CS 579. Compuatational Complexity
Problem Set 1

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due February 3, 2017

Collaboration Policy: The homework can be worked on in groups of up to 3 students each (2 would be optimal, but 1 and 3 are both accepted).

One submission per team is sufficient. Please write the solution for each of the problems on a separate sheet of paper. Write your team names and netids on each submission and please **staple** all the sheets together.

Submissions should be written in LaTeX, unless your handwriting is indistinguishable from LaTeX.

Homework is due before the end of class, February 3. Only one late homework per person will be allowed. If you submit more than one homework late, you will get no grade for the excess late homeworks.

Problem 1 (20 pts.)

Recall that **NEXP** is defined by

$$\mathbf{NEXP} = \bigcup_{c \geq 1} \mathbf{NTIME}(2^{n^c}).$$

Give a definition of **NEXP** that does not involve non-deterministic Turing machines, analogous to the verifier definition of **NP** seen in class, and prove that your definition is equivalent to the above definition using non-deterministic Turing machines.

Problem 2 (20 pts.)

Recall that $\mathbf{E} = \mathbf{DTIME}\left(2^{O(n)}\right)$ is the class of problems solvable by a deterministic Turing machine in time $2^{O(n)}$, where n is the length of the input. We say that a language A has a many-to-one polynomial time reduction to a language B, written $A \leq_m^p B$ if there is a polynomial time computable function $f(\cdot)$ such that for every instance x we have $x \in A \iff f(x) \in B$.

- Show that **NP** is closed under polynomial many-to-one reductions, that is $A \leq_m^p B$ and $B \in \mathbf{NP}$ implies $A \in \mathbf{NP}$.
- Show that if **E** were closed under many-to-one reductions, we would have a contradiction to the time hierarchy theorem. Conclude that $\mathbf{NP} \neq \mathbf{E}$.

Problem 3 (20 pts.)

Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$.

Problem 4 (20 pts.)

Prove that if $\mathbf{P} = \mathbf{NP}$, then $\mathbf{EXP} = \mathbf{NEXP}$.

Problem 5 (20 pts.)

Suppose $L_1, L_2 \in \mathbf{NP} \cap \mathbf{coNP}$. Show that the language

$$L_1 \triangle L_2 = \{x \mid x \text{ is in exactly one of } L_1, L_2\}$$

is in $\mathbf{NP} \cap \mathbf{coNP}$.