Probabilistic Computation

Lecture 13

Understanding BPP

Recap

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- Probabilistic computation

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- NTM (on "random certificates") for L:

Recap

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  - Pr[M(x)=yes]:

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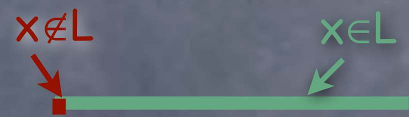

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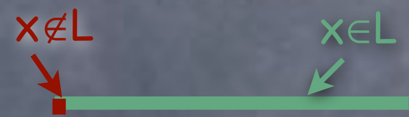
  - Pr[M(x)=yes]: 

- PTM for L: Pr[yes]: 

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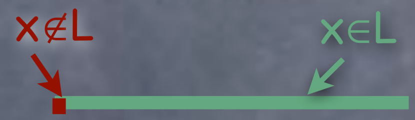
  - Pr[M(x)=yes]: 

- PTM for L: Pr[yes]: 

- BPTM for L: Pr[yes]: 

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- NTM (on "random certificates") for L:
  - Pr[M(x)=yes]: 

- PTM for L: Pr[yes]: 

- BPTM for L: Pr[yes]: 

- RTM for L: Pr[yes]: 

Recap

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- PP, RP, co-RP, BPP

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  - PP too powerful: NP $\subseteq$ PP

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    - Gap can be boosted from 1/poly to 1-1/exp

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    - A realistic/useful computational model

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  - PP too powerful: NP ⊆ PP
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- Today:

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  - PP too powerful: NP $\subseteq$ PP
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- Today:
  - NP $\not\subseteq$ BPP, unless PH collapses

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- PP, RP, co-RP, BPP
  - PP too powerful: NP ⊆ PP
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    - Gap can be boosted from 1/poly to 1-1/exp
    - A realistic/useful computational model
- Today:
  - NP ⊄ BPP, unless PH collapses
  - BPP ⊆ $\Sigma_2^P \cap \Pi_2^P$

BPP vs. NP

BPP vs. NP

- Can randomized algorithms efficiently decide all NP problems?

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  - Unlikely: NP $\subseteq$ BPP $\Rightarrow$ PH = Σ_2^P

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  - Unlikely: NP $\subseteq$ BPP $\Rightarrow$ PH = Σ_2^P
  - Will show BPP $\subseteq$ P/poly
    - Then NP $\subseteq$ BPP $\Rightarrow$ NP $\subseteq$ P/poly

BPP vs. NP

- Can randomized algorithms efficiently decide all NP problems?
  - Unlikely: NP $\subseteq$ BPP $\Rightarrow$ PH = Σ_2^P
  - Will show BPP $\subseteq$ P/poly
    - Then NP $\subseteq$ BPP $\Rightarrow$ NP $\subseteq$ P/poly
      - $\Rightarrow$ PH = Σ_2^P

BPP $\subseteq$ P/poly

BPP ⊆ P/poly

- If error probability is sufficiently small, will show there should be at least one random tape which works for all 2^n inputs of length n

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r \ x						
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BPP ⊆ P/poly

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r \ x						
	✓	✗	✗	✓	✓	✓
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BPP $\subseteq$ P/poly

- If error probability is sufficiently small, will show there should be at least one random tape which works for all 2^n inputs of length n
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- One such random tape if average (over x) error probability is less than 2^{-n}

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BPP ⊆ P/poly

- If error probability is sufficiently small, will show there should be at least one random tape which works for all 2^n inputs of length n
  - Then, can give that random tape as advice
- One such random tape if average (over x) error probability is less than 2^{-n}
  - BPP: can make worst error probability $< 2^{-n}$

r \ x						
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BPP vs. PH

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- BPP $\subseteq \Sigma_2^P$
  - So BPP $\subseteq \Sigma_2^P \cap \Pi_2^P$

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- L = { x| for almost all r, M(x,r)=yes }

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    - Note: Neighborhood of z is small (polynomially large), so can go through all of them in polynomial time

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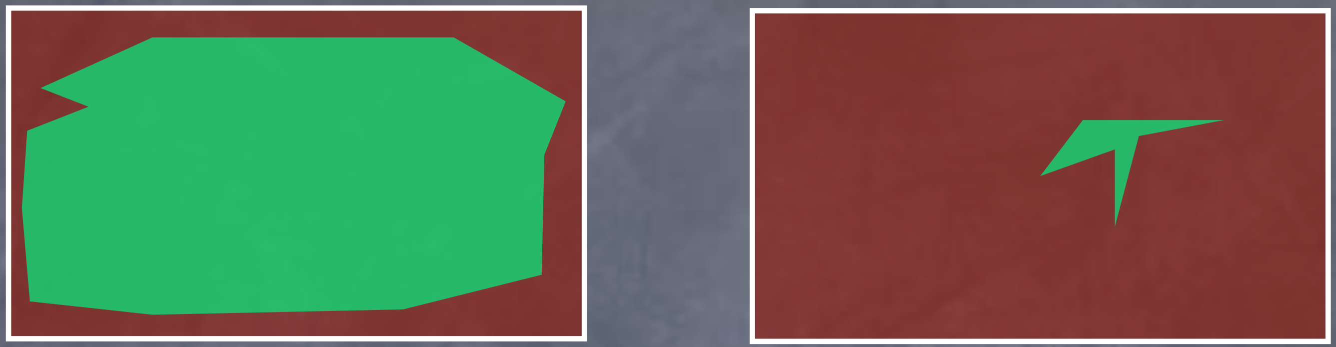

Space of random tapes = $\{0,1\}^m$

Yes_x = {r| M(x,r)=yes }

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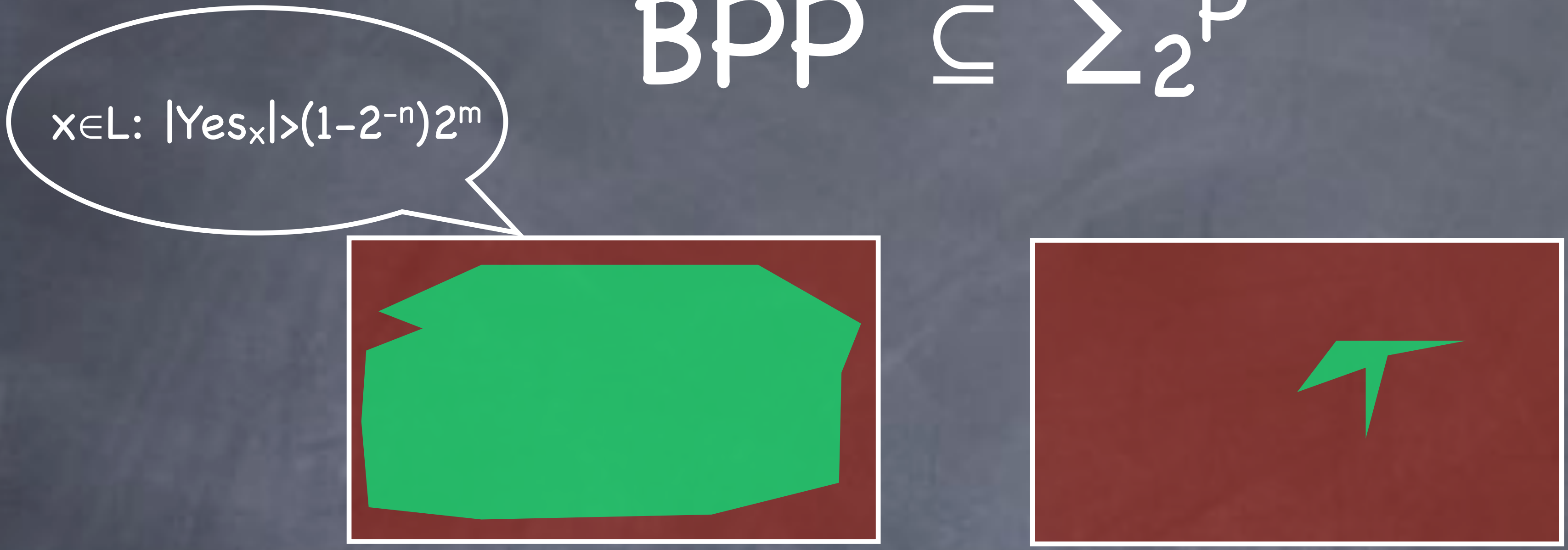

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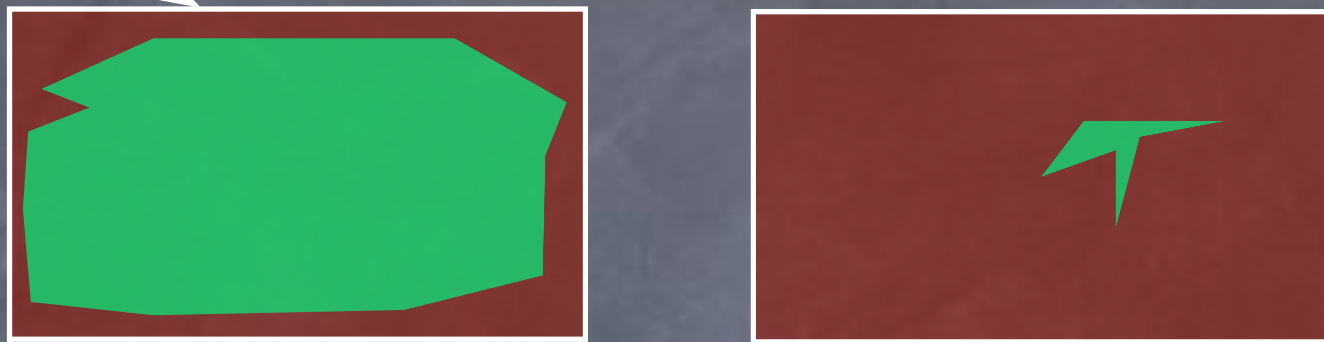

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- $x \notin L$: Yes_x very small, so its few shifts cover only a small region

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- "A small set of shifts": $P = \{u_1, u_2, \ldots, u_k\}$
  - $P(r) = \{ r \oplus u_1, r \oplus u_2, \ldots, r \oplus u_k\}$ where r, u_i are m-bit strings, and k is "small" (poly(n))

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    - In fact, most P work (if k big enough)!

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  - Define a probability distribution over all candidate P's and prove that the property holds with positive probability (often even close to one)

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- Probabilistic Method (finding hay in haystack)
  - To prove ∃P with some property
  - Define a probability distribution over all candidate P's and prove that the property holds with positive probability (often even close to one)
    - Distribution s.t. easy to prove positive probability of property holding

BPP $\subseteq \Sigma_2^P$

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- Probabilistic method to find $P = \{u_1, u_2, \ldots, u_k\}$, s.t. for all large S, $P(S) = \{0,1\}^m$

$BPP \subseteq \Sigma_2^P$

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  - $Pr_{(over\ P)}[P(S) \neq \{0,1\}^m] = Pr_{(over\ P)}[\exists z\ z \notin P(S)]$
    $\leq \Sigma_z\ Pr_{(over\ P)}[z \notin P(S)]$

BPP $\subseteq \Sigma_2^P$

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    $\leq \Sigma_z\ Pr_{(\text{over } P)}[z \notin P(S)] \quad = \Sigma_z\ Pr_{(\text{over } u1..uk)}[\forall i \quad z \oplus u_i \notin S]$

$BPP \subseteq \Sigma_2^P$

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    - Pick each u_i independently, and uniformly at random from $\{0,1\}^m$
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    $= \Sigma_z\ \Pi_i\ \ Pr_{\ (over\ ui)}[z \oplus u_i\ \notin S]$

$\mathrm{BPP} \subseteq \Sigma_2^P$

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  - $\Pr_{(\text{over } P)}[P(S) \neq \{0,1\}^m] = \Pr_{(\text{over } P)}[\exists z\ z \notin P(S)]$

    $\leq \Sigma_z \Pr_{(\text{over } P)}[z \notin P(S)] \quad = \Sigma_z \Pr_{(\text{over } u1..uk)}[\forall i \quad z \oplus u_i \notin S]$

    $= \Sigma_z \Pi_i \Pr_{(\text{over } ui)}[z \oplus u_i \notin S] = \Sigma_z \Pi_i \Pr_{(\text{over } ui)}[u_i \notin z \oplus S]$

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    $= \Sigma_z\ \Pi_i\ Pr_{(\text{over }ui)}[z \oplus u_i \notin S] = \Sigma_z\ \Pi_i\ Pr_{(\text{over }ui)}[u_i \notin z \oplus S]$

    $= \Sigma_z\ \Pi_i\ (|S^c|/2^m)$

$BPP \subseteq \Sigma_2^P$

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    $= \Sigma_z\ \Pi_i\ (|S^c|/2^m) \quad < \Sigma_z\ \Pi_i\ 2^{-n}$

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    $\leq \Sigma_z\ Pr_{(over\ P)}[z \notin P(S)] \quad = \Sigma_z\ Pr_{(over\ u1..uk)}[\forall i \quad z \oplus u_i \notin S]$

    $= \Sigma_z\ \Pi_i\ Pr_{(over\ ui)}[z \oplus u_i \notin S] = \Sigma_z\ \Pi_i\ Pr_{(over\ ui)}[u_i \notin z \oplus S]$

    $= \Sigma_z\ \Pi_i\ (|S^c|/2^m) \quad < \Sigma_z\ \Pi_i\ 2^{-n} \quad = 2^m.(2^{-n})^k = 1$

$BPP \subseteq \Sigma_2^P$

- Probabilistic method to find $P = \{u_1,u_2,\ldots,u_k\}$, s.t. for all large S, $P(S) = \{0,1\}^m$
  - Distribution over P's: randomized experiment to generate P
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    $= \Sigma_z\ \Pi_i\ Pr_{(over\ ui)}[z \oplus u_i \notin S] = \Sigma_z\ \Pi_i\ Pr_{(over\ ui)}[u_i \notin z \oplus S]$

    $= \Sigma_z\ \Pi_i\ (|S^c|/2^m) \quad < \Sigma_z\ \Pi_i\ 2^{-n} \quad = 2^m.(2^{-n})^k = 1$
  - So (with $|S|>(1-2^{-n})2^m$ and $k=m/n$), $\exists P$, $P(S) = \{0,1\}^m$

$BPP \subseteq \Sigma_2^P$

Space of random strings = $\{0,1\}^m$

Yes_x = {r| M(x,r)=yes }

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- For each $x \in L$, $\exists P$ (of size $k=m/n$) s.t. $P(Yes_x)=\{0,1\}^m$

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$x \in L$: $|Yes_x| > (1-2^{-n})2^m$

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- For each $x \in L$, $\exists P$ (of size $k=m/n$) s.t. $P(Yes_x)=\{0,1\}^m$
- For each $x \notin L$, $P(Yes_x) \subsetneq \{0,1\}^m$

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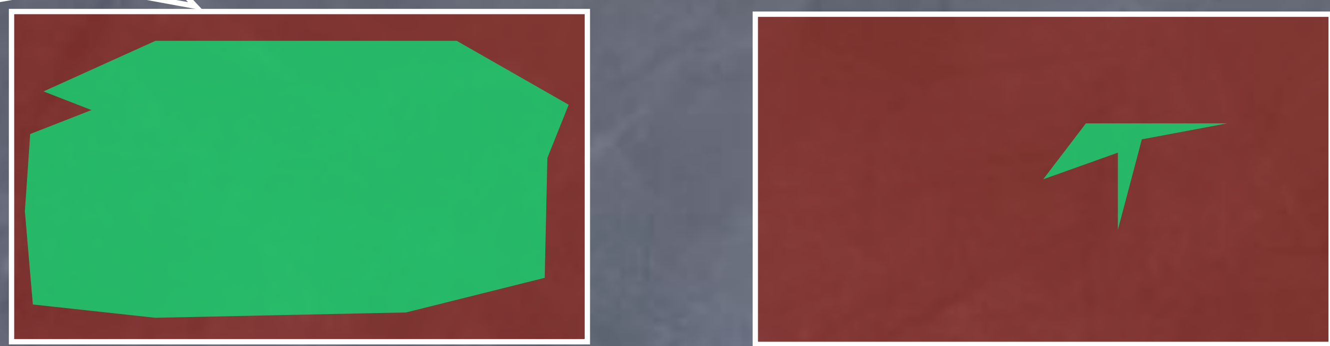
$BPP \subseteq \Sigma_2^P$



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      - If $(M,x,1^t)$ in L, we will indeed accept with prob. > 2/3
      - But M may not have a bounded gap. Then, if $(M,x,1^t)$ not in L, we may accept with prob. very close to 2/3.

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  - But is true for BPTIME(T)/1

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