Computational Complexity

Lecture 3
in which we come across
Diagonalization and Time-hierarchies

co-Class

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no counter-example

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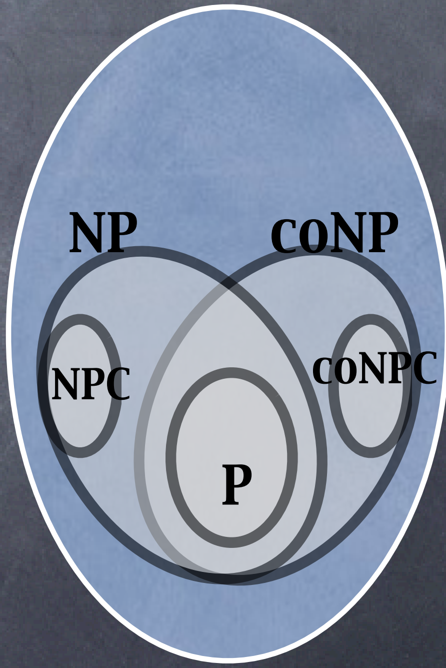
NP, P, co-NP and NPC



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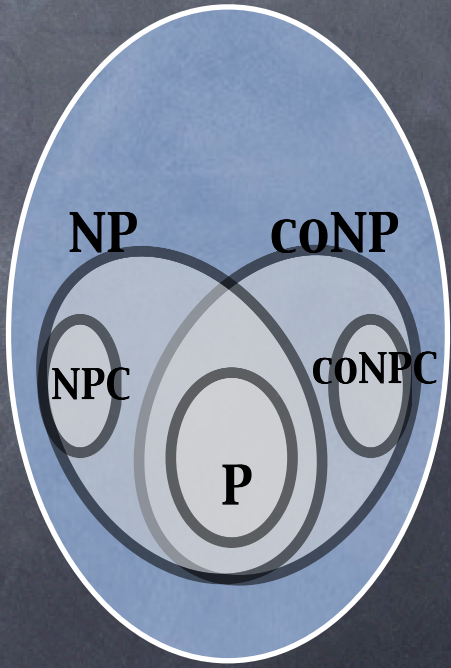

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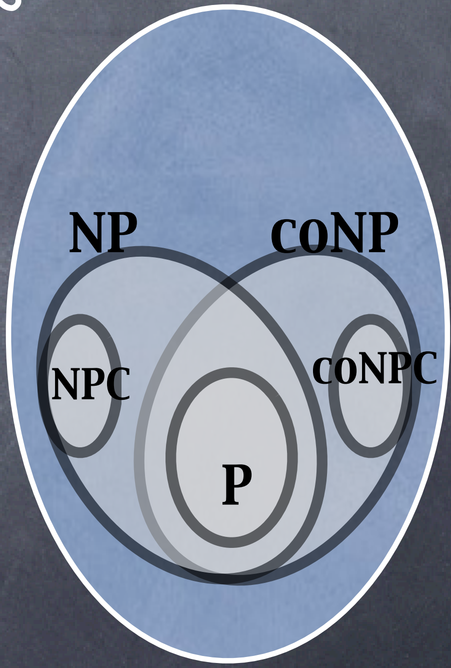

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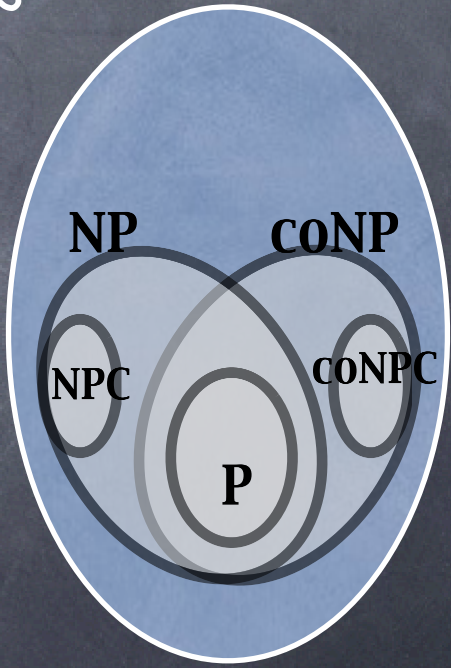

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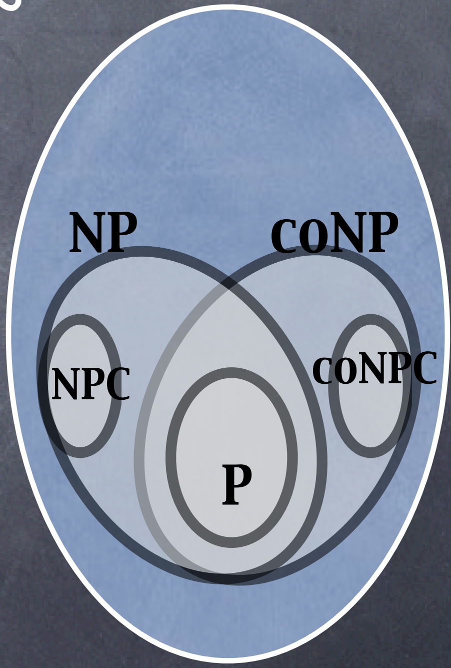

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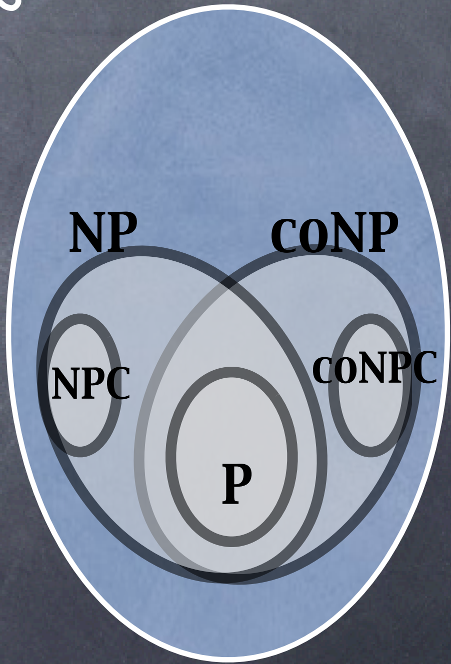

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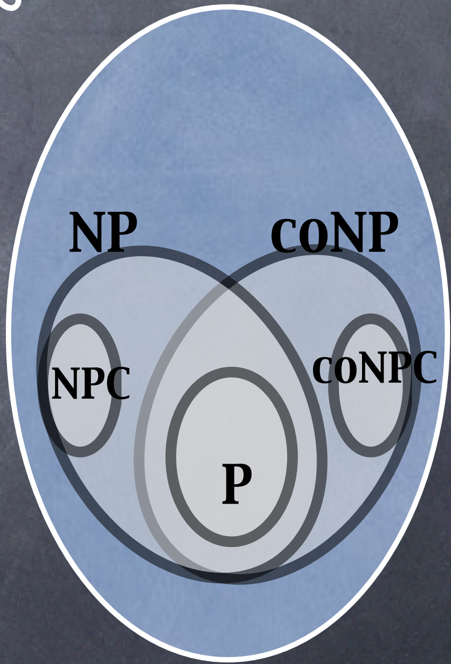

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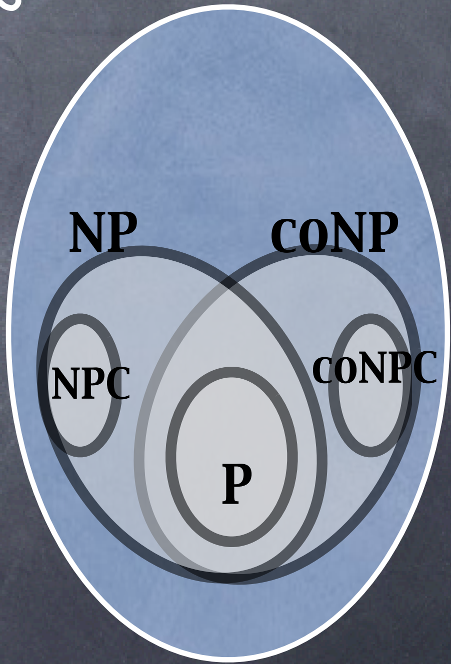

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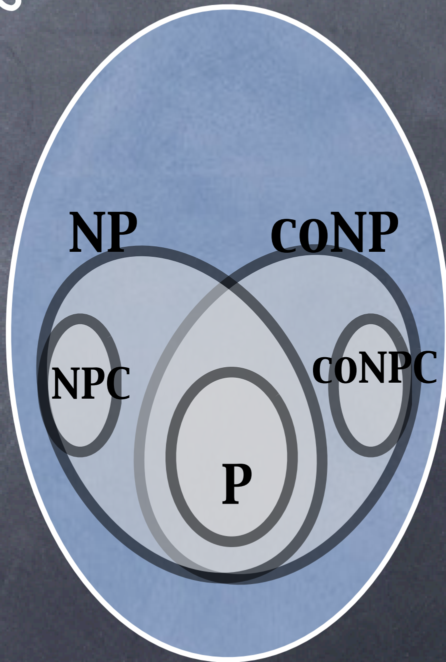
  - co-NP complete = co-(NP-complete)



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  - Count? What if both infinite?!
  - Comparing infinite sets: diagonalization!

Cantor’s Diagonal Slash

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R_1 =	1	0	0	1	0	0	0	0	1
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Undecidable Languages

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Diagonalization to Separate Classes

- Diagonalization can separate the class of decidable languages (from the class of all languages)
  - Plan: Use similar techniques to separate complexity classes

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- Consequences, for e.g., $P \subsetneq EXP$
  - $P \subseteq \mathrm{DTIME}(2^n) \subsetneq \mathrm{DTIME}(2^{2n}) \subseteq EXP$

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	0	0	1	0	1
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Simulate up to T steps of M_i in T′ steps. T′ large and nice enough to allow simulation

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Think DTIME(T) $\subseteq$ rows

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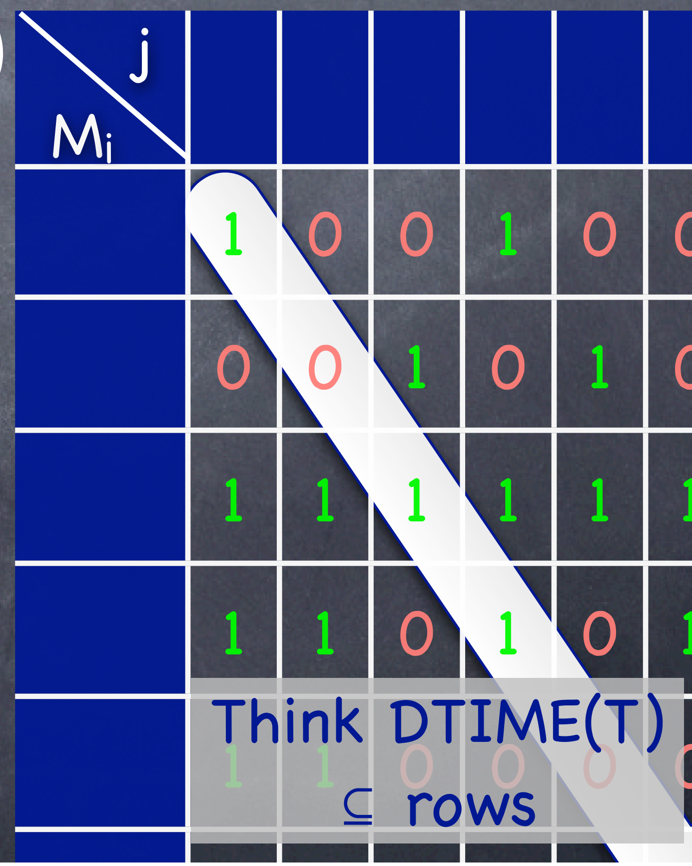

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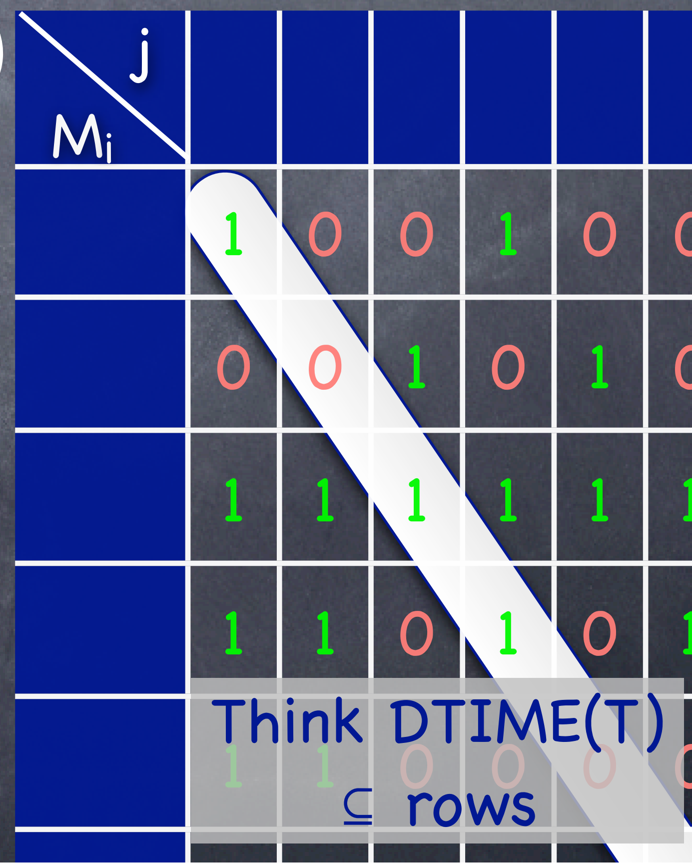

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- L′ in DTIME(T′)

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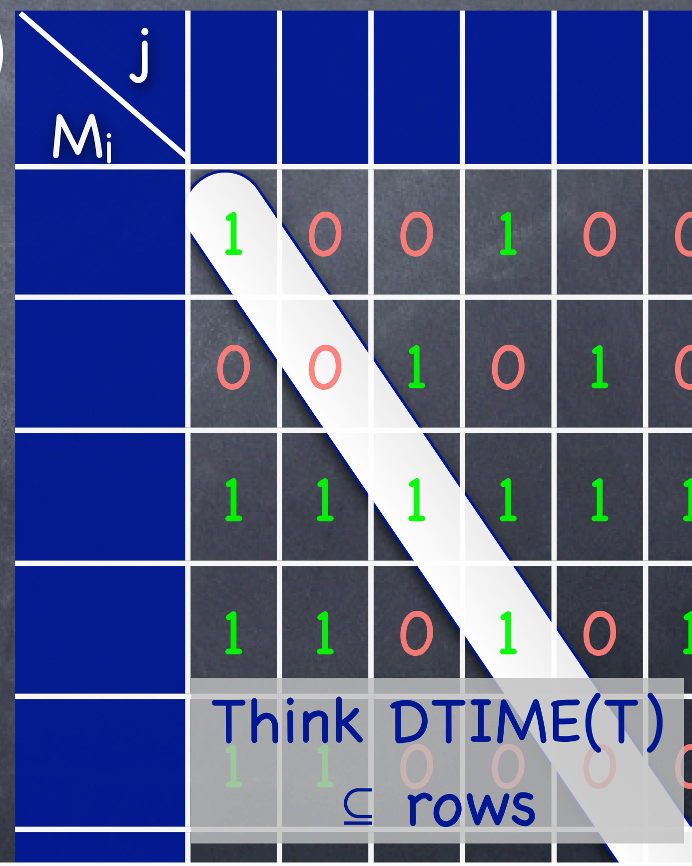

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Simulate up to T steps of M_i in T′ steps. T′ large and nice enough to allow simulation

- Let L′ = inverted diagonal.
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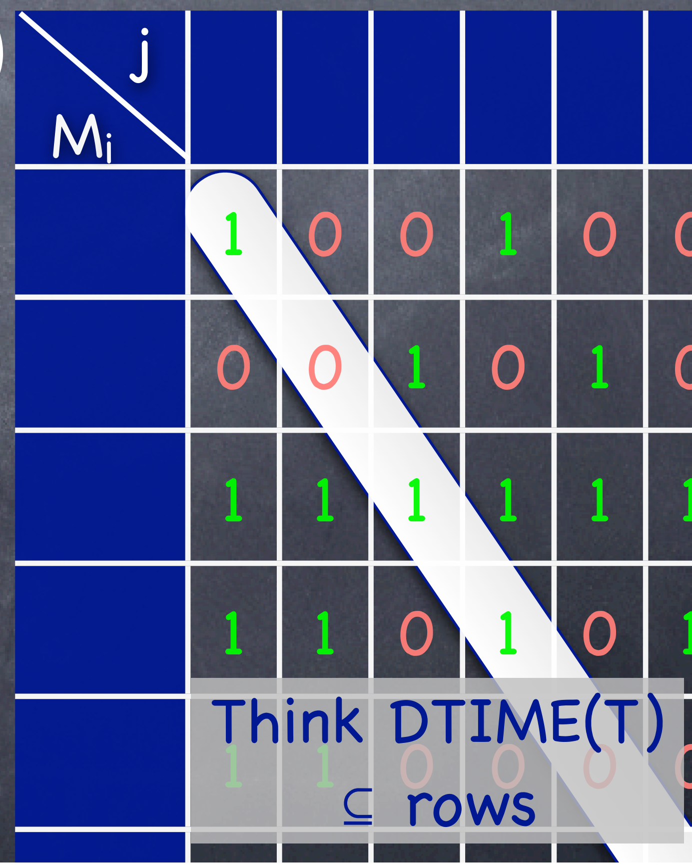

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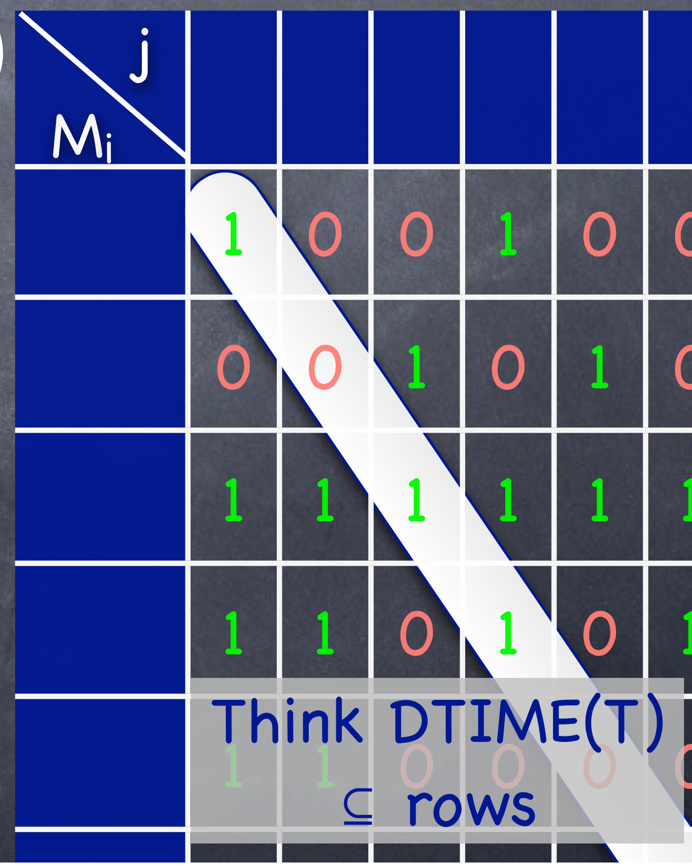

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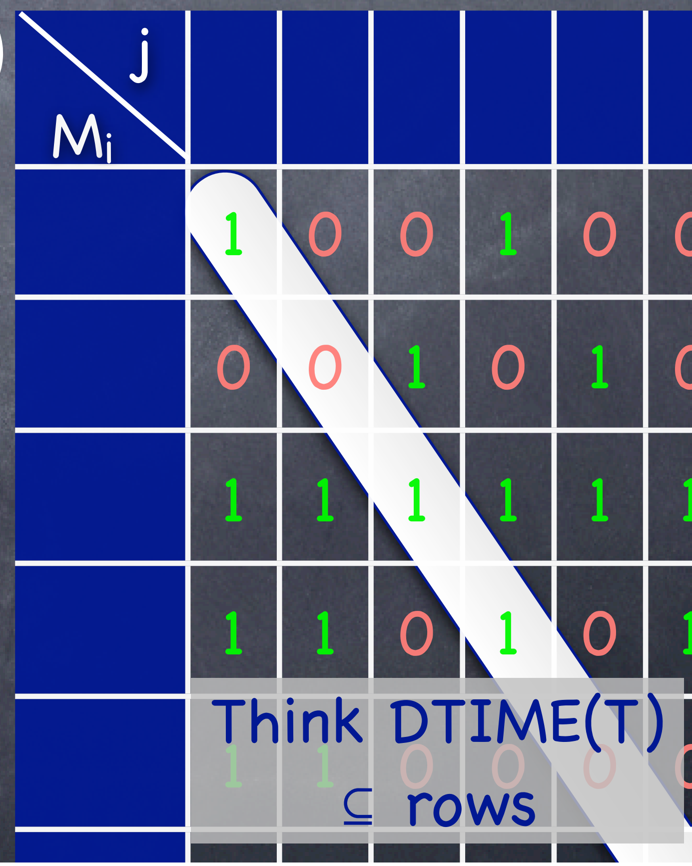
  - If M accepts L′ in time T, then for sufficiently large i s.t. M_i=M, UTM can finish simulating $M_i(i)$. Then table(i,i)=L′(i)!



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  - Issue: NTIME(T′) enough to simulate NTIME(T), but not to simulate co-NTIME(T)!

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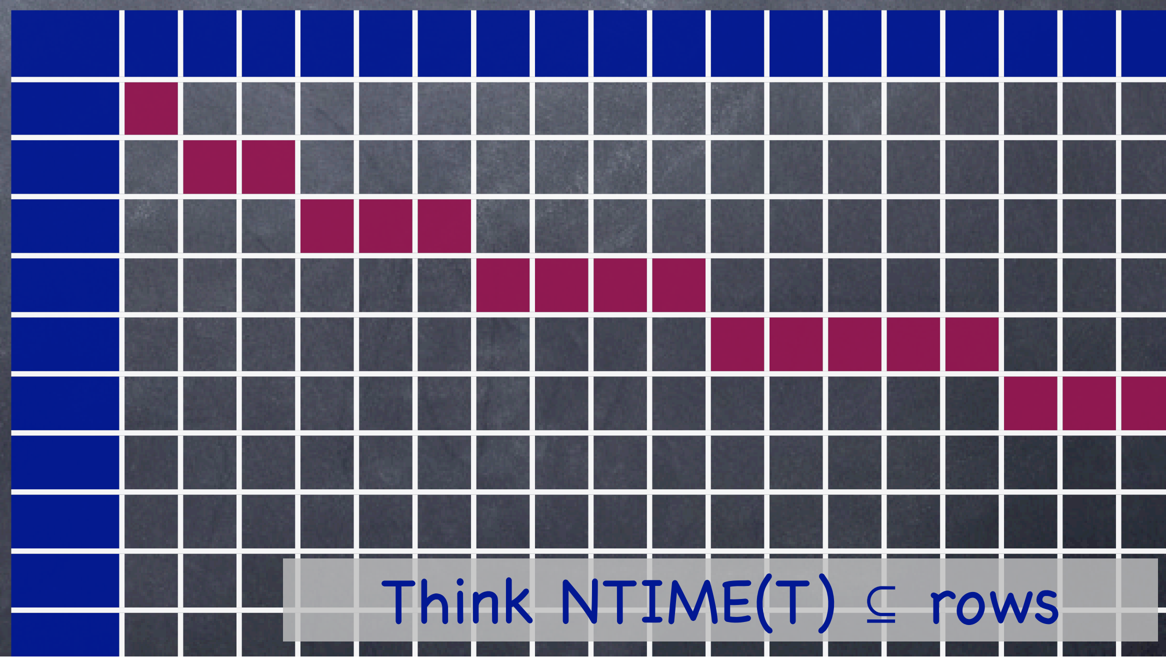

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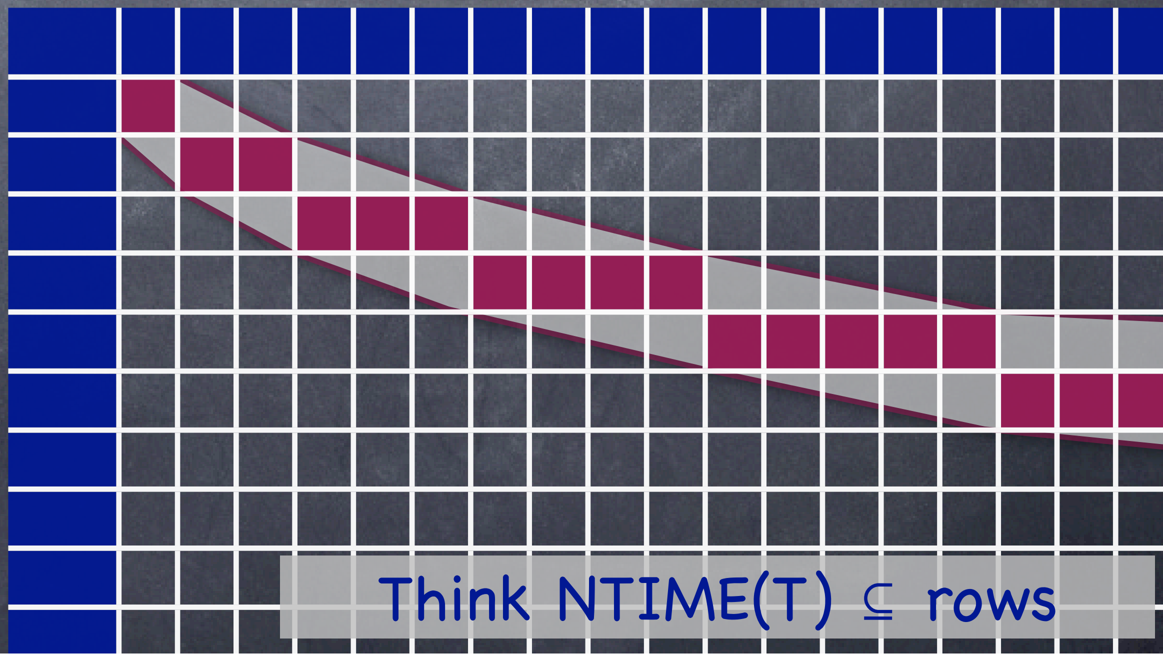

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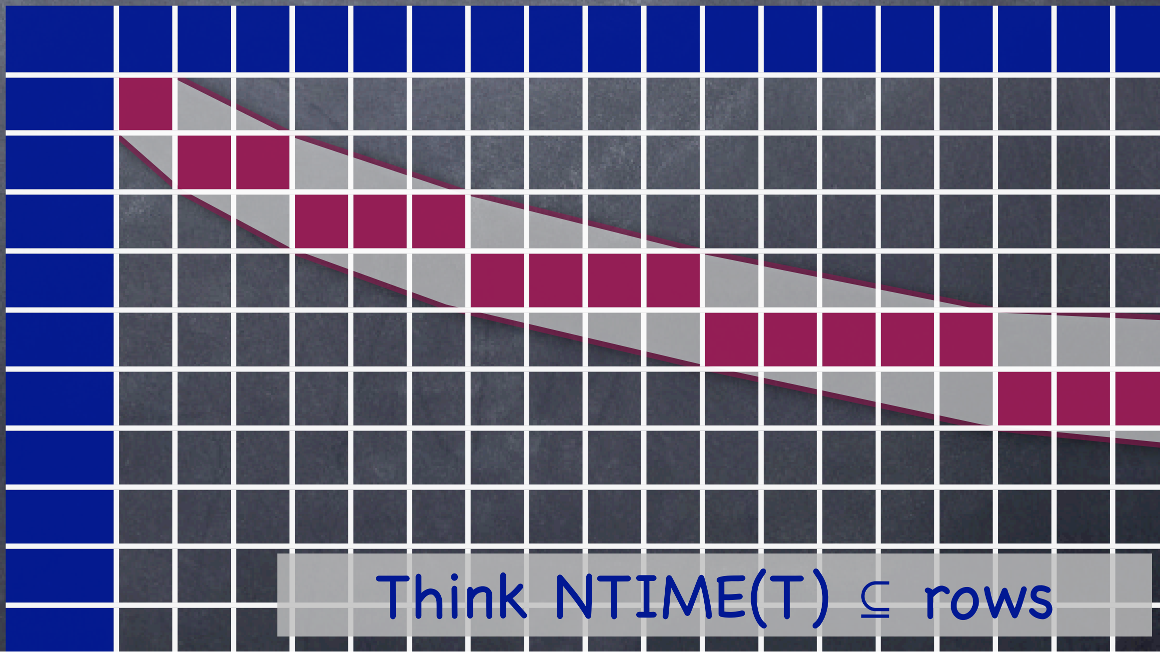

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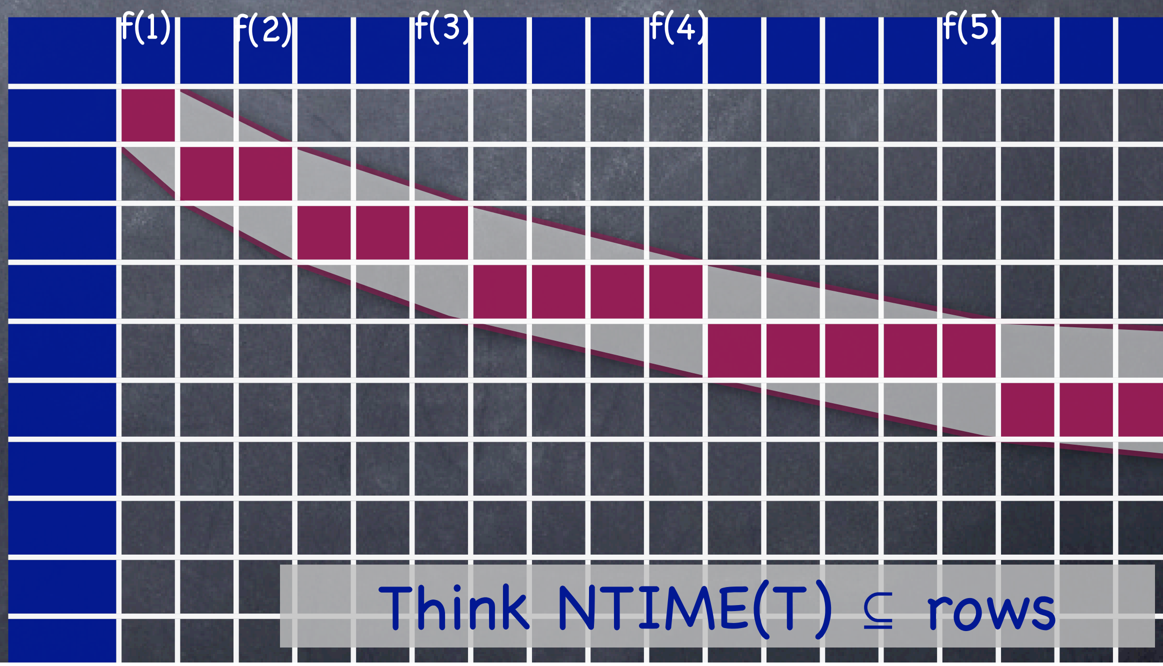

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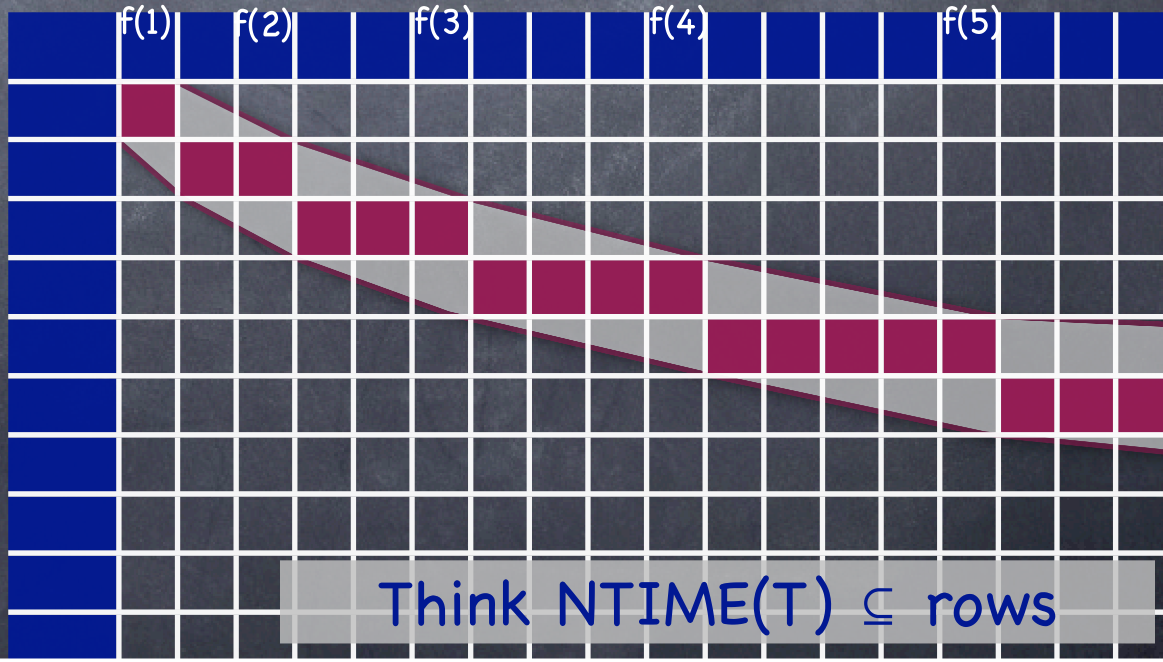

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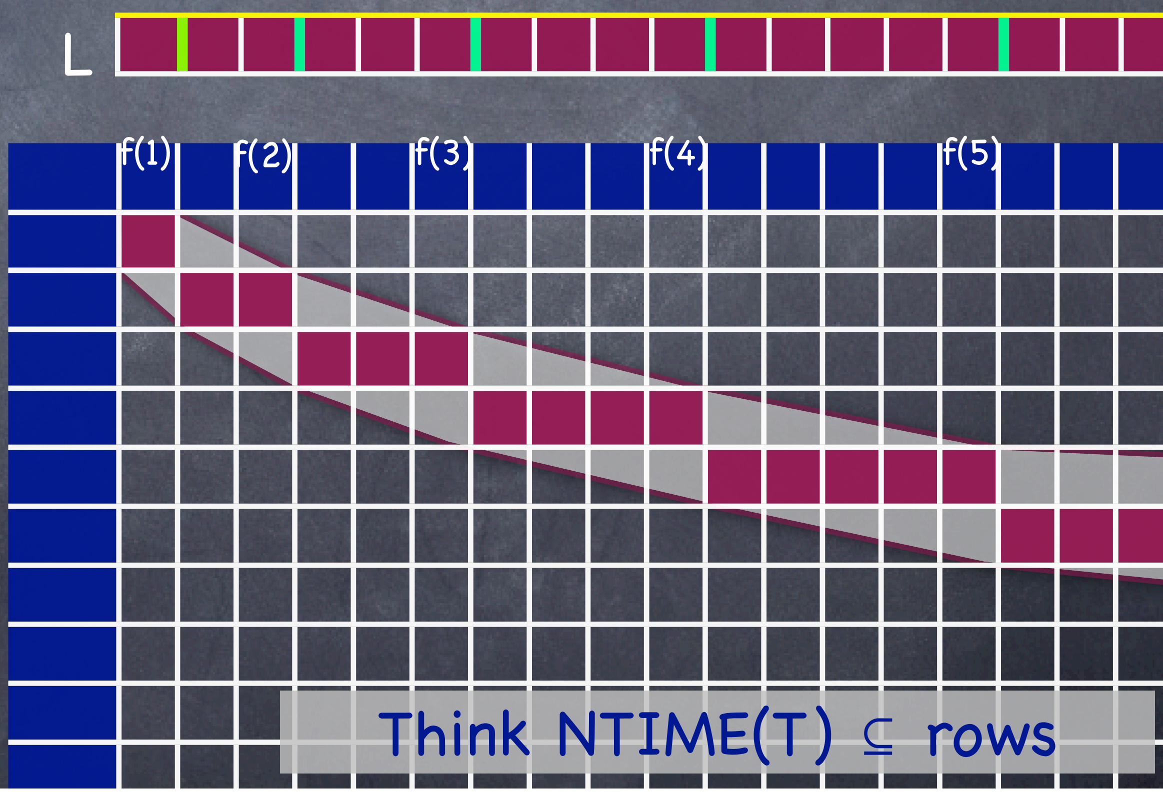

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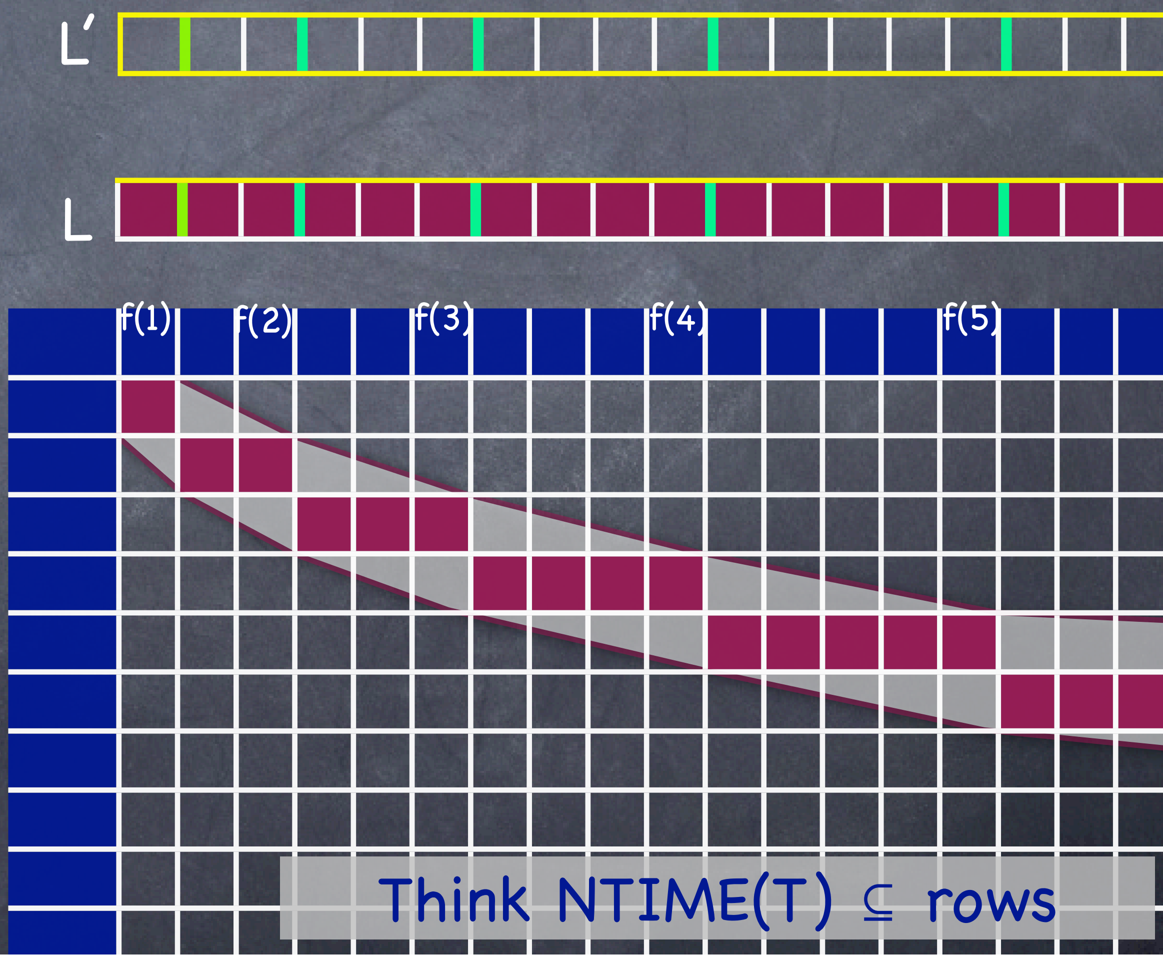

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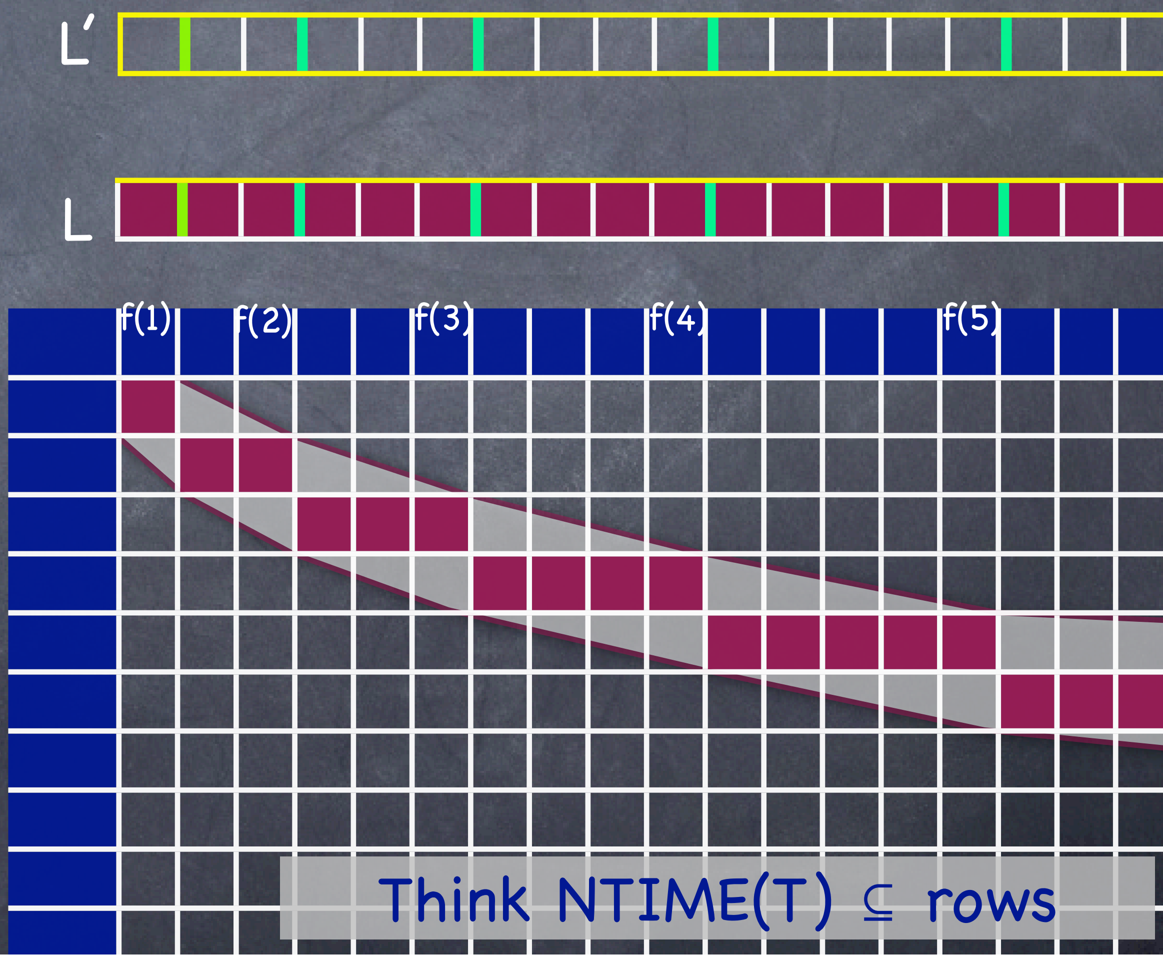

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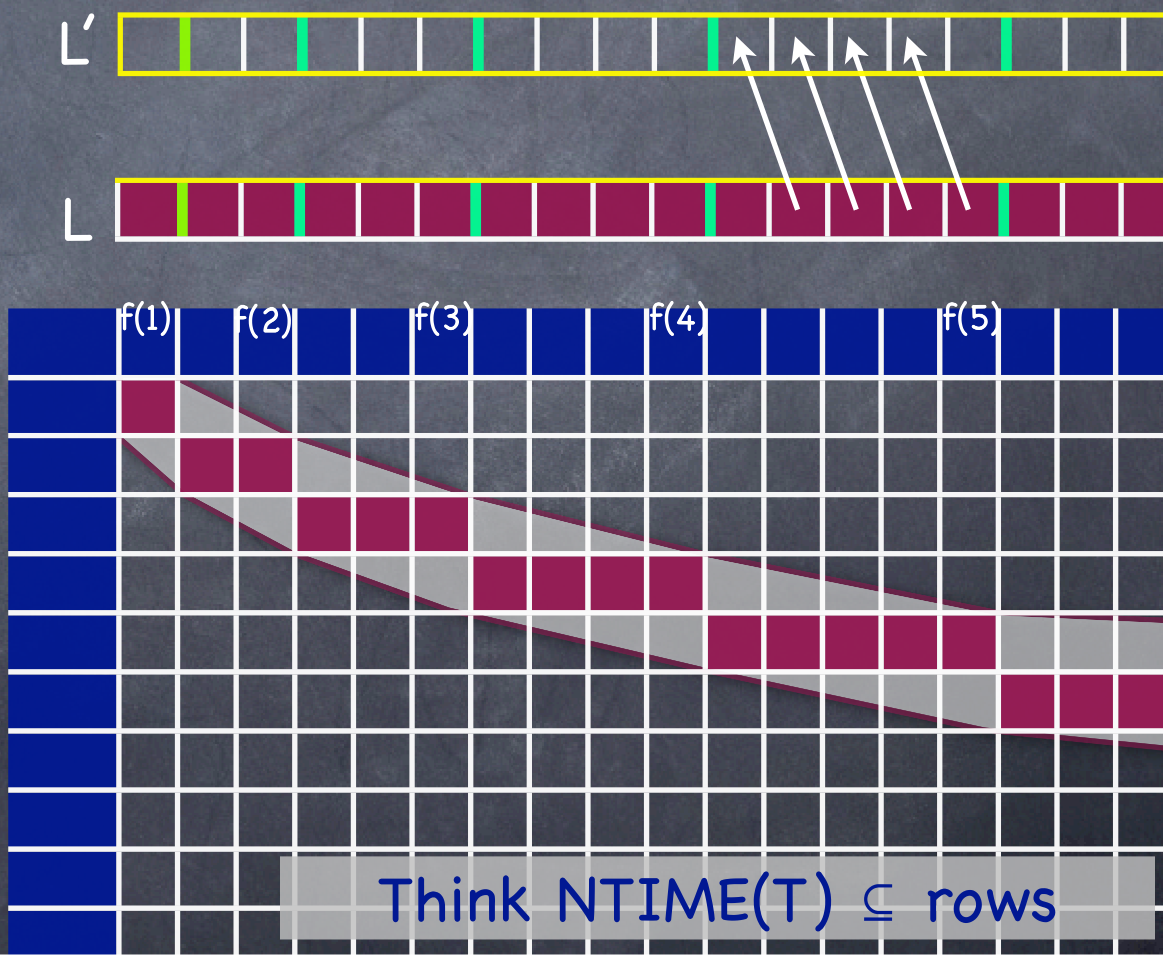

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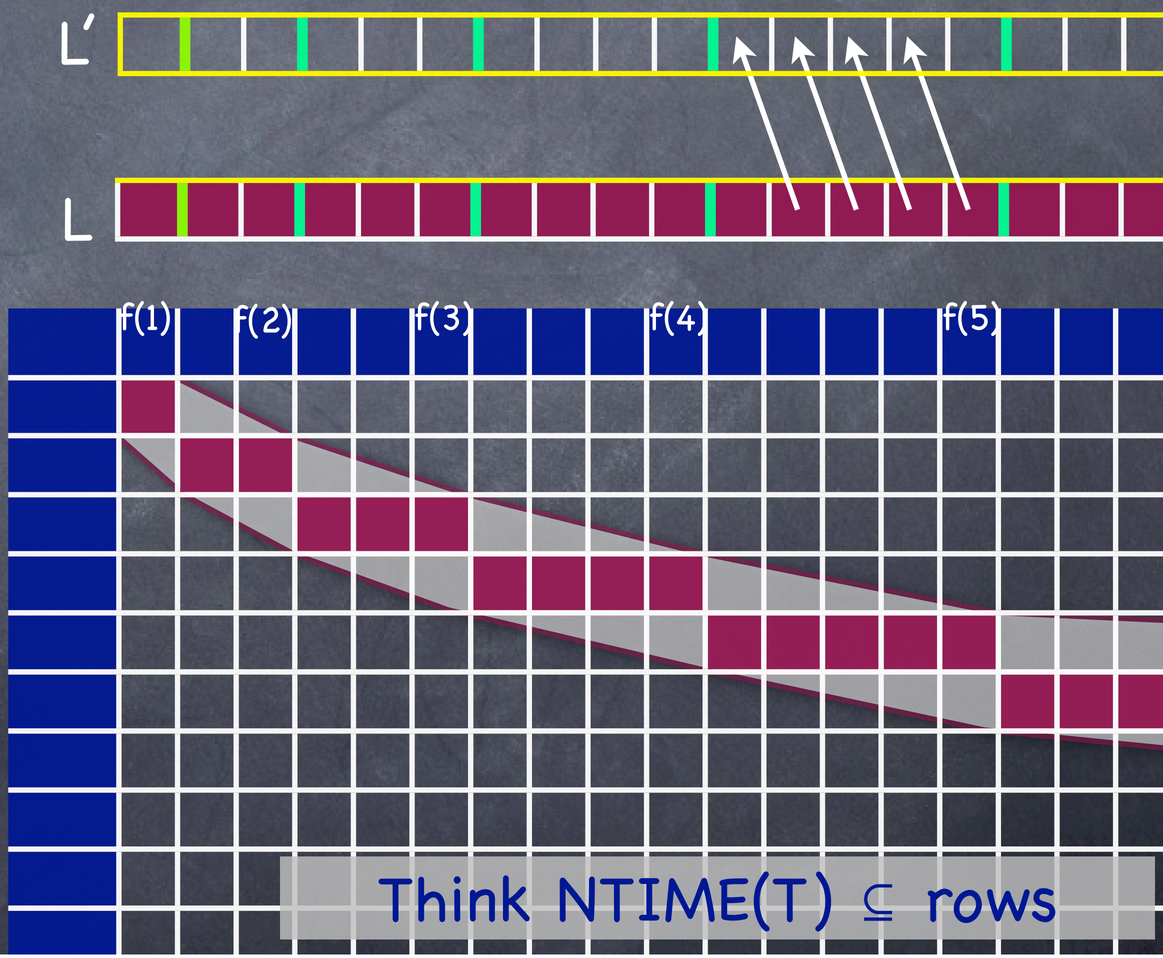

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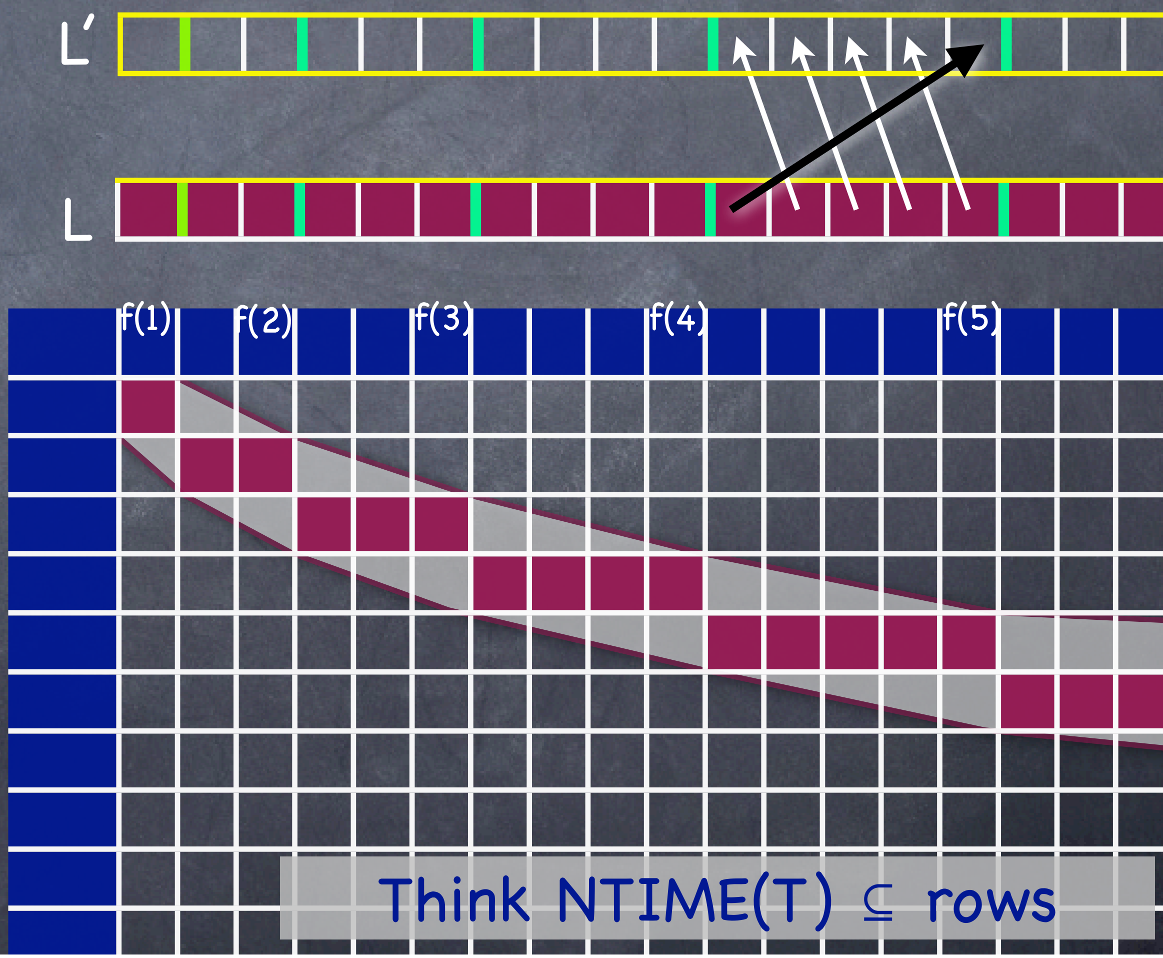

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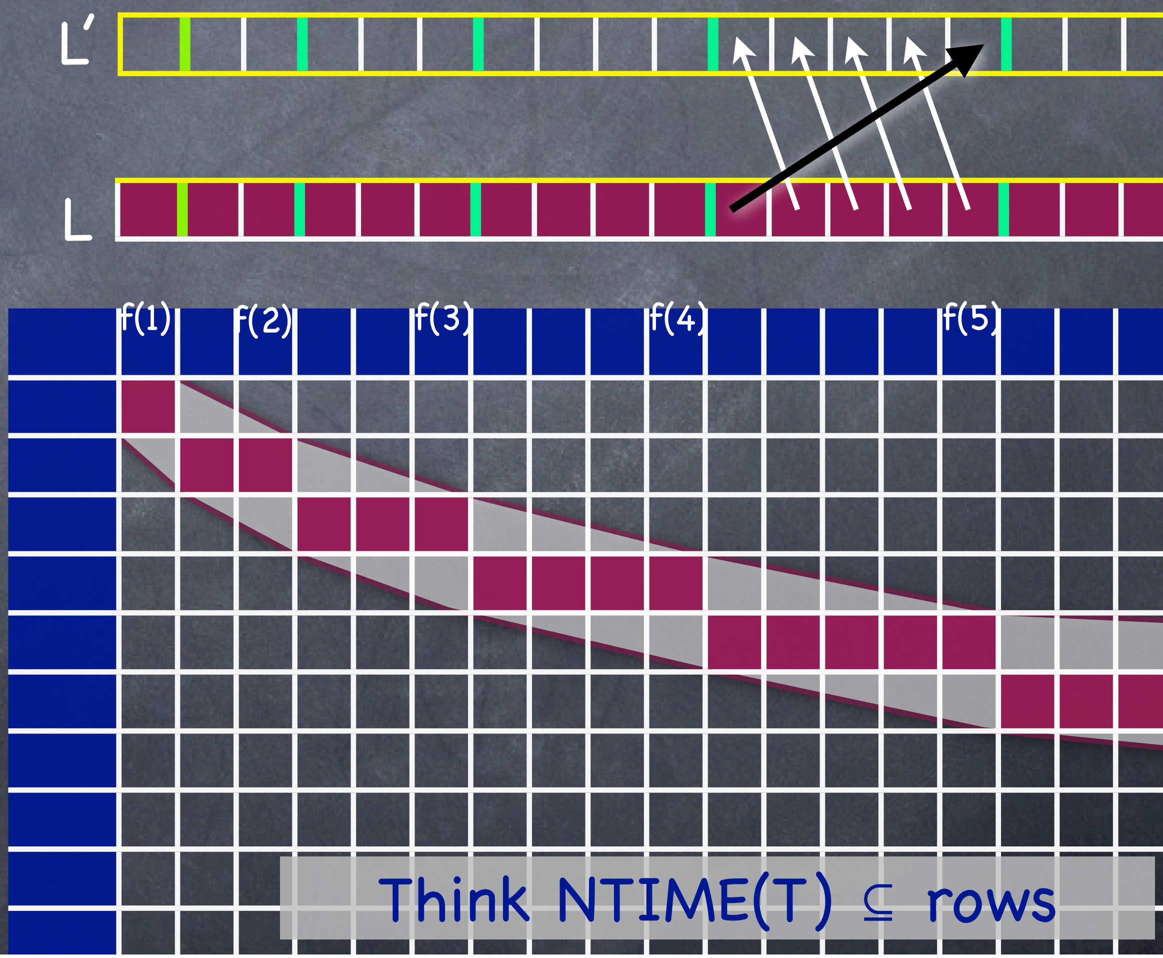

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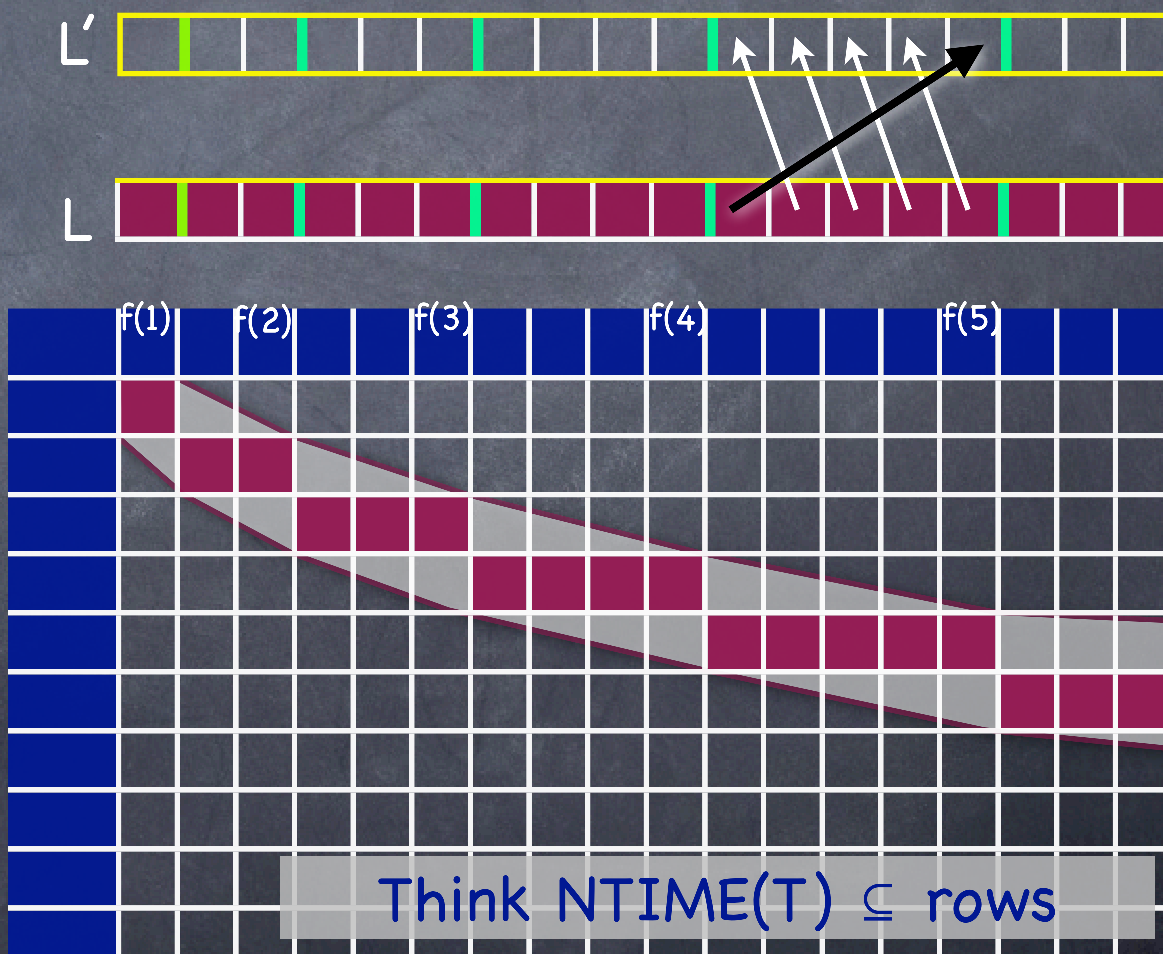

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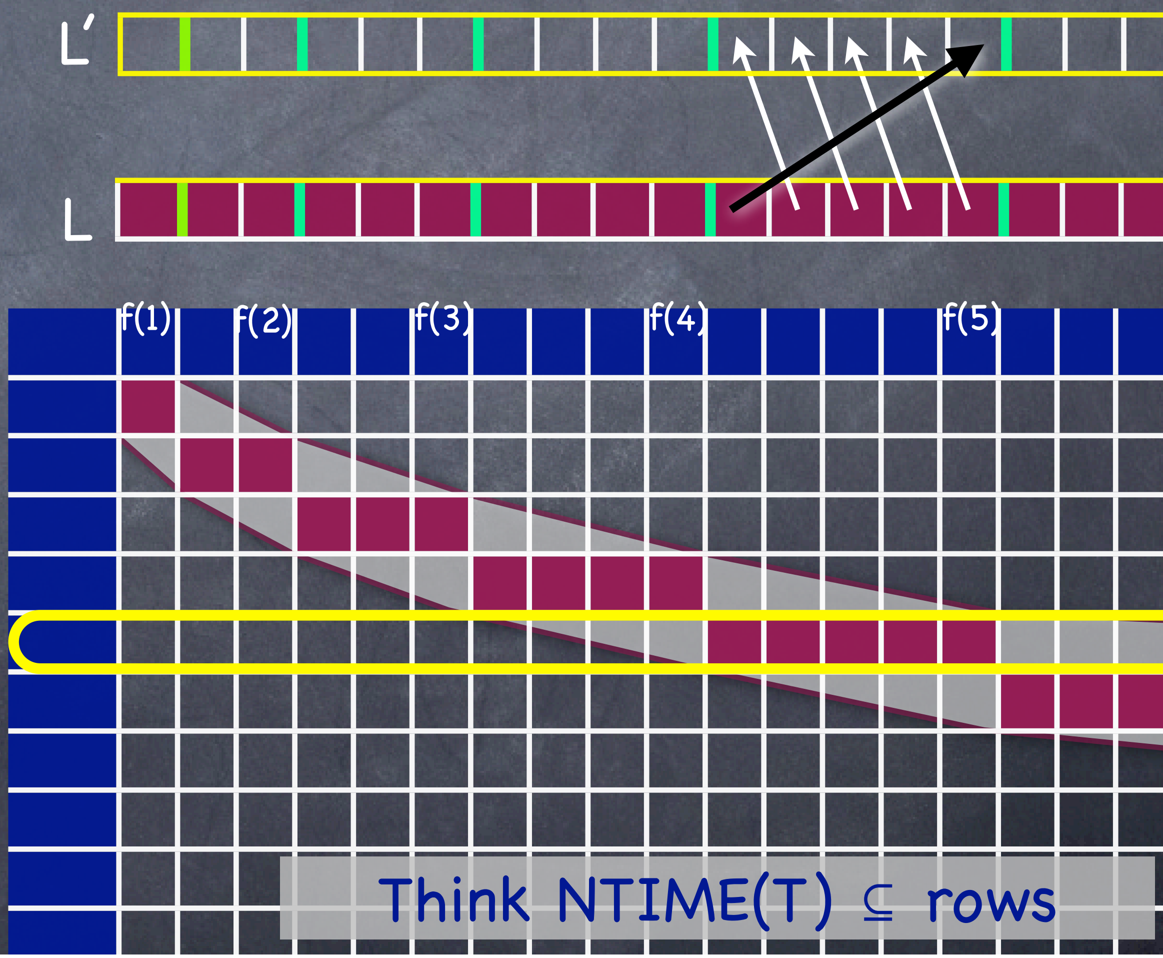

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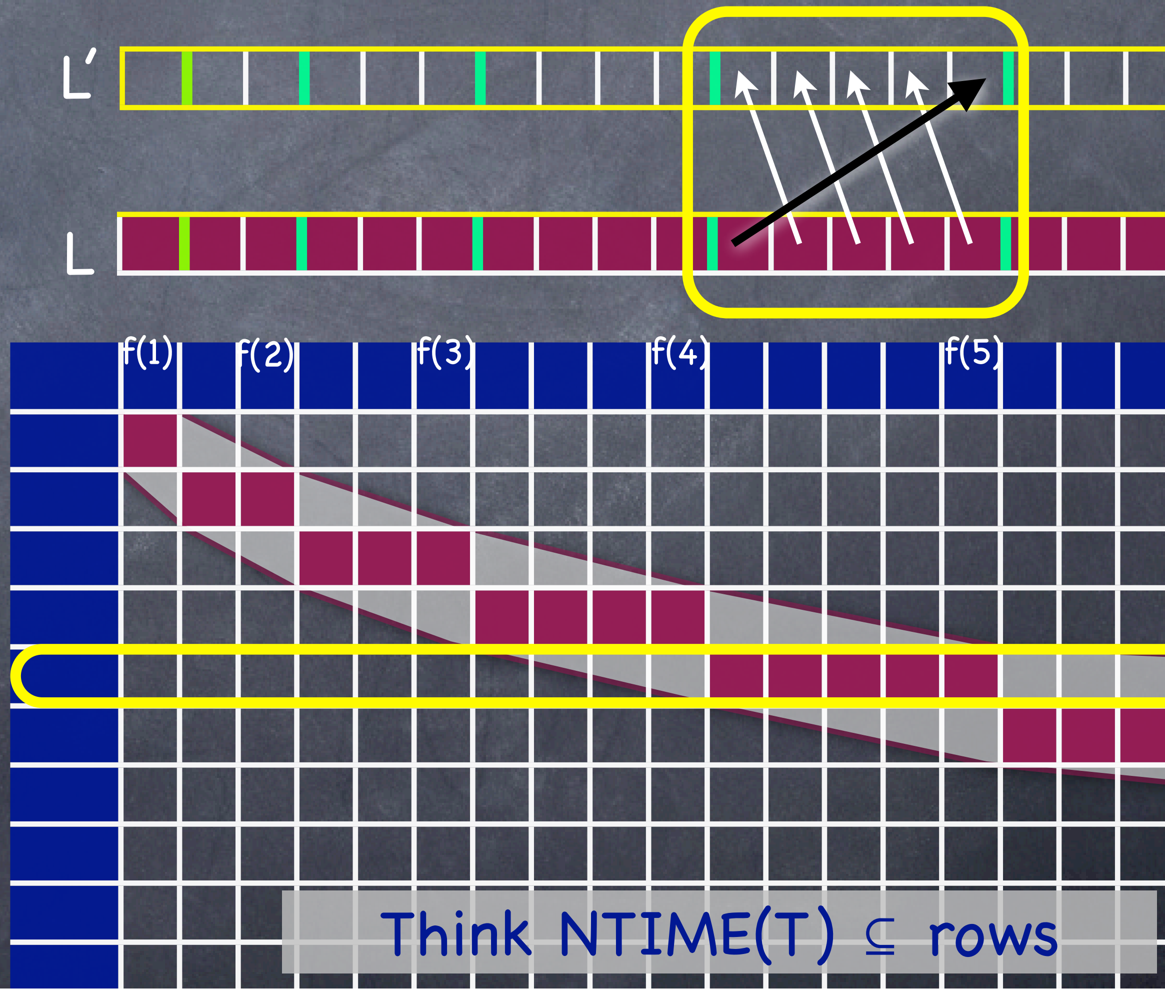

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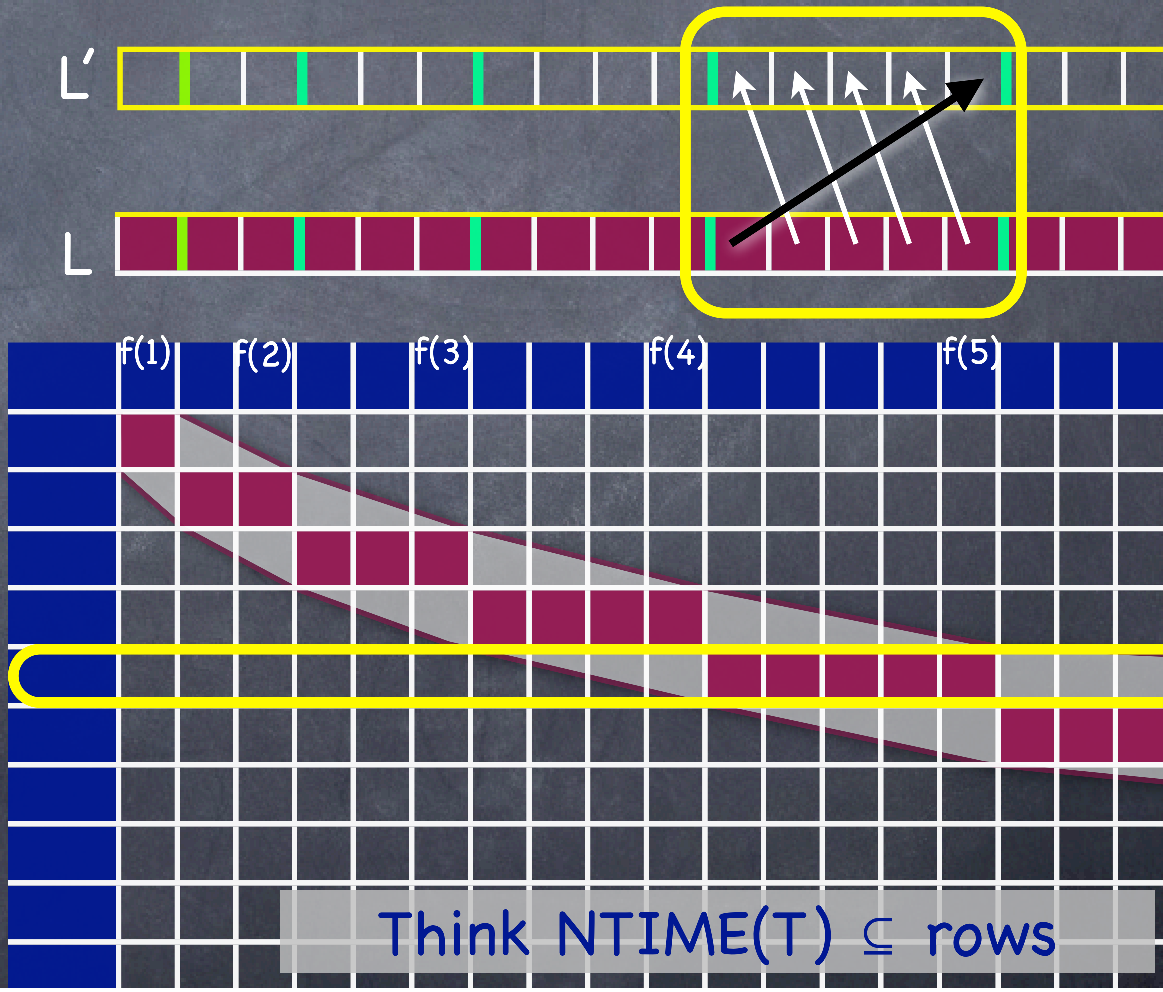
Delay, Rapid thickening



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        - Just diagonalization won't help (next lecture)

Today

- DTIME Hierarchy
  - DTIME(T) $\subsetneq$ DTIME(T′) if T log T = o(T′)
- NTIME Hierarchy
  - NTIME(T) $\subsetneq$ NTIME(T′) if T = o(T′)
- Using diagonalization

Next Lecture

- Another application of diagonalization
  - Ladner's Theorem: If P≠NP, NP language which is neither in P nor NP-complete
- Limits of Diagonalization
- Starting Space Complexity