Interactive Proofs

Lecture 19

And Beyond

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  - Similar to BPP ⊆ Σ_2^P (yields MAM protocol; MAM=AM)

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- Similarly, $MA \subseteq \Sigma_2^P$

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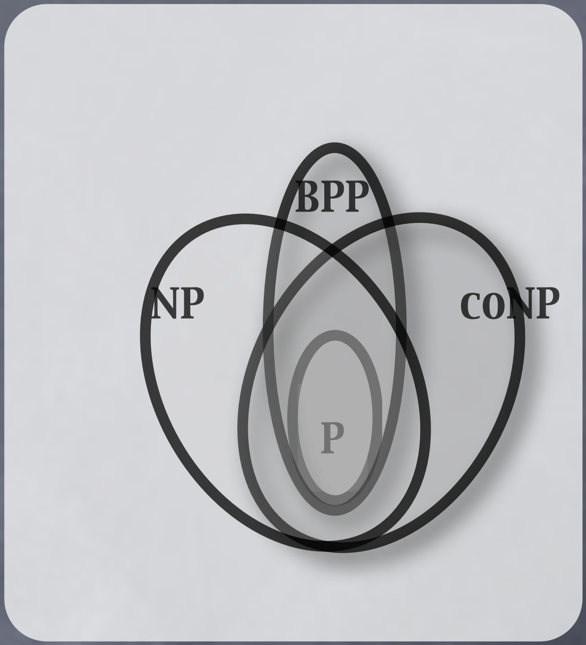

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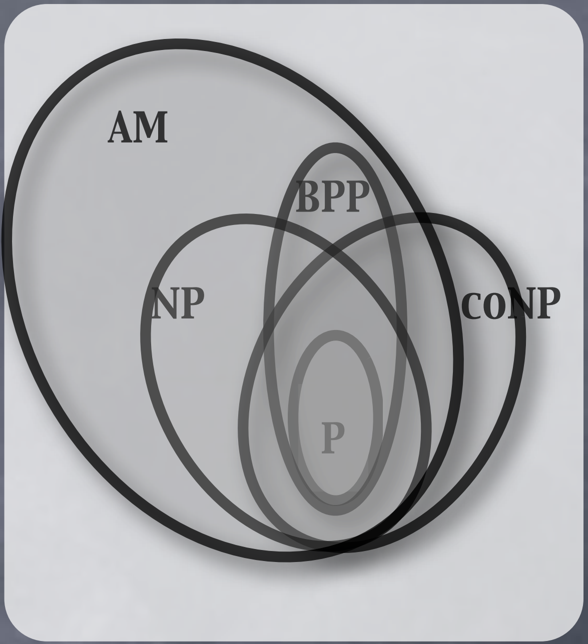

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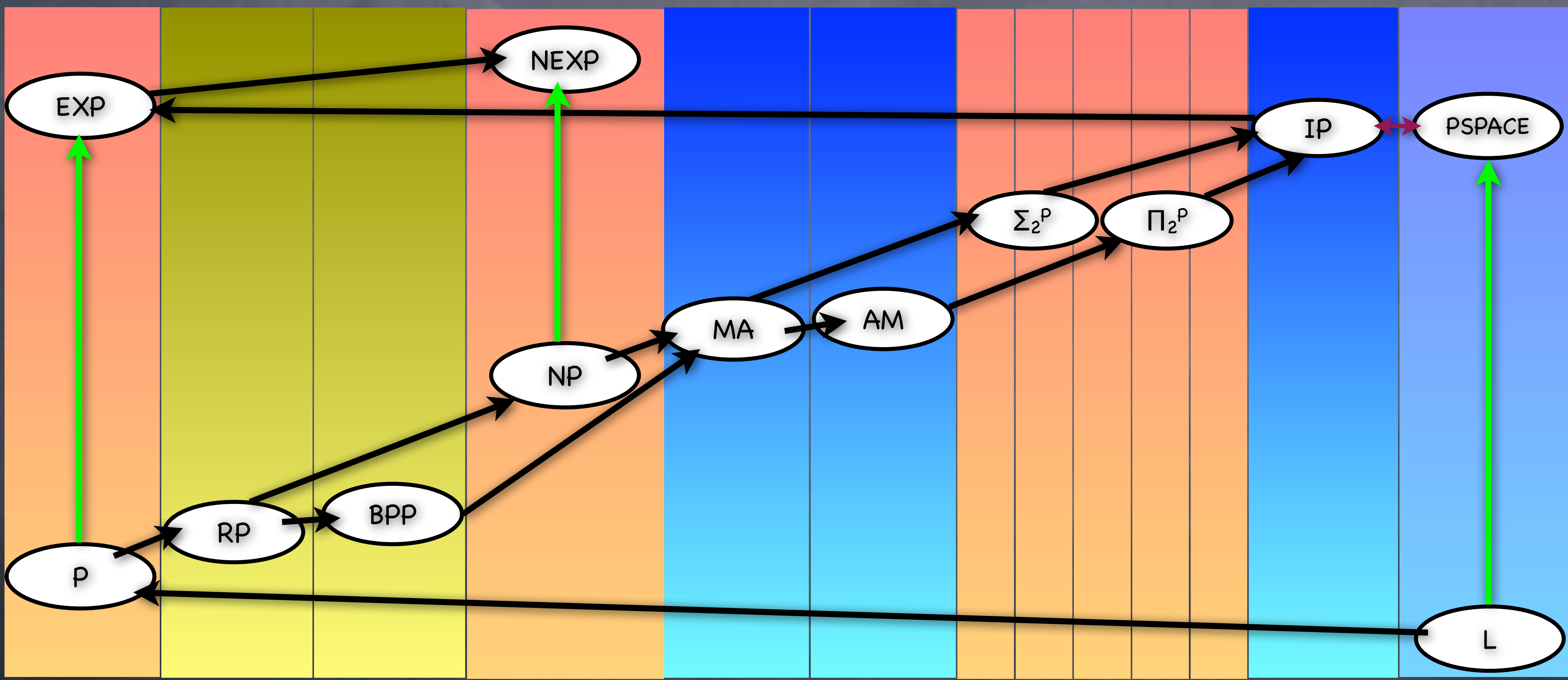
Zoo



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- User: In fact I just care if it works correctly on the inputs I want to solve. Maybe for each input I have, your machine could prove correctness using an IP protocol?
- Vendor: But I don't have a (nano-bio-quantum) implementation of the prover's program...

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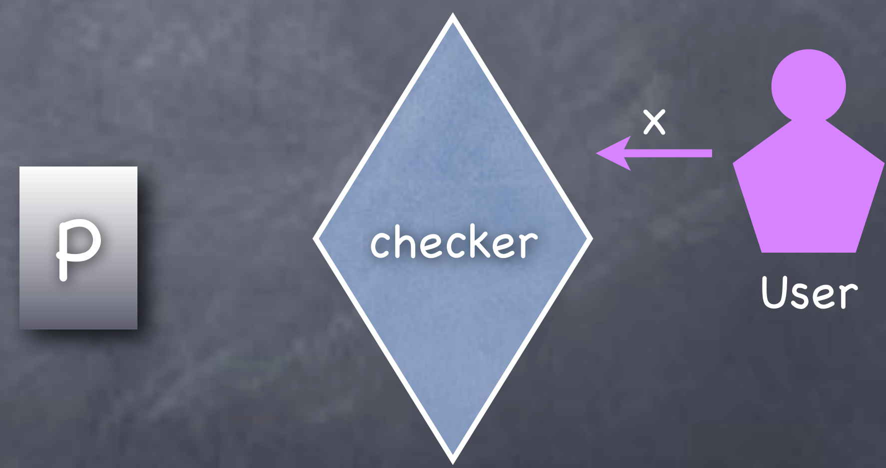

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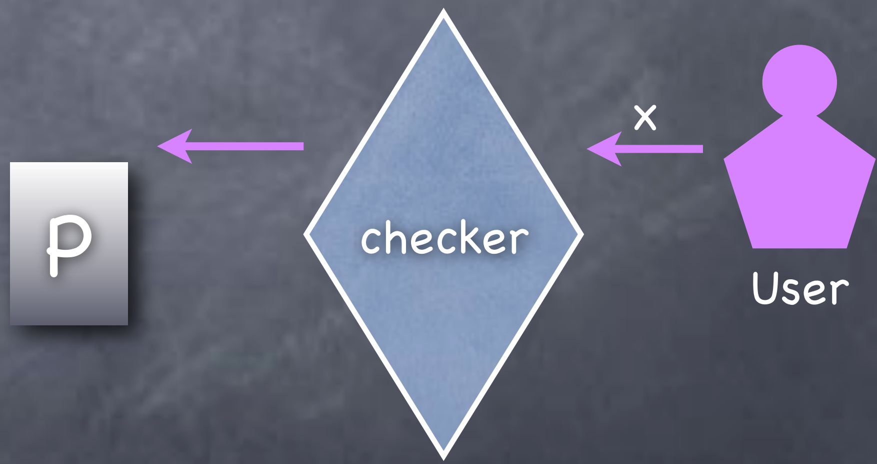

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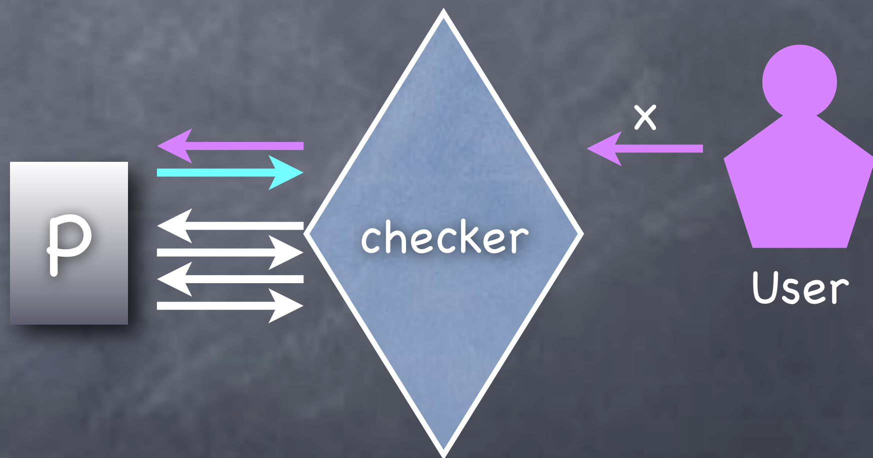

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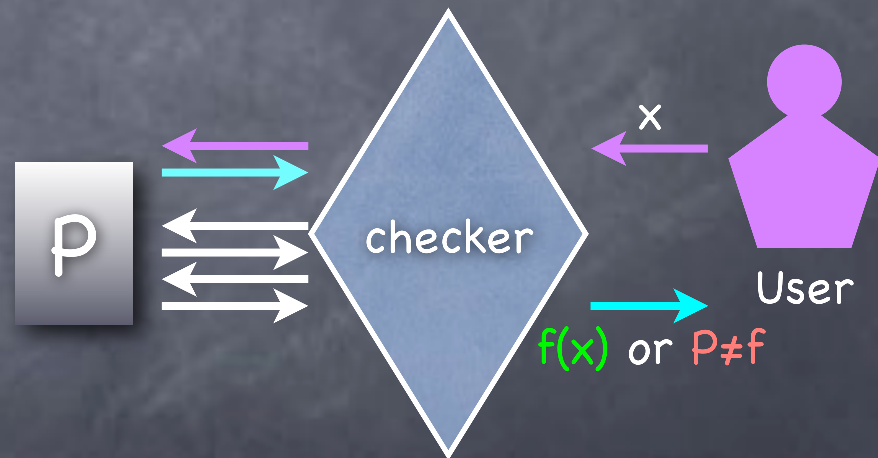

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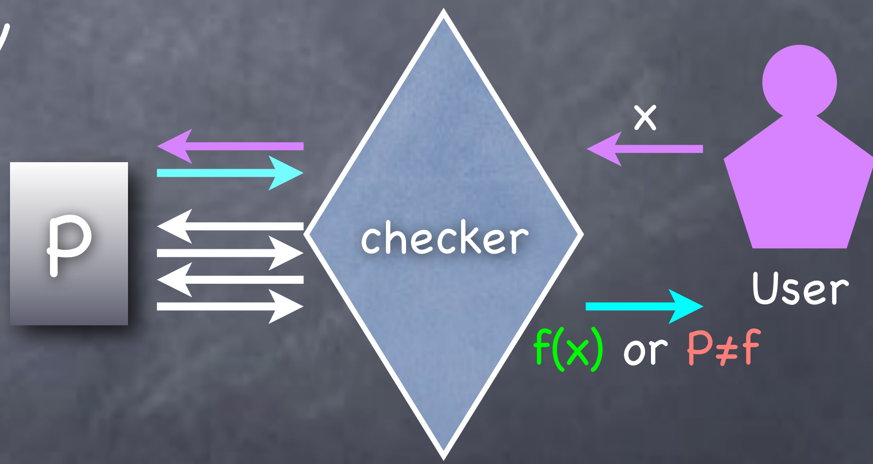
  - On each input, either ensures (w.h.p) that P's output is correct, or finds out that P≠f, efficiently



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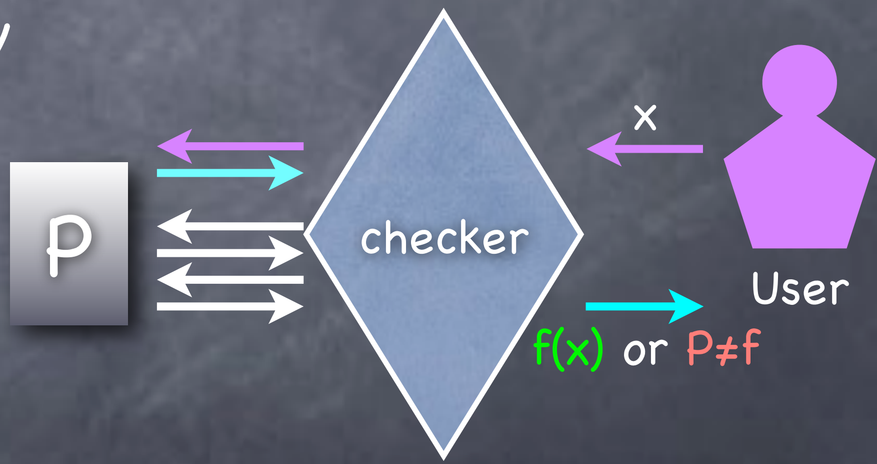

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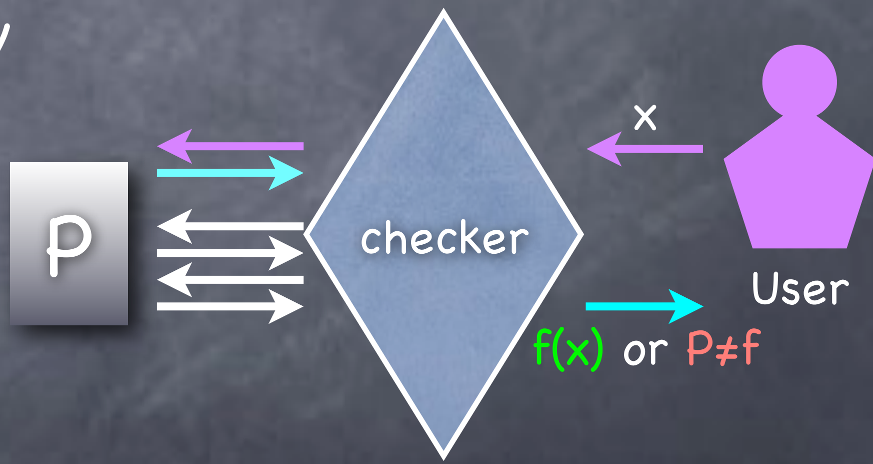

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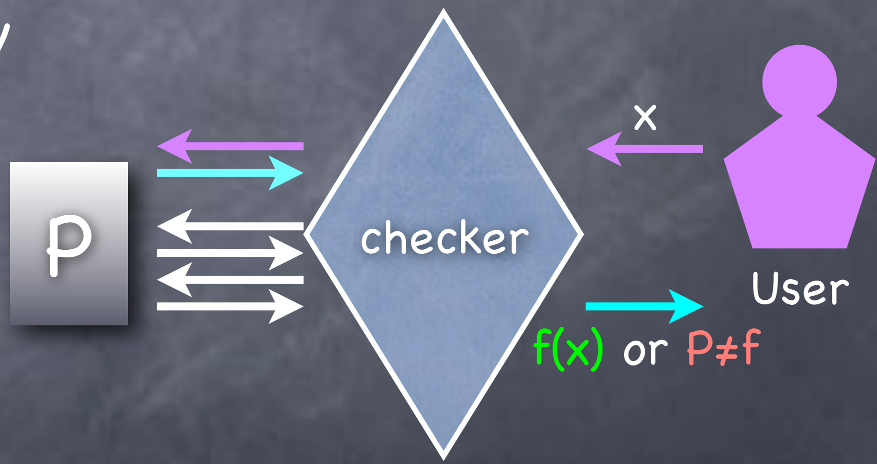
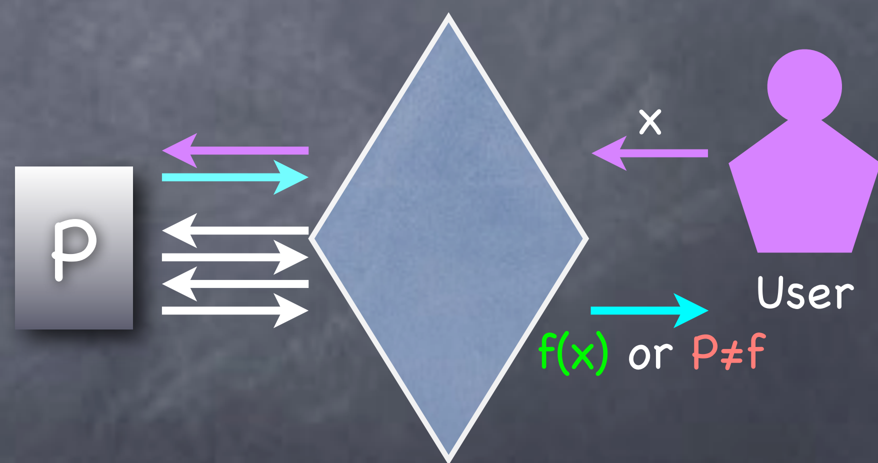
- Will consider boolean f (i.e., a language L)



Program Checking and IP



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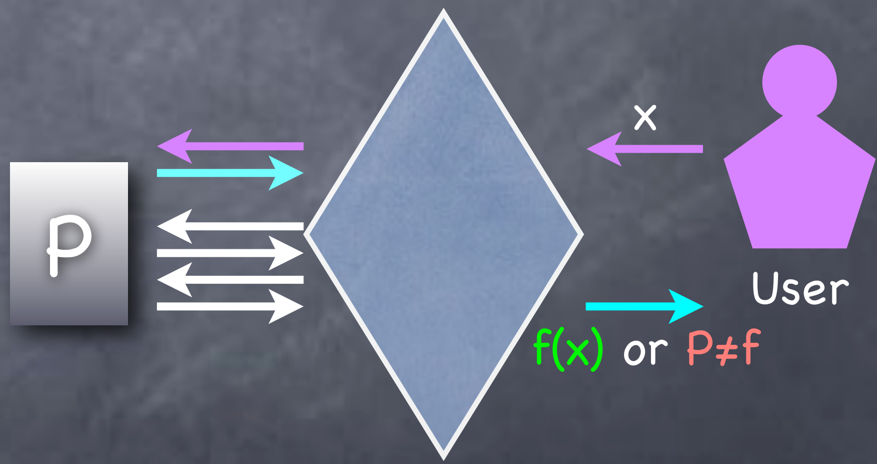

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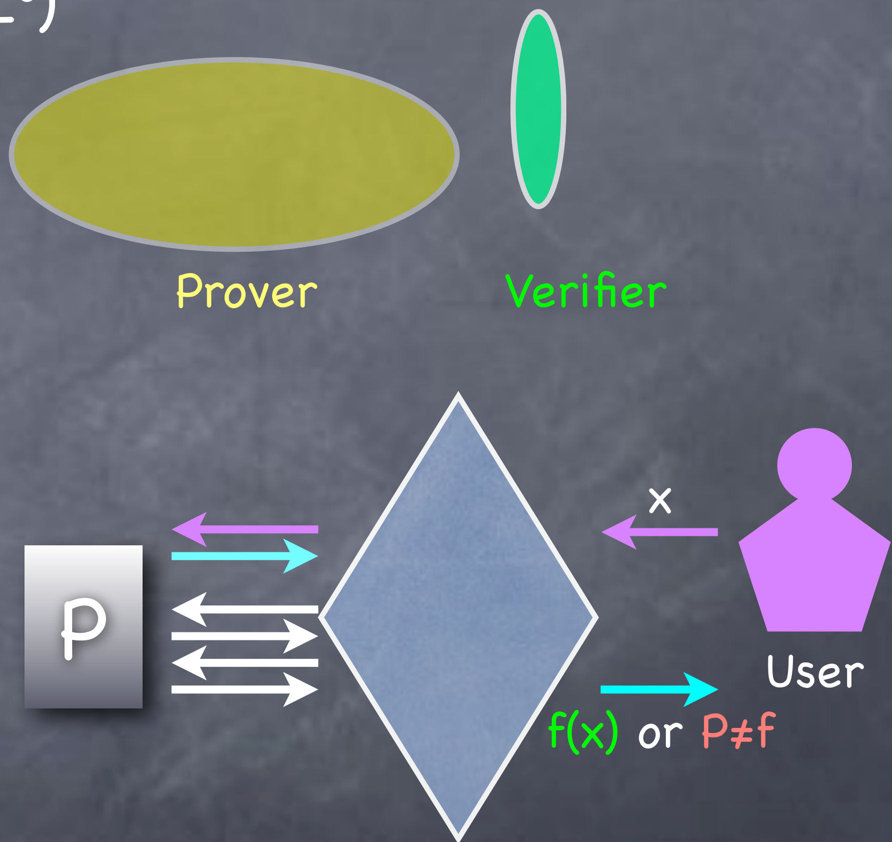

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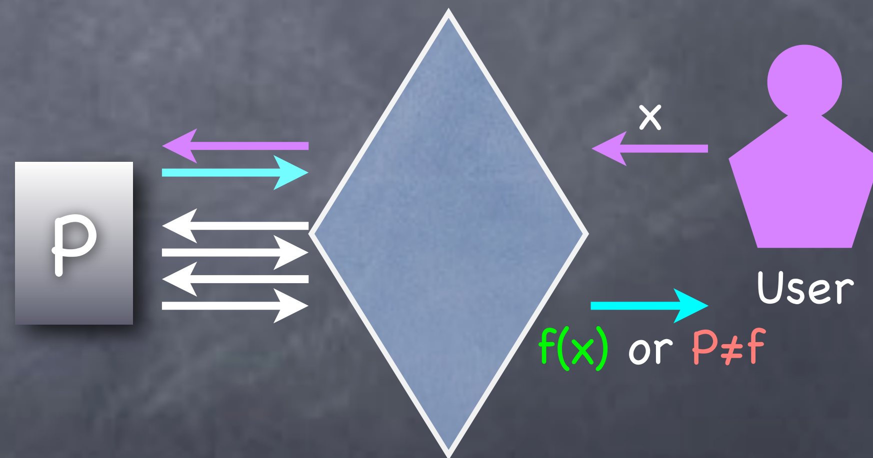

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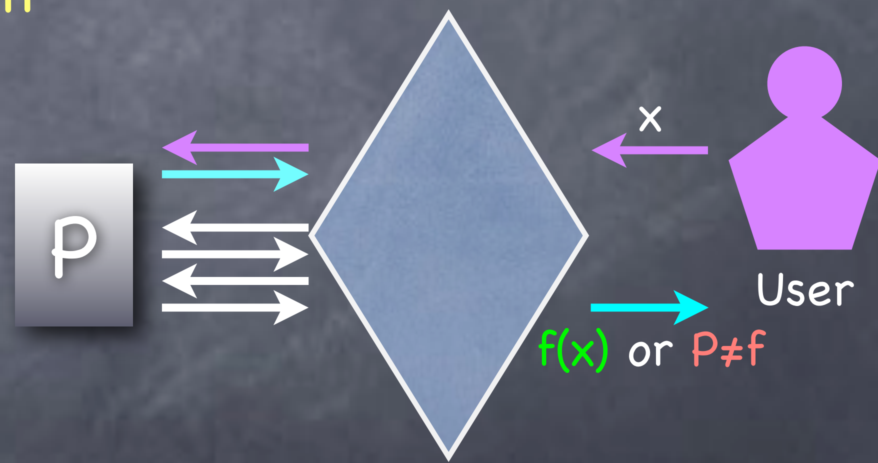

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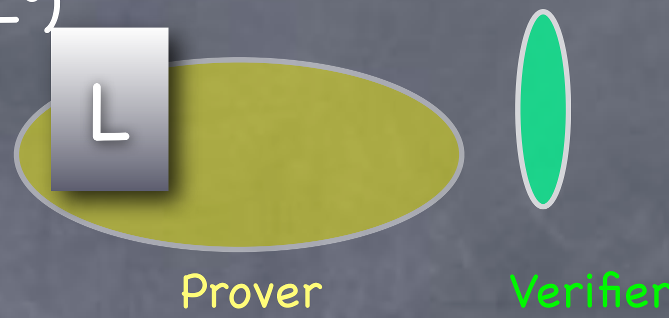

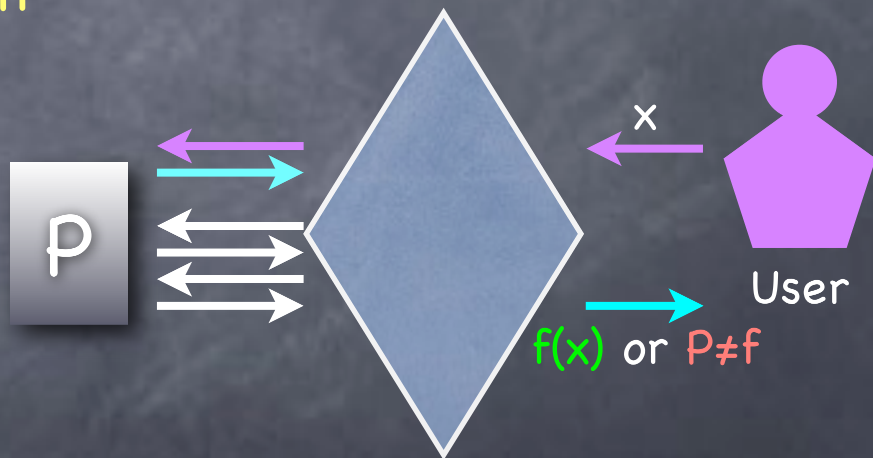

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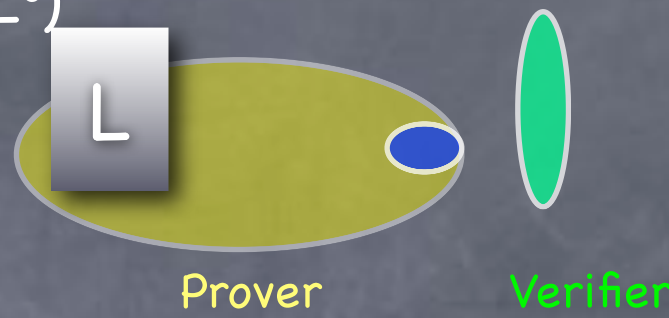

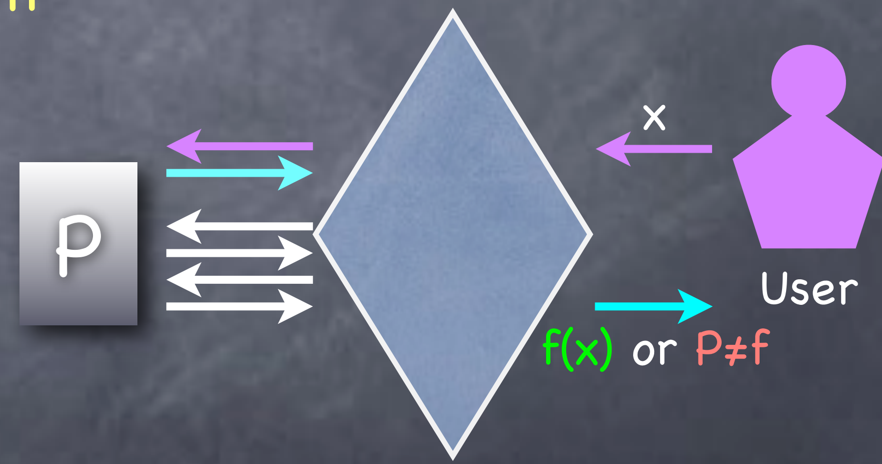

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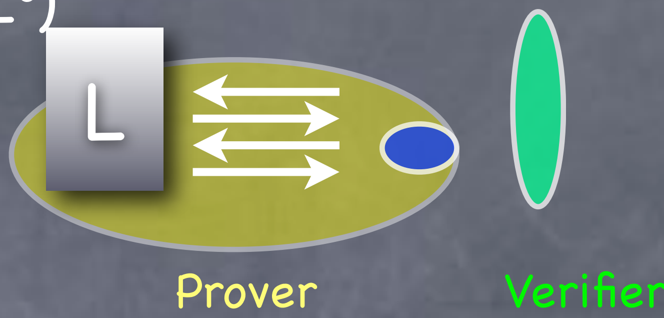

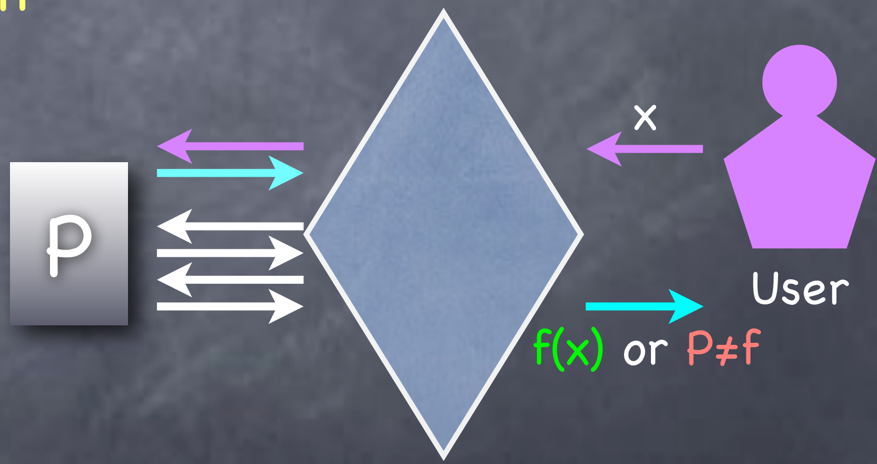

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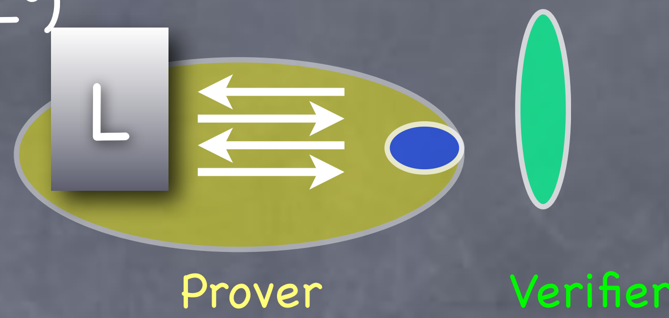

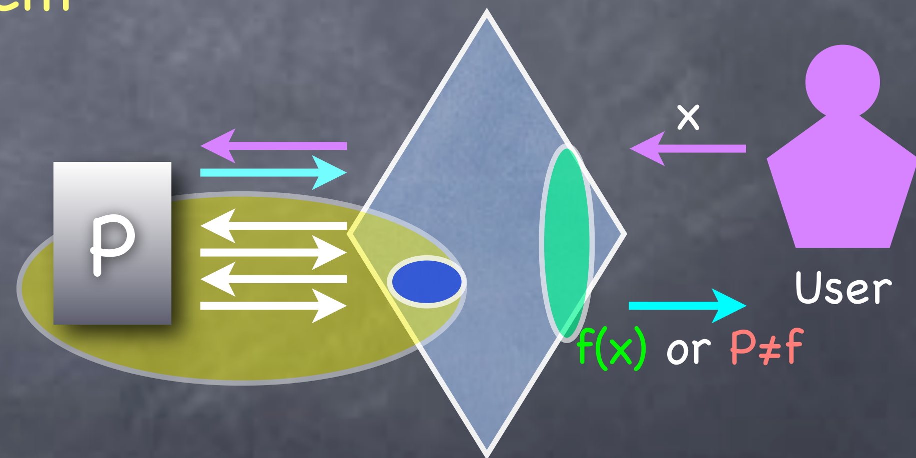

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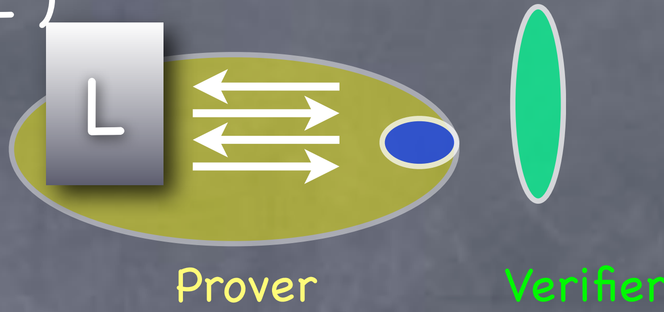

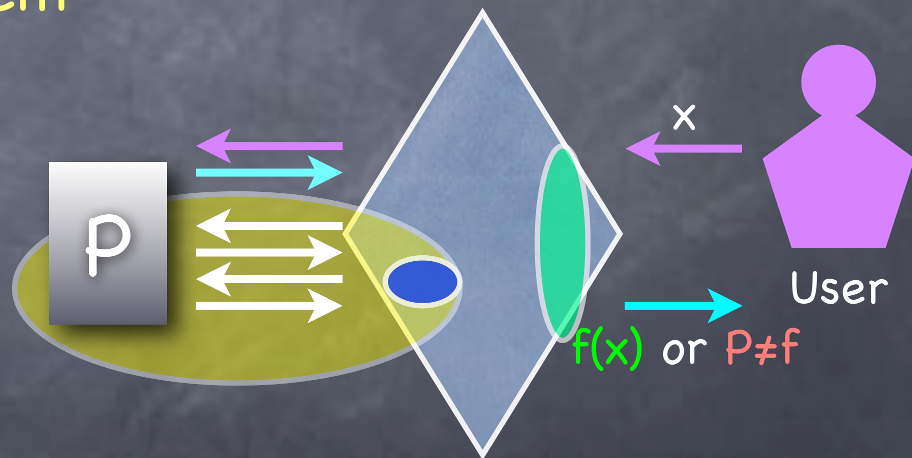

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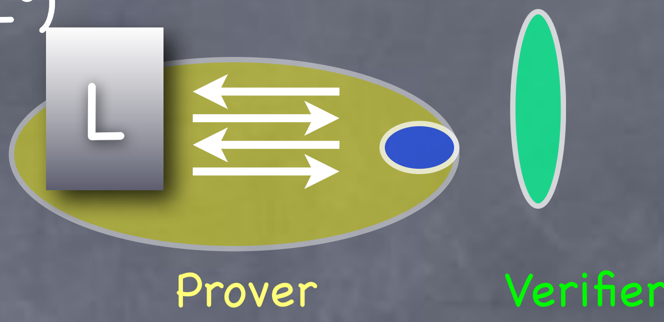

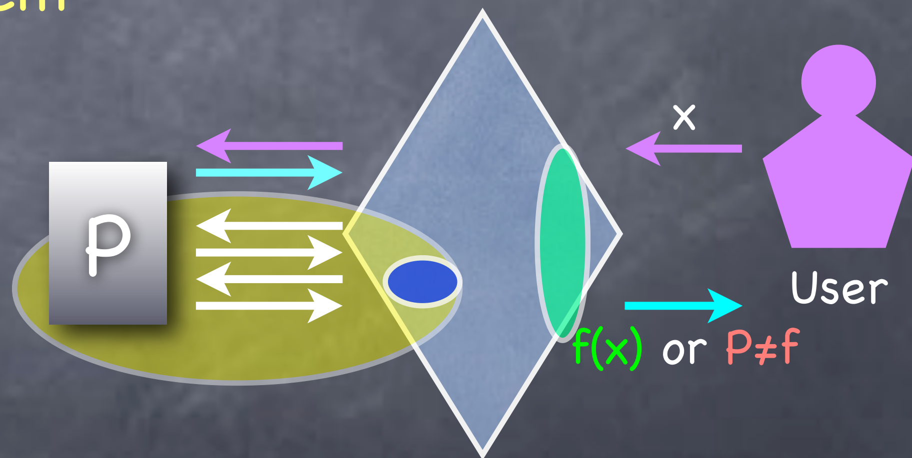

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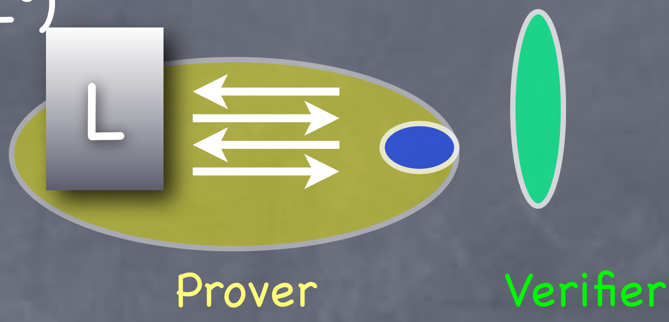
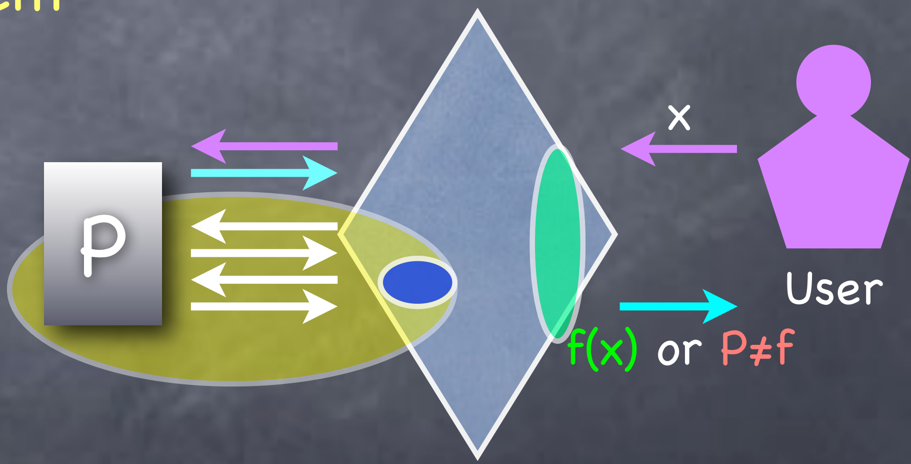
- How about Graph Isomorphism?





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- Note: An IP protocol (i.e., an NP proof) for GI, where prover is in P^{GI}

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Program Checking for GI

- If $P(G_0,G_1)$ says $G_0 \not\equiv G_1$, test P similar to in IP protocol for GNI (coke from can/bottle)
  - Let $H = \pi(G_b)$ where π is a random permutation and $b = 0$ or 1 at random
  - Run $P(G_0,H)$ many times
  - If P says $G_0 \equiv H$ exactly whenever $b=0$, output $G_0 \not\equiv G_1$
  - Else output "Bad P"
- Note: Prover in the IP protocol for GNI is in P^{GI}

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  - Parallel repetition theorem highly non-trivial!

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    - Also useful in certain cryptographic protocols

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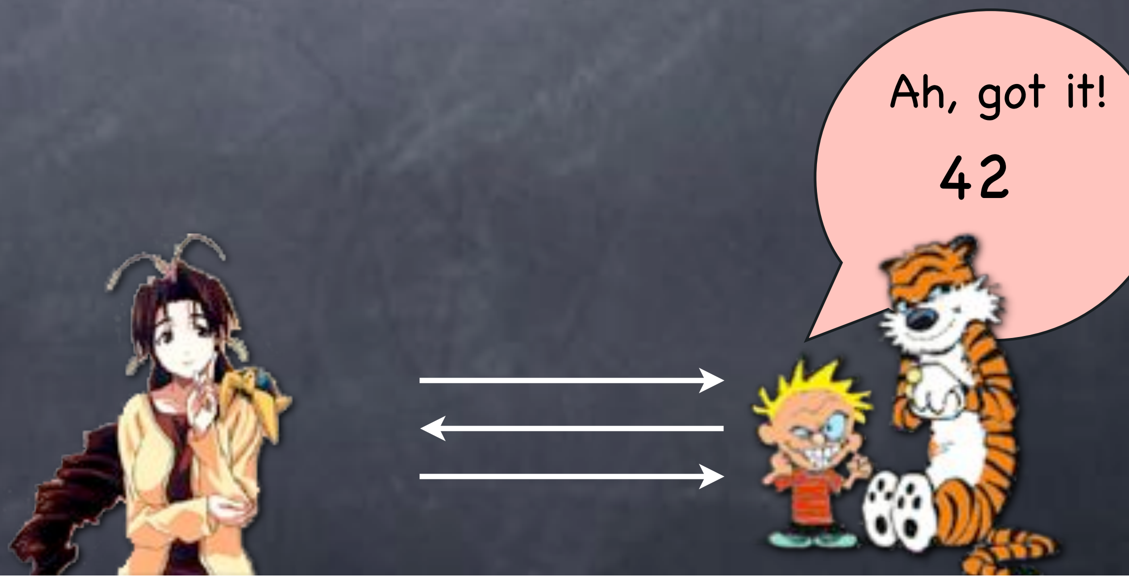

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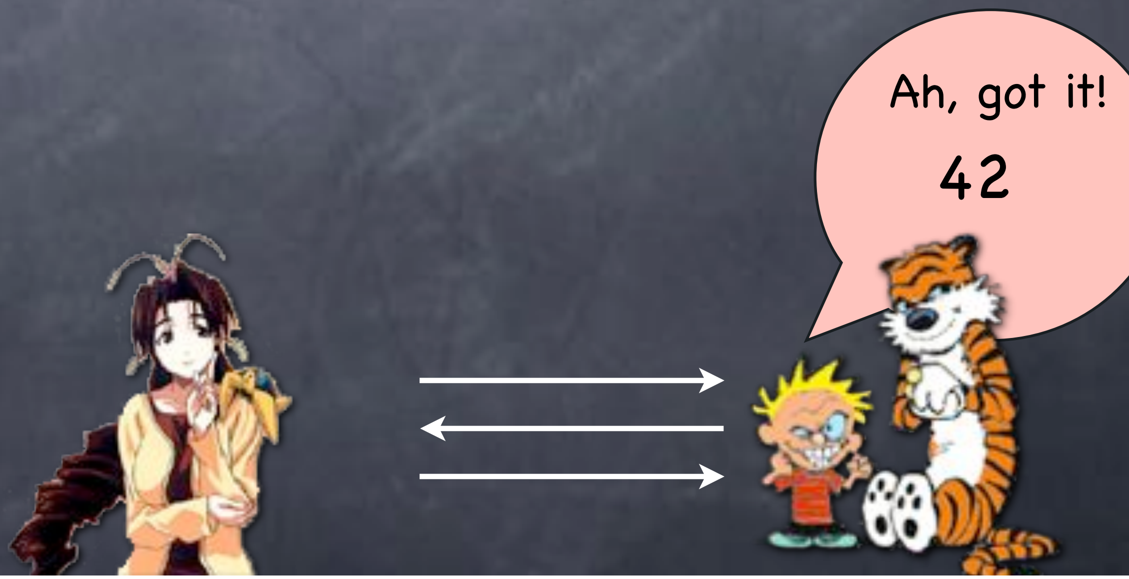

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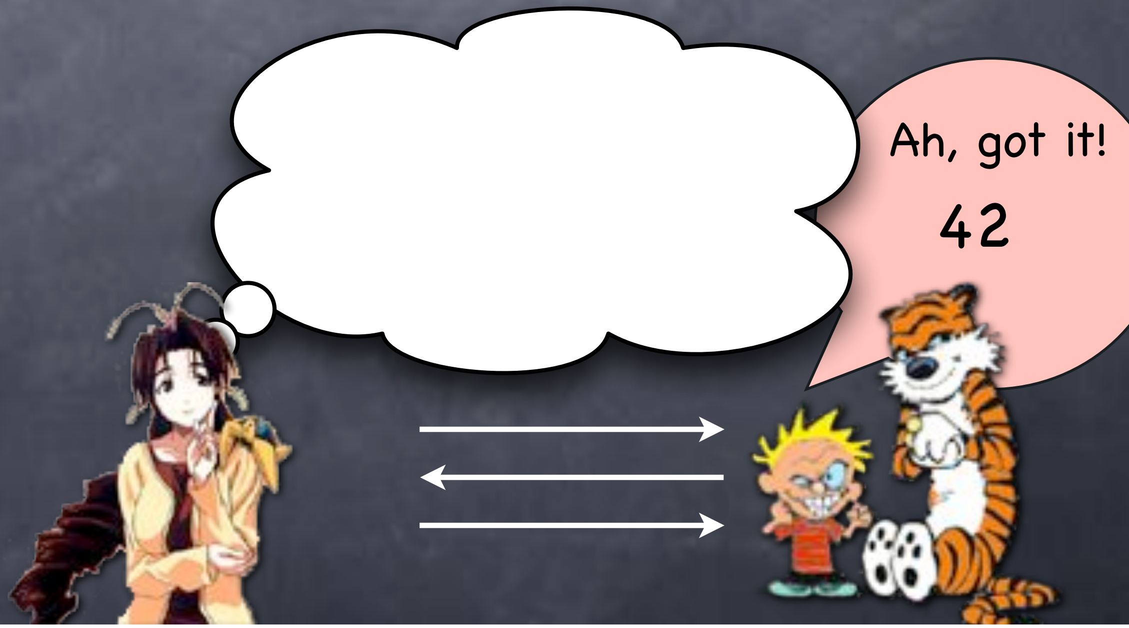

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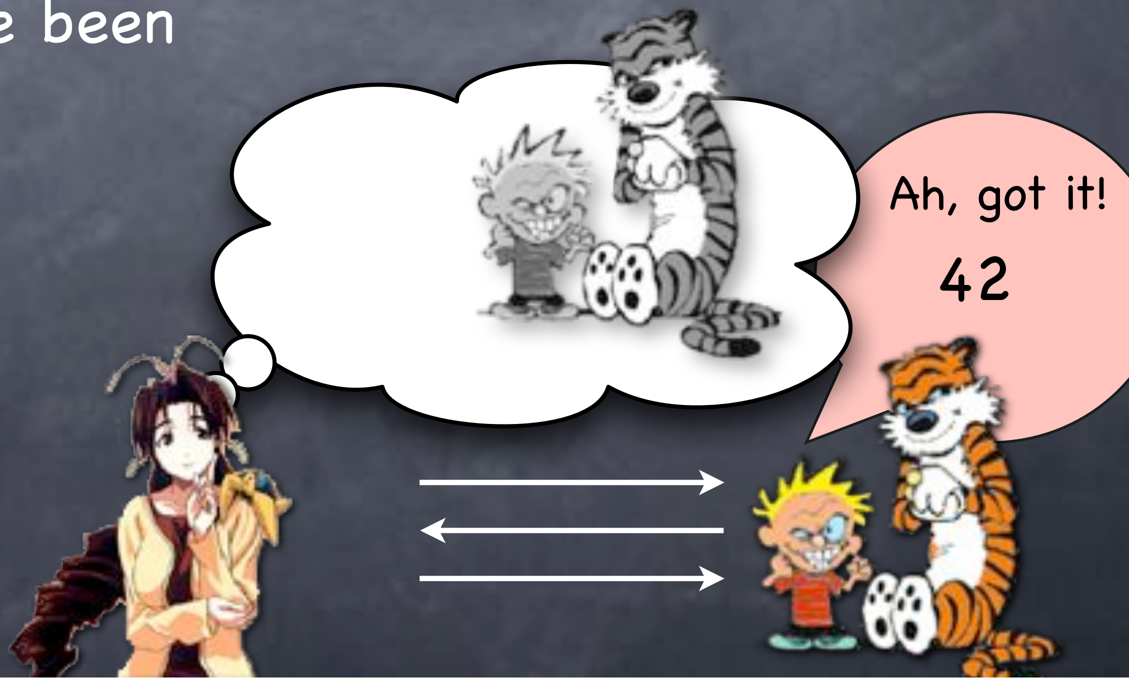

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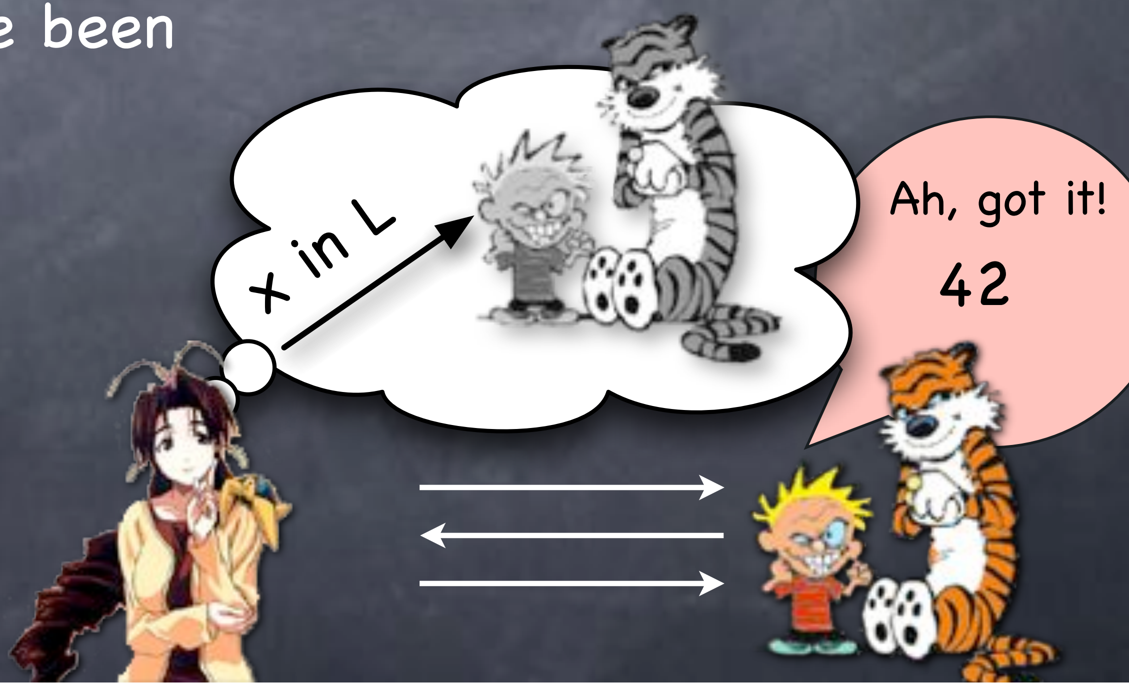

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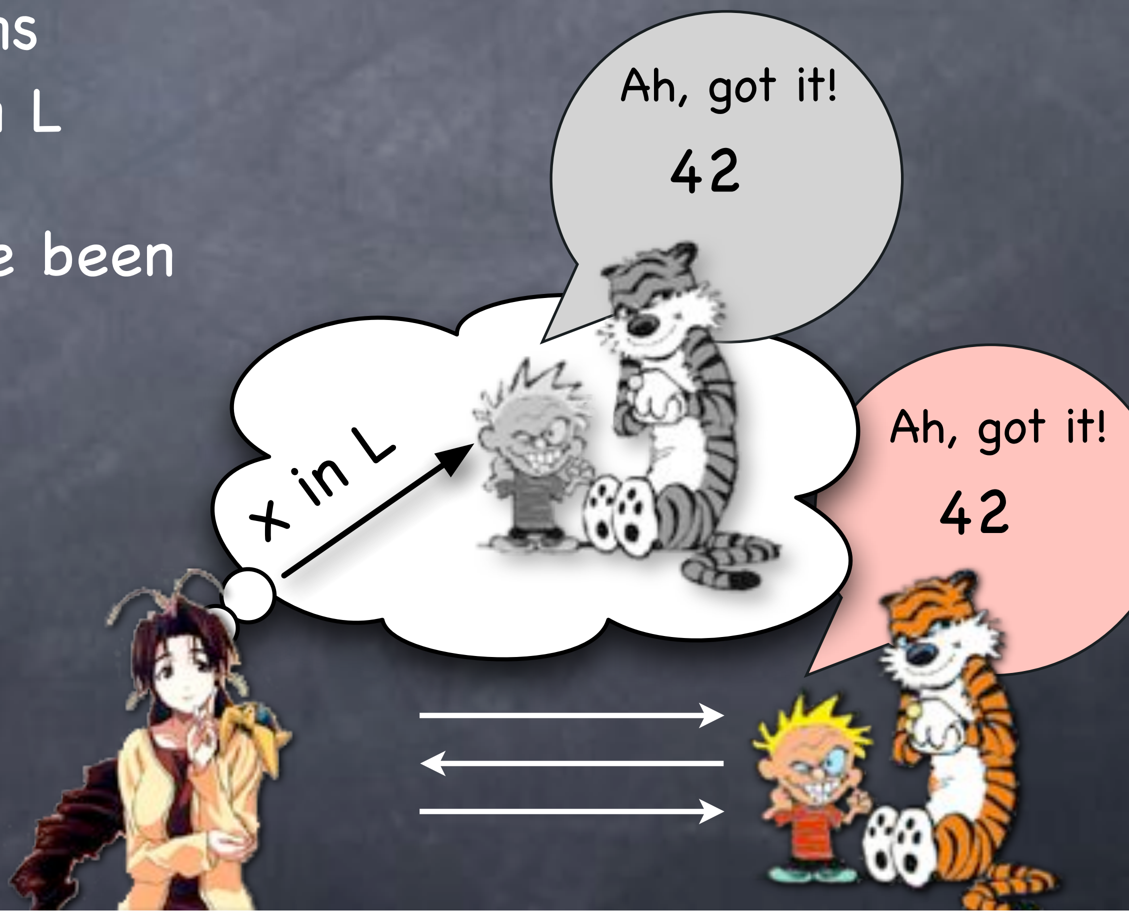

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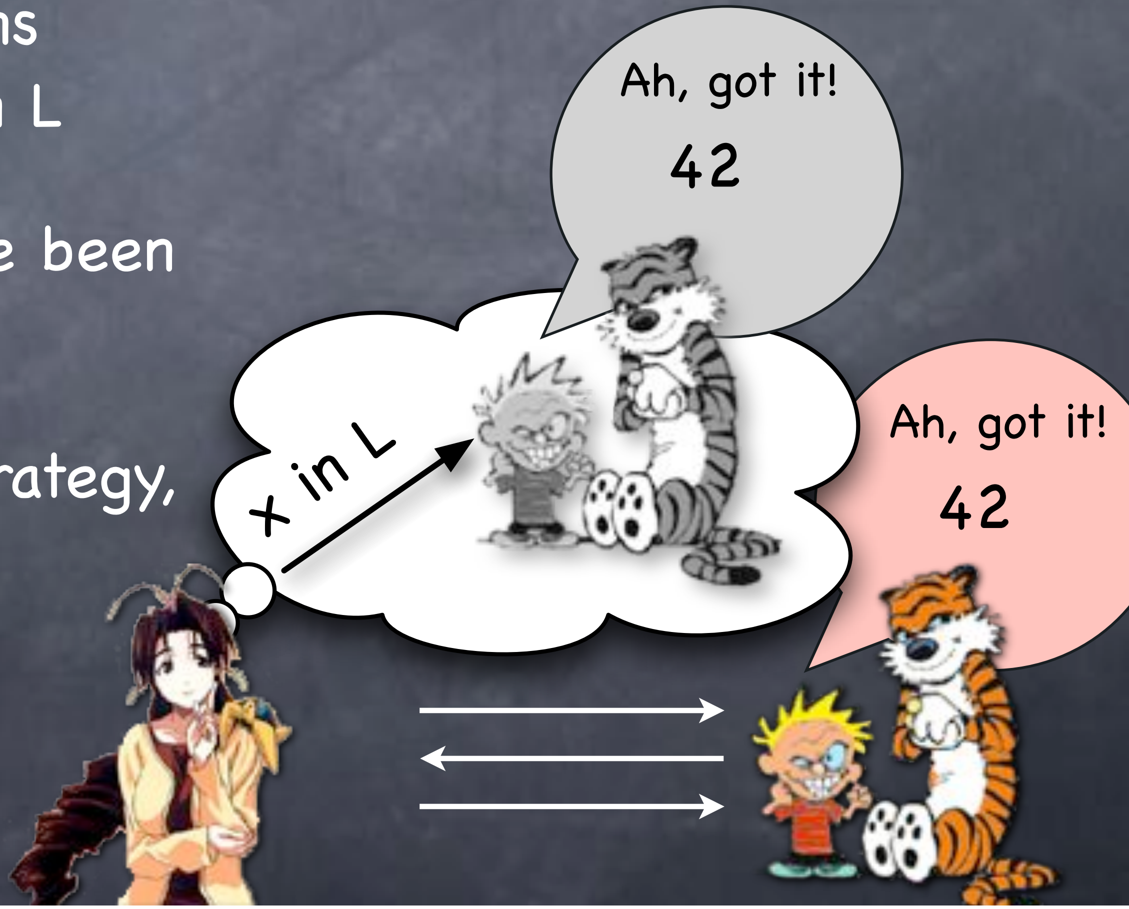
  - For every adversarial strategy, there exists a simulation strategy



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  - MIP, PCP, ZK proofs
- Understanding power of interaction/non-determinism and randomness
  - Useful in "hardness of approximation", in cryptography, ...