Interactive Proofs

Lecture 16

What the all-powerful can convince mere mortals of

Recap

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- Non-deterministic Computation

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- Non-deterministic Computation
- Polynomial Hierarchy

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  - Non-determinism on steroids!

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- Non-deterministic Computation
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- Today: Interactive Proofs
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Interactive Proofs

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- **Prover** wants to convince **verifier** that x has some property

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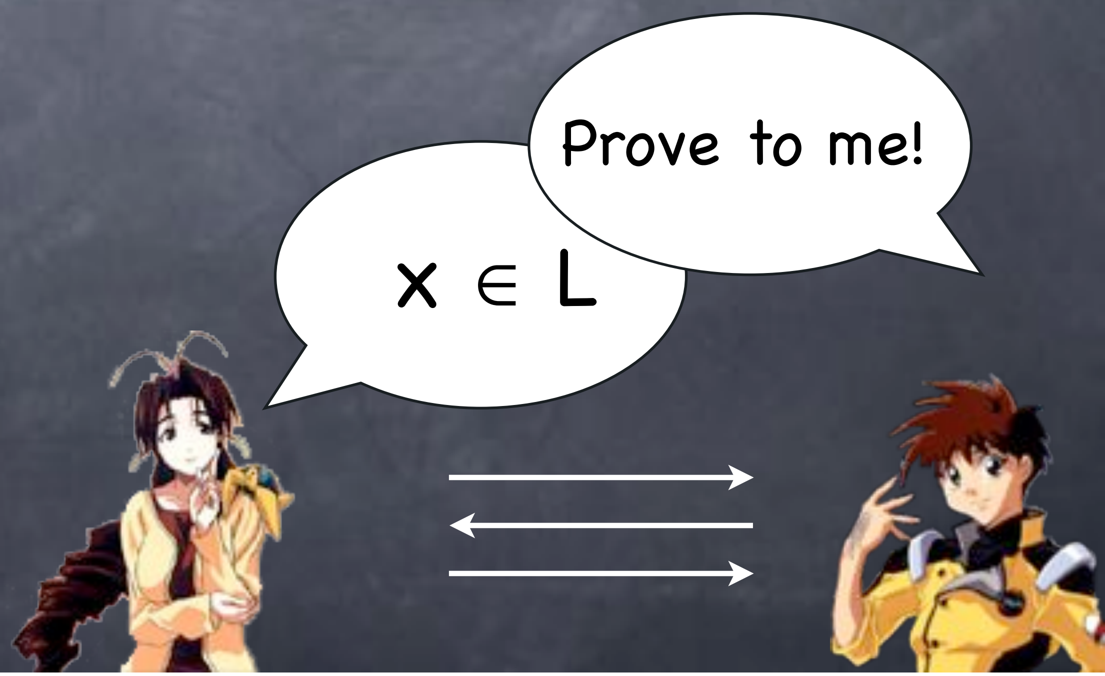

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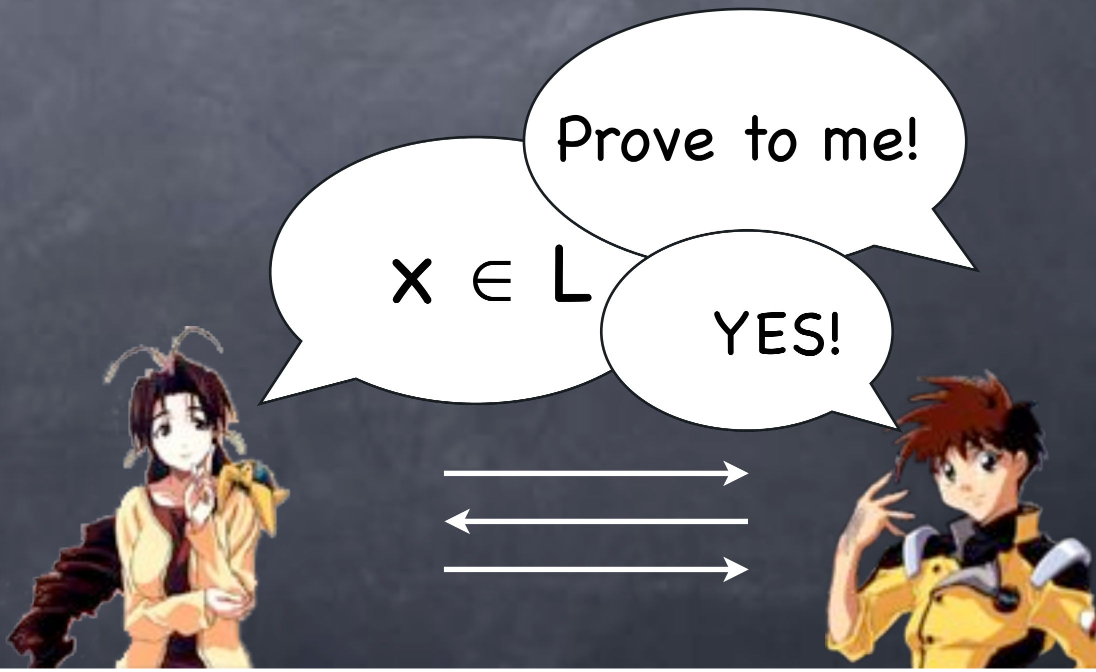

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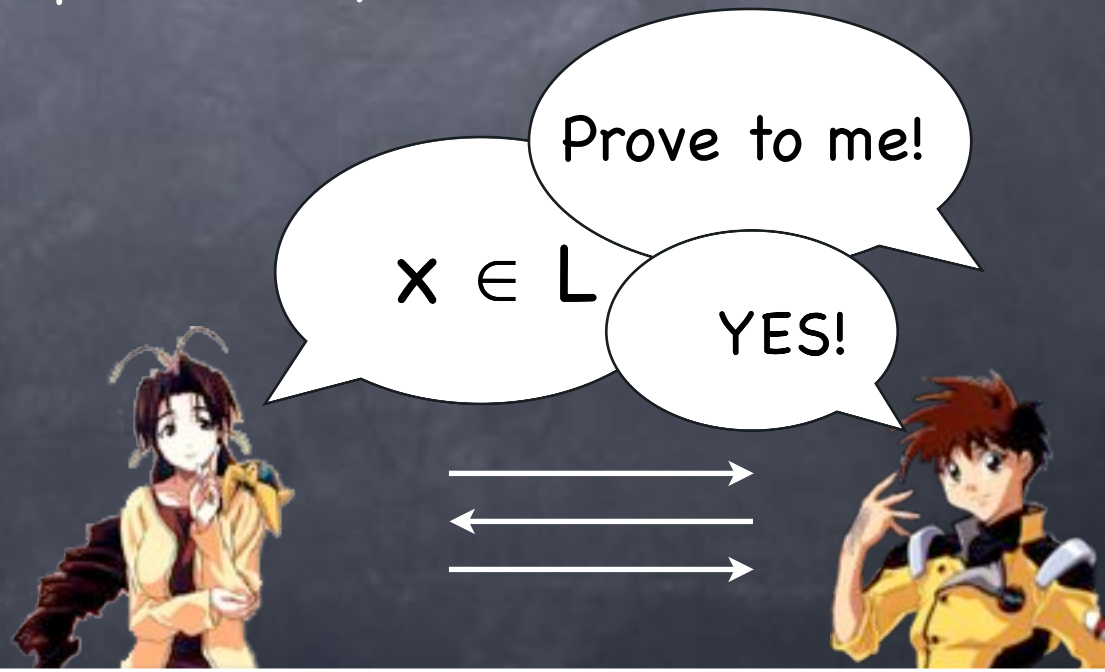

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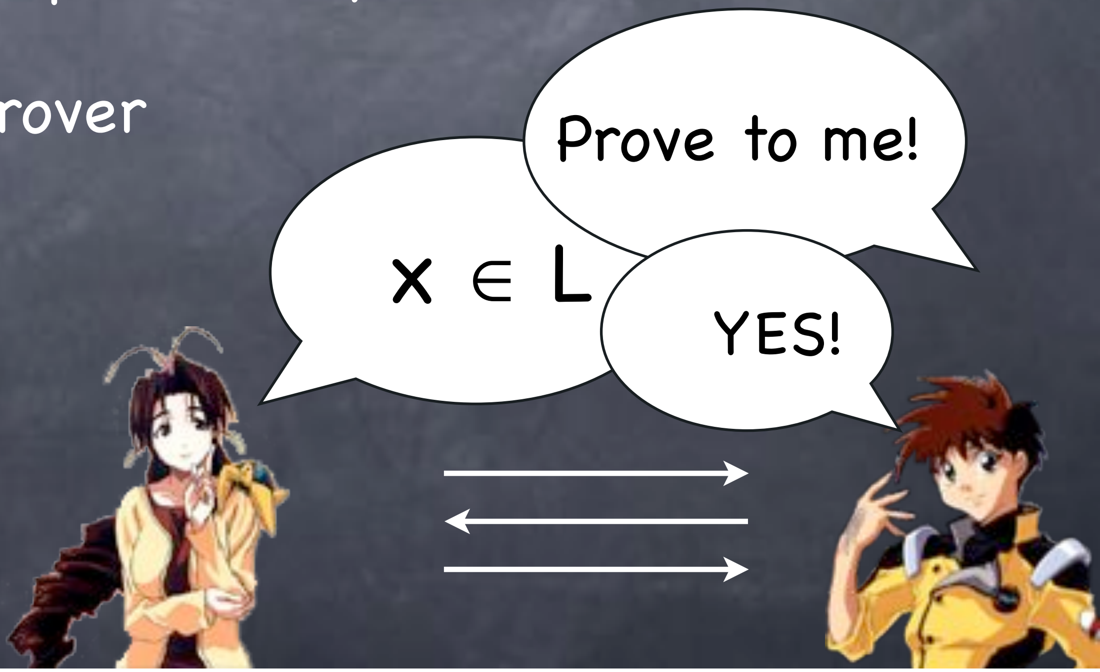

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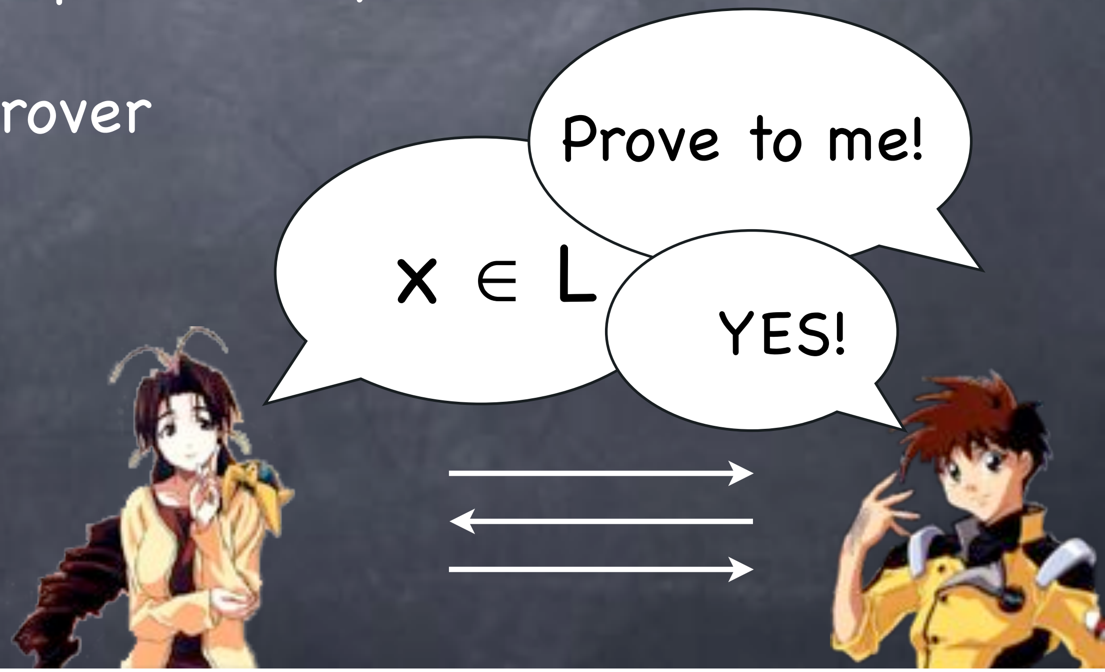

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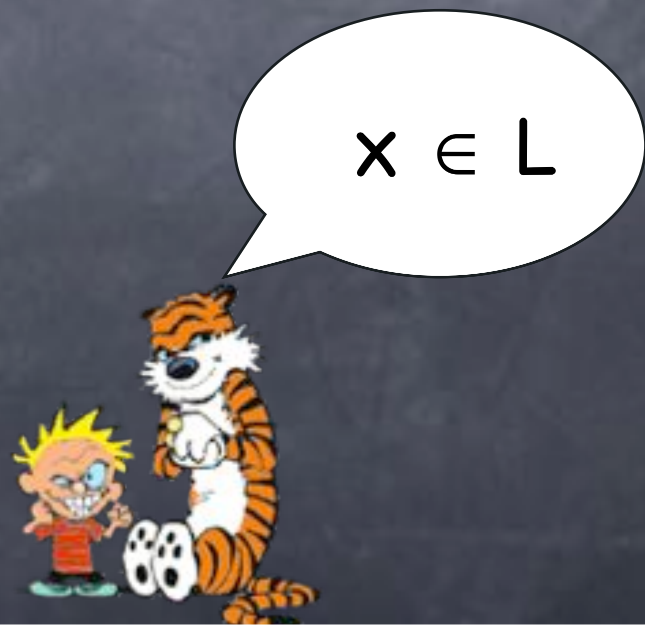

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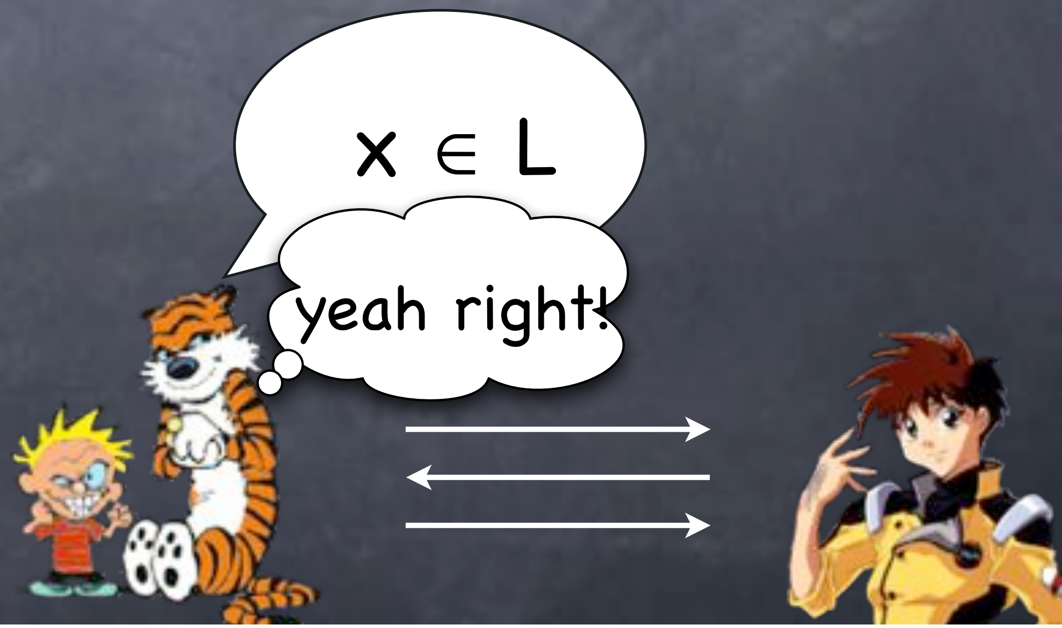

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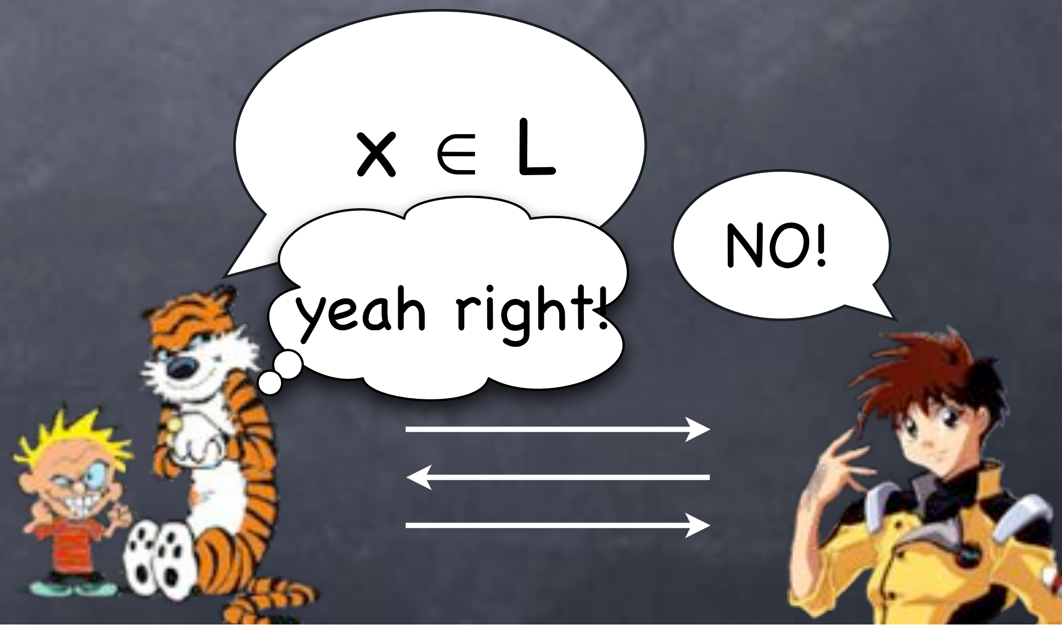

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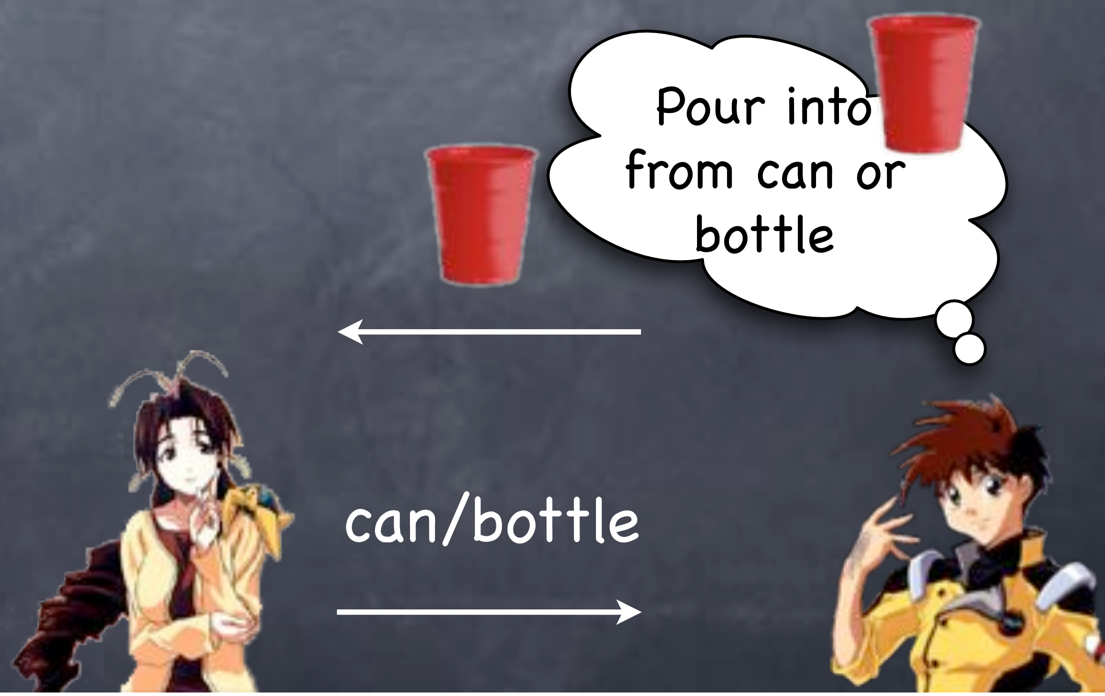

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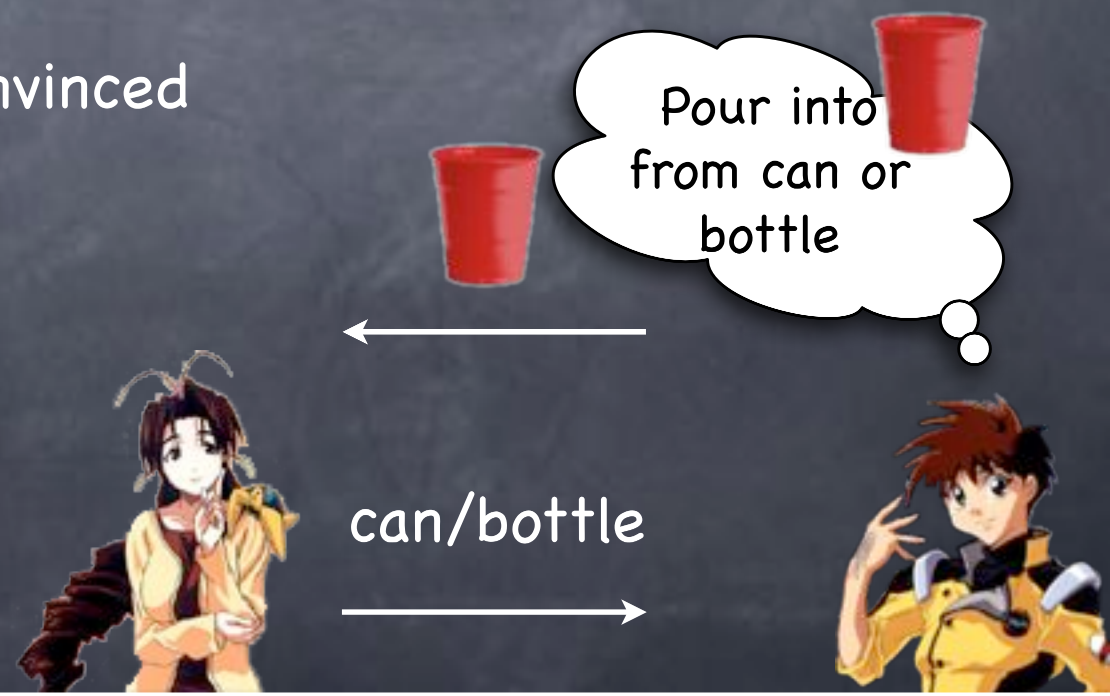

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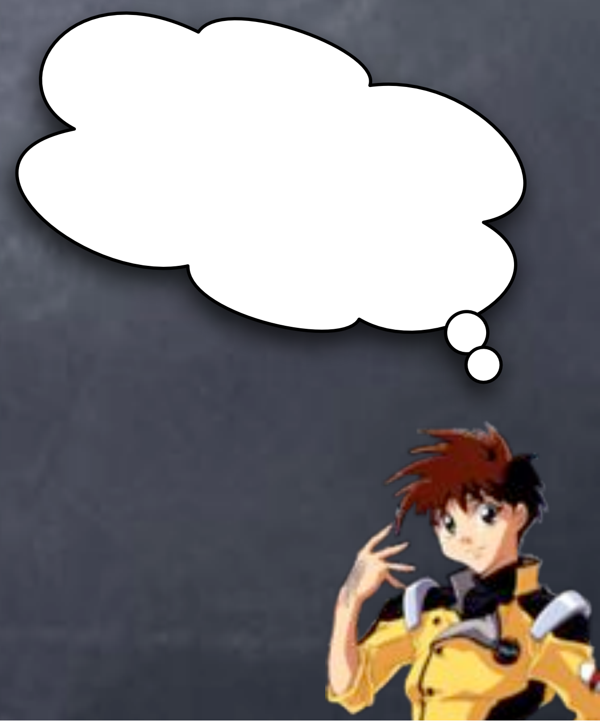

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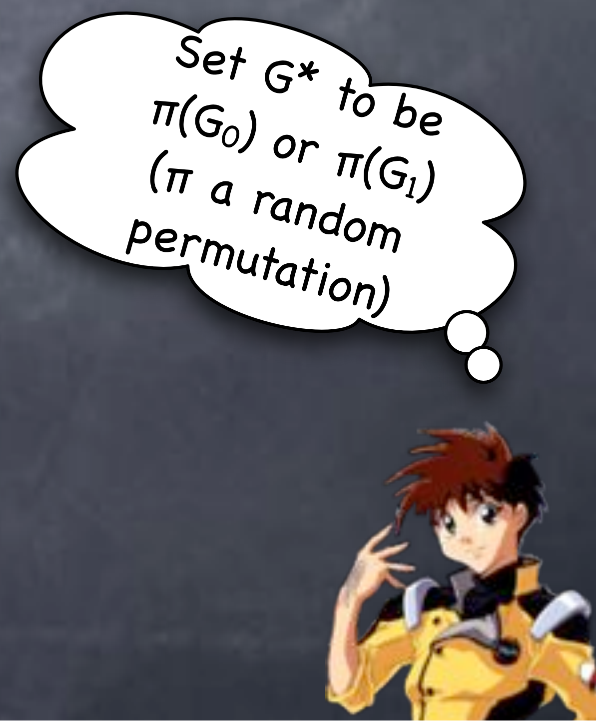

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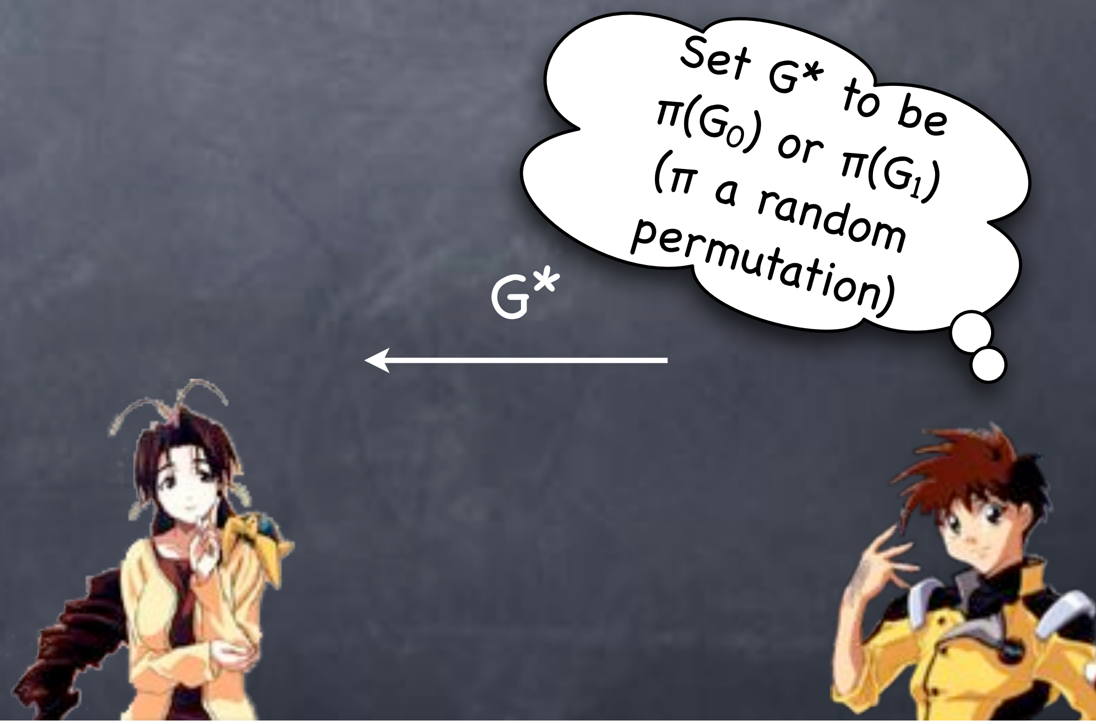

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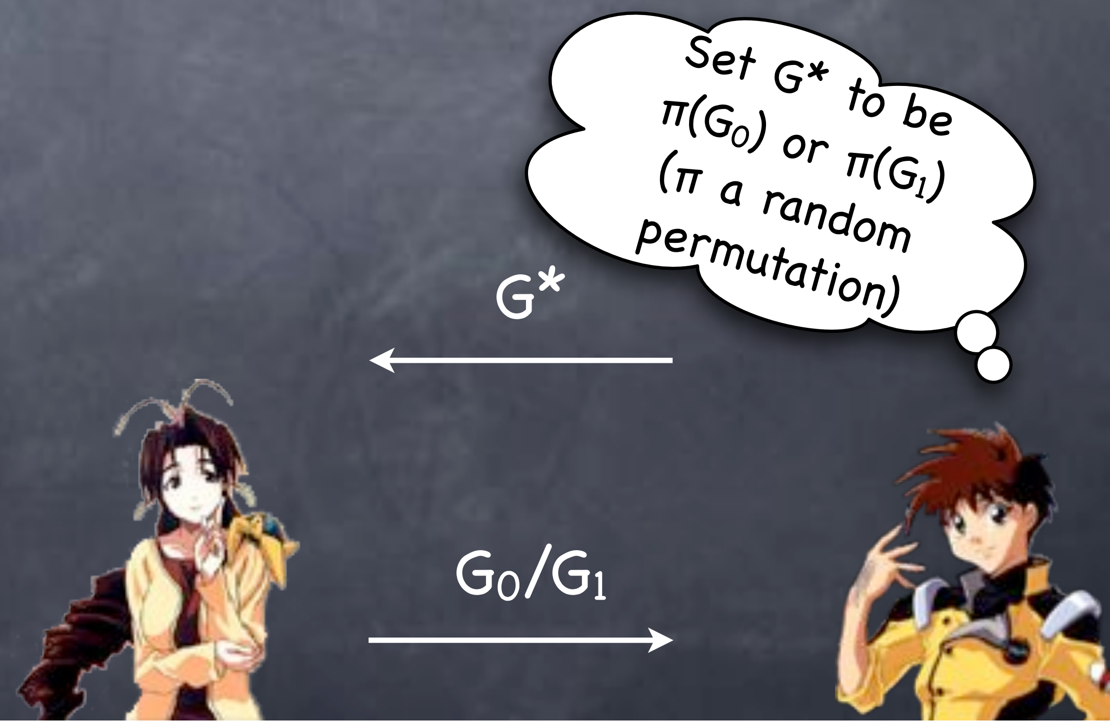

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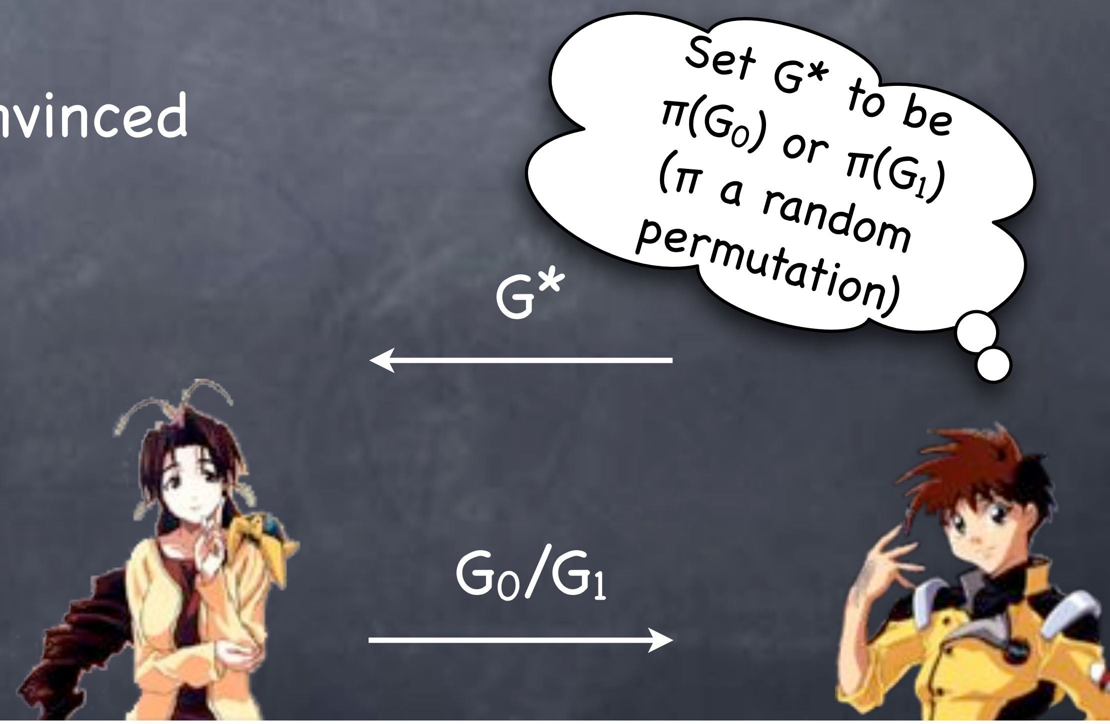

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  - Except with probability at most 1/3

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- Deterministic Prover IP = IP
  - For each input prover can choose the random tape which maximizes Pr[yes] (probability over honest verifier's randomness)

Public and Private Coins

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- Private coins: Verifier does not send everything about the coins
  - e.g. GNI protocol: verifier keeps coin tosses hidden; uses it to create challenge

Arthur Merlin Proofs

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- Arthur-Merlin proof-systems

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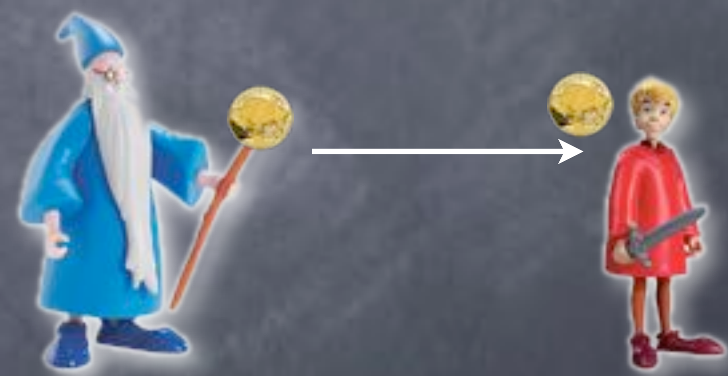

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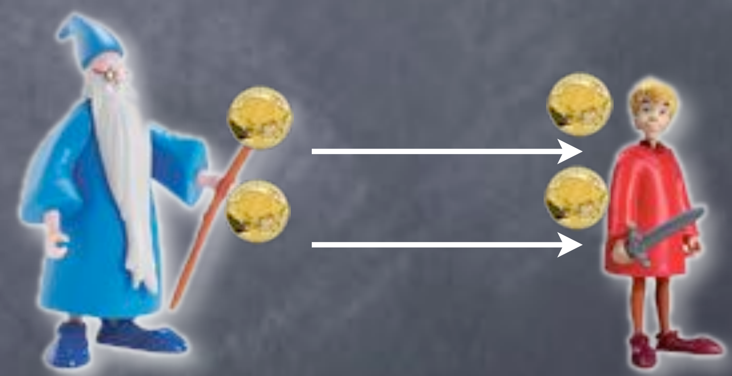

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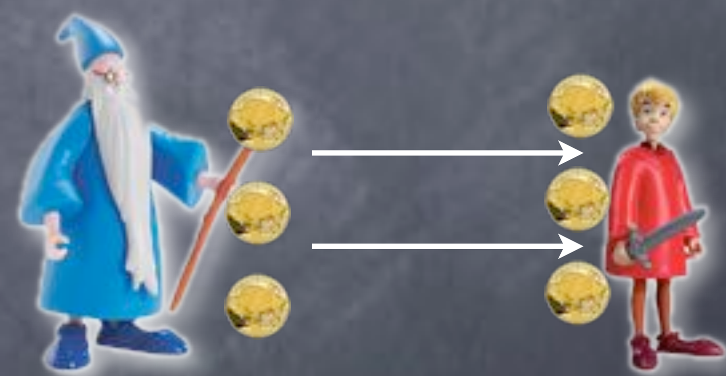

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  - If G_0 and G_1 isomorphic, same set of n! isomorphic graphs
  - Else 2(n!) isomorphic graphs
  - Prover to prove that $|\{H: H \equiv G_0 \text{ or } H \equiv G_1\}| > n!$

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- But what if $K/|U|$ is exponentially small?

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  - Clearly no single function for all S!

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- Hash collision probability = $1/|R|$

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- Pr[Yes] has a constant gap between |S| > 2K and |S| < K [Exercise]