CS 579. Computational Complexity
Problem Set 2

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due February 17 , 2017

Collaboration Policy: The homework can be worked on in groups of up to 3 students each (2 would be optimal, but 1 and 3 are both accepted).

One submission per team is sufficient. Please write the solution for each of the problems on a separate sheet of paper. Write your team names and netids on each submission and please **staple** all the sheets together.

Submissions should be written in LaTeX, unless your handwriting is indistinguishable from LaTeX.

Homework is due before the end of class, February 17. Only one late homework per person will be allowed. If you submit more than one homework late, you will get no grade for the excess late homeworks.

Problem 1 (40 pts.)

Recall that $\mathbf{EXP} = \mathbf{DTIME}\left(2^{n^{O(1)}}\right)$.

- Prove that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{\Sigma_k} = \mathbf{P}$ for all k.
- Prove that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{EXP} \not\subseteq \mathbf{P/poly}$.

Problem 2 (20 pts.)

Prove that if $\mathbf{NP} \subseteq \mathbf{BPP}$ then $\mathbf{NP} = \mathbf{RP}$.

Problem 3 (20 pts.)

Define a language L which belongs to $\mathbf{SIZE}(O(1))$ and is undecidable.

Problem 4 (20 pts.)

Define a language L to be downward-self-reducible if there is a polynomial time algorithm R that for any n and $x \in \{0,1\}^n$, $R^{L_{n-1}}(x) = L(x)$ where by L_k we denote an oracle that solves L on inputs of size at most k. Prove that if L is downward self-reducible, then $L \in \mathbf{PSPACE}$.