Quantum Computation

Lecture 27 And that's all we got time for!



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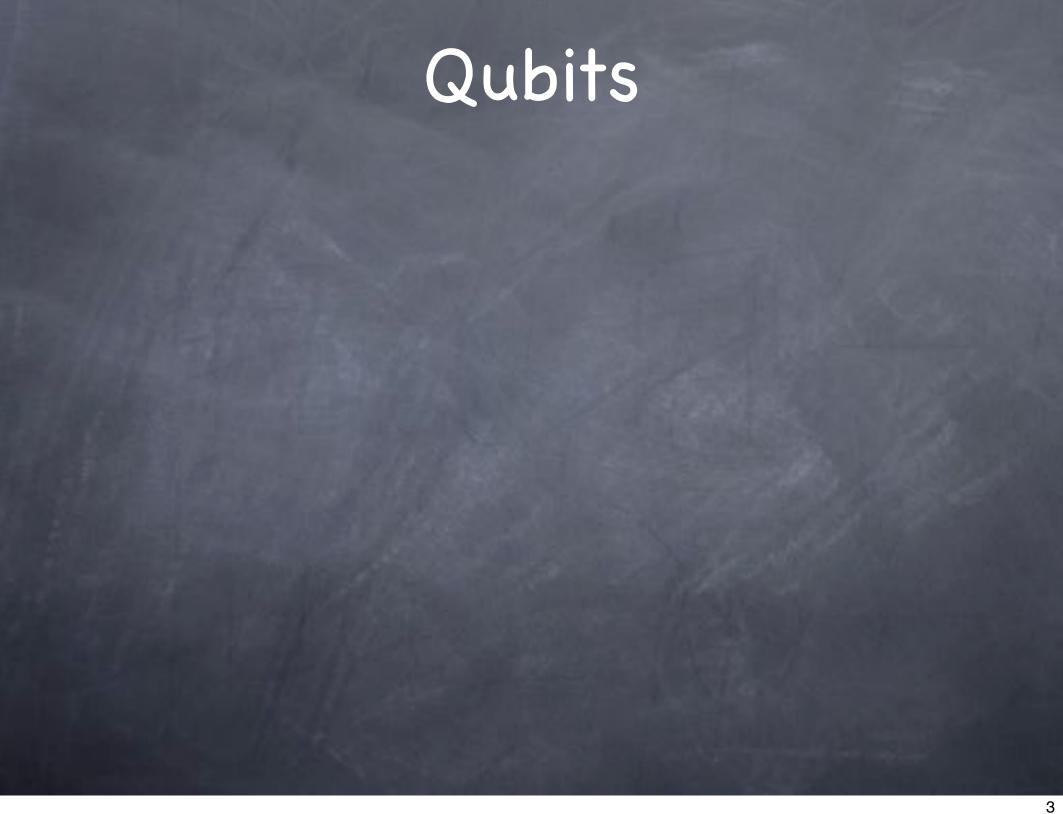
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 - qs is the "amplitude" of basis state s



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 - e.g. $\sqrt{\frac{1}{2}} = \frac{1}{1} = \frac{1}{1$
- (Also, state can be "mixed": a probability distribution over amplitude vectors. Doesn't change power of quantum computing)

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- Can choose "non-standard" bases for measurement. But again, can do without it

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0 1 0 0 (on 2 qubits)

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 - Contrast with classical case: probabilities can only add

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 - Toffoli gate has a classical analog (on 3 bits) that can be described as $T(a,b,c) = (a,b,c \oplus a \land b)$

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 - "Copy" the output to unused qubits, and run the reverse computation to return the rest to original state

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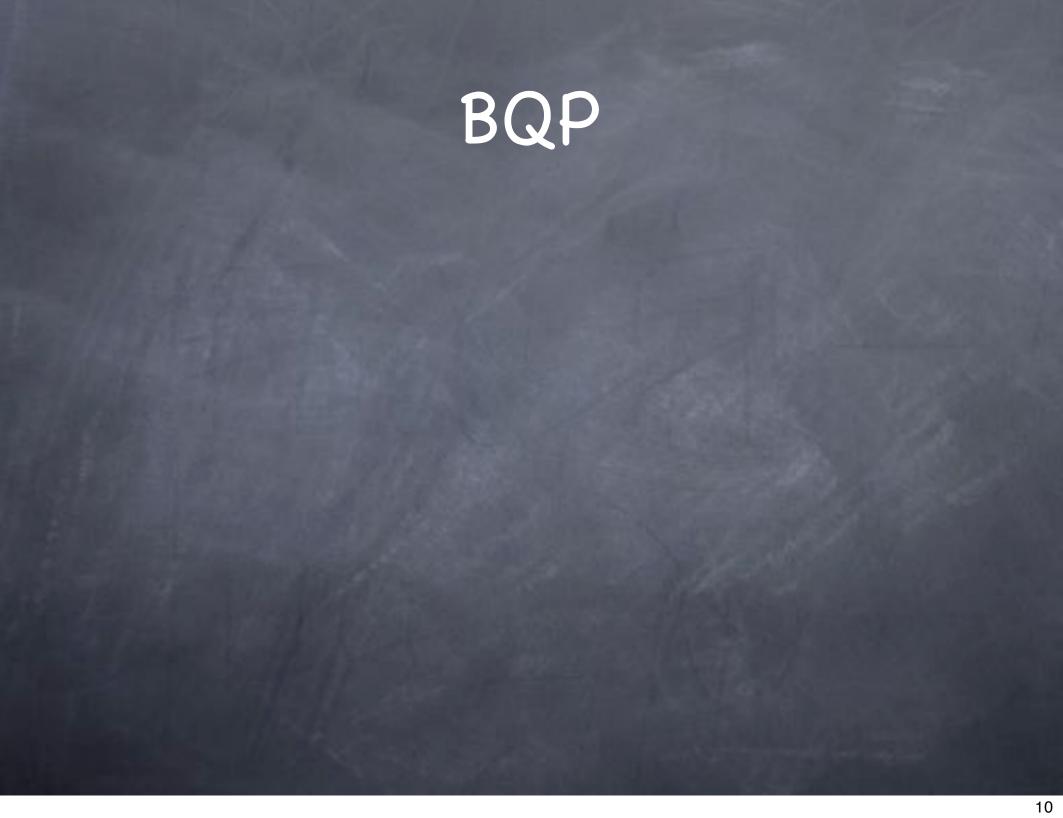
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 - \otimes $x \in L \Rightarrow C_n(|x0^m\rangle) = 1 \text{ w.p. } > 2/3; x \notin L \Rightarrow C_n(|x0^m\rangle) = 1 \text{ w.p. } < 1/3$



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- How about BQP and NP?

Two Quantum Algorithms

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 - In $O(2^{n/2})$ iterations, amplitude of $|z\rangle$ becomes large (i.e., constant)

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 - Tool used: Quantum Fourier Transform

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 - If f is periodic, then $\dot{f}(x)$ (coefficient of X_x in f's FT) will be large for some x which is related to f's period
- @ QFT: initial state = Σ_x f(x) |x> and final state = Σ_x f(x) |x>
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- - Basis vectors: $X_x(y) = (-1)^{xy}$ (normalized)
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- @ QFT: initial state = Σ_x f(x) |x> and final state = Σ_x f(x) |x>
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- Measuring the final state gives x with large coefficients with good probability. Enough to retrieve f's period.

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