CS 579. Computational Complexity
Problem Set 6

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due May 3, 2017

Collaboration Policy: The homework can be worked on in groups of up to 3 students each (2 would be optimal, but 1 and 3 are both accepted).

One submission per team is sufficient. Please write the solution for each of the problems on a separate sheet of paper. Write your team names and netids on each submission and please **staple** all the sheets together.

Submissions should be written in LaTeX, unless your handwriting is indistinguishable from LaTeX.

Homework is due before the end of class, May 3. Only one late homework per person will be allowed. If you submit more than one homework late, you will get no grade for the excess late homeworks.

Problem 1 (35 pts.)

Alice and Bob share an arbitrarily long common string S. Alice is given as input a random bit x_A and Bob a random bit x_B. Without communicating with each other, Alice and Bob wish to output bits a and b respectively such that $x_A \wedge x_B = a \oplus b$. Prove that any protocol that Alice and Bob follow has success probability at most 3/4.

Problem 2 (30 pts.)

Consider a CNOT gate whose second input is $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$. Describe the action of the CNOT gate on the first input. Now show that if the CNOT gate is applied in the Hadamard basis, i.e., apply the Hadamard gate to the inputs and outputs of the CNOT gate - then the result is a CNOT gate with the control and target qubits swapped

Problem 3 (35 pts.)

You are given one of two quantum states of a single qubit: either $|\phi\rangle = |0\rangle$ *or* $|\psi\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$. What measurement best distinguishes between these two states? If the state you are presented is either $|\phi\rangle$ or $|\psi\rangle$ with .5 probability each, what is the probability that your measurement correctly identifies the state? Can you generalize your result to distinguish between two arbitrary quantum states $|\phi\rangle$ and $|\psi\rangle$ on two qubits?