly connected if every two nodes are connected by a directed path in each direction. Let $\mathsf{STRONGLYCONNECTED} = \{\langle G\rangle : G$ is a strongly connected graph$\}$. Show that STRONGLYCONNECTED is NL-complete.