Computational Complexity

Lecture 2
in which we talk about
NP-completeness
(reductions, reductions)

Recap

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- Today: Hardest problems in NP

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  - if can decide L_2, can decide L_1

Turing and Many-One

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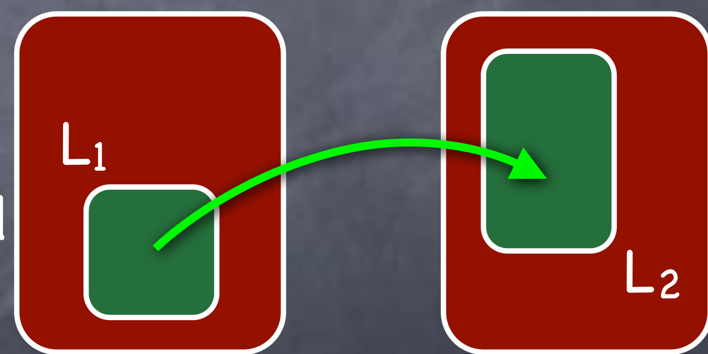

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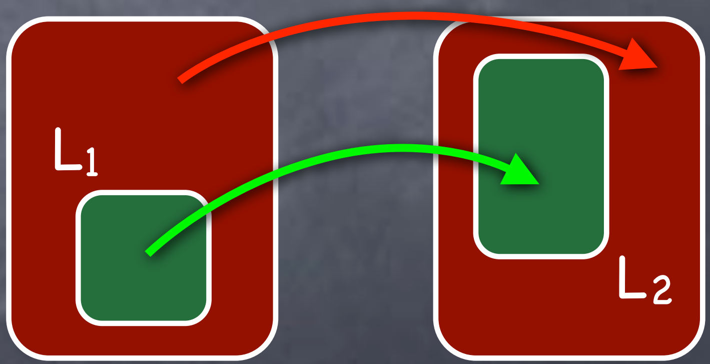

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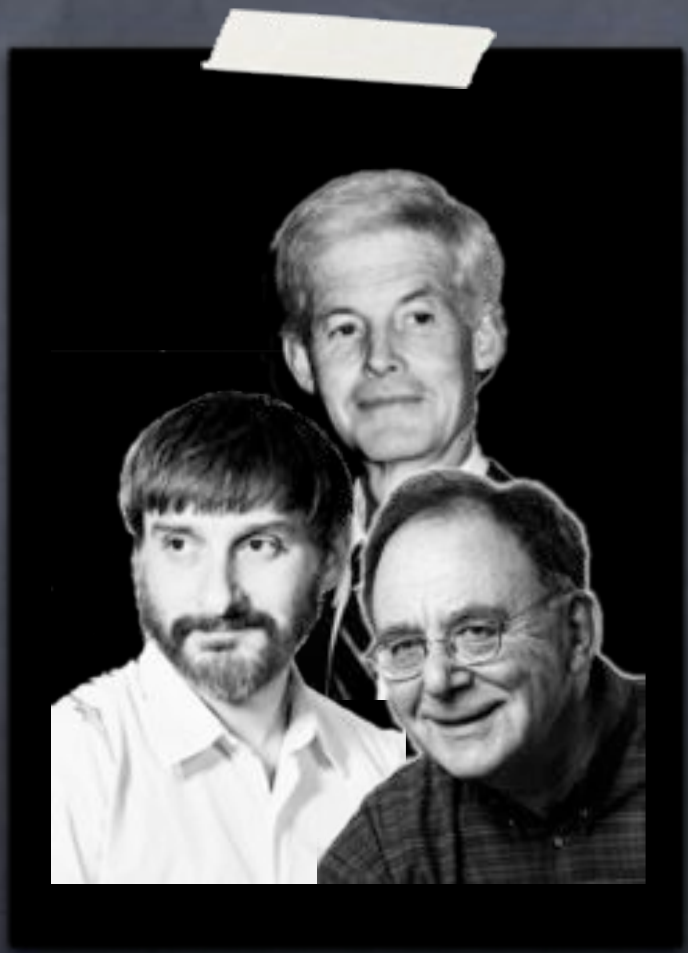
  - L_2 may be way harder

Cook, Karp, Levin



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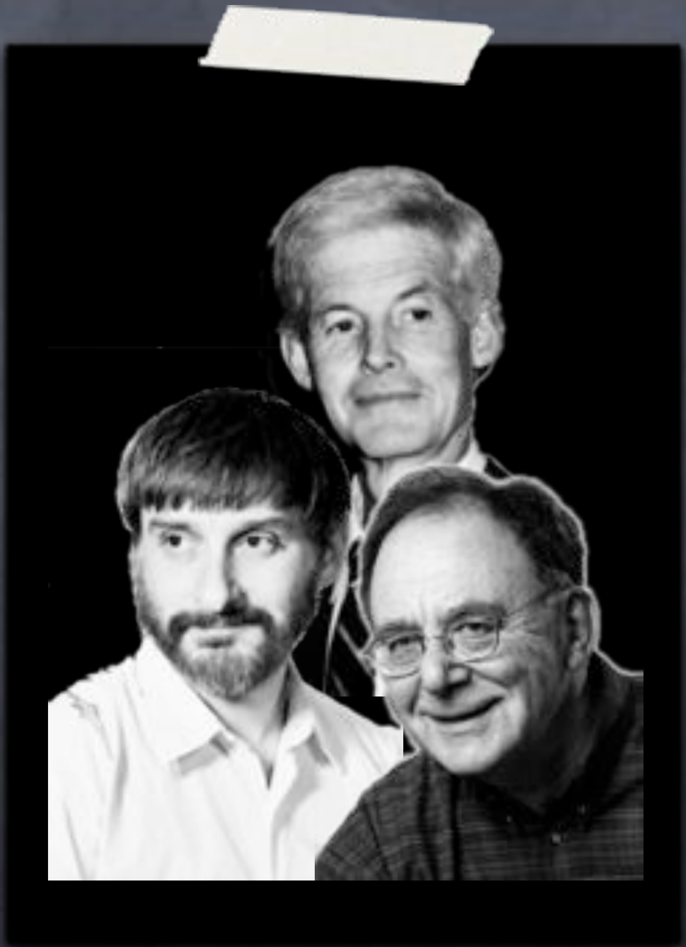

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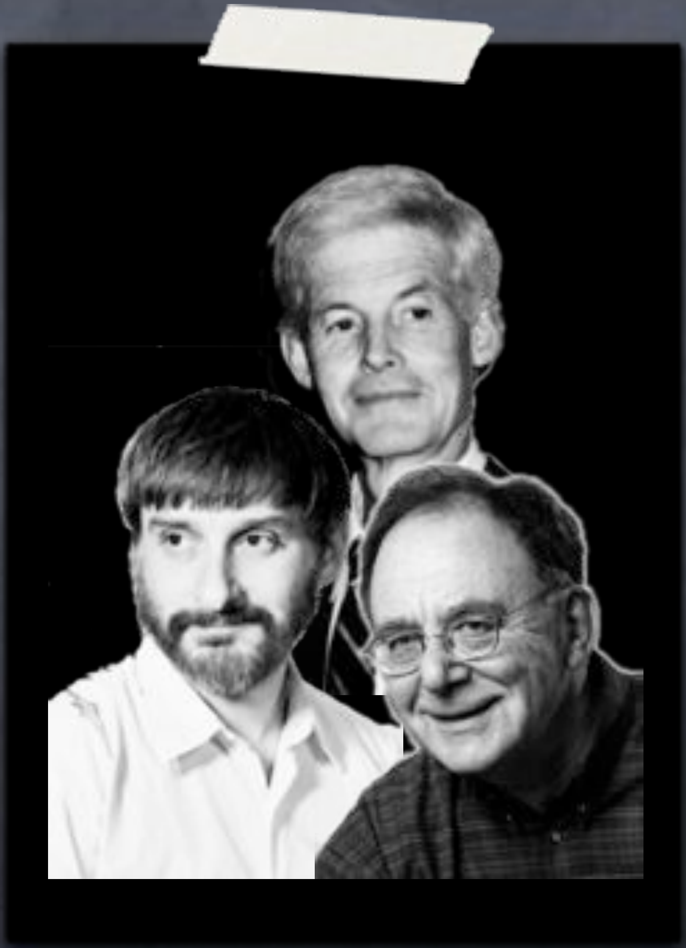

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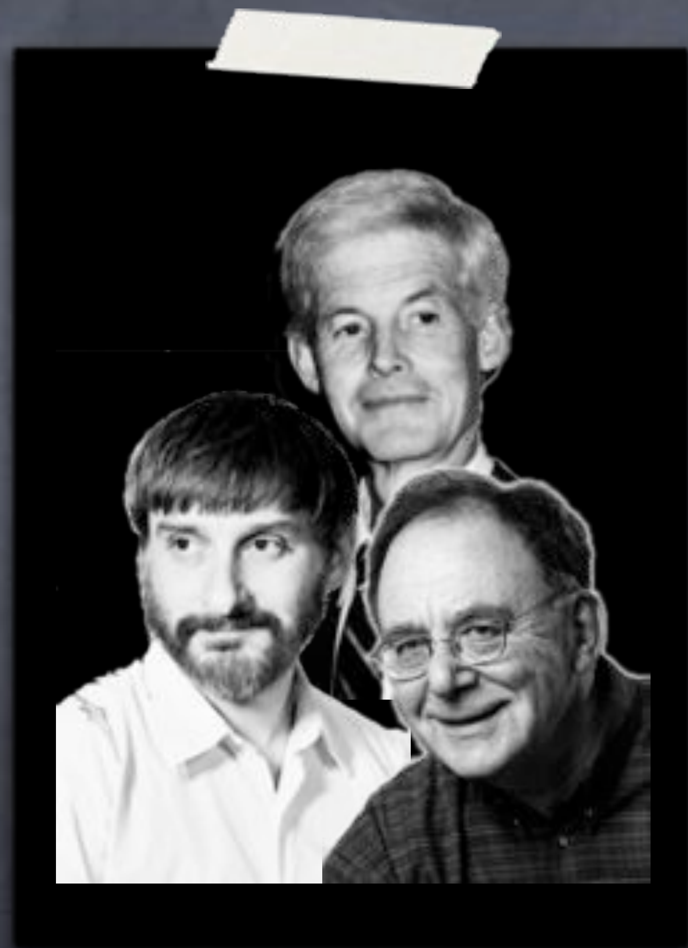

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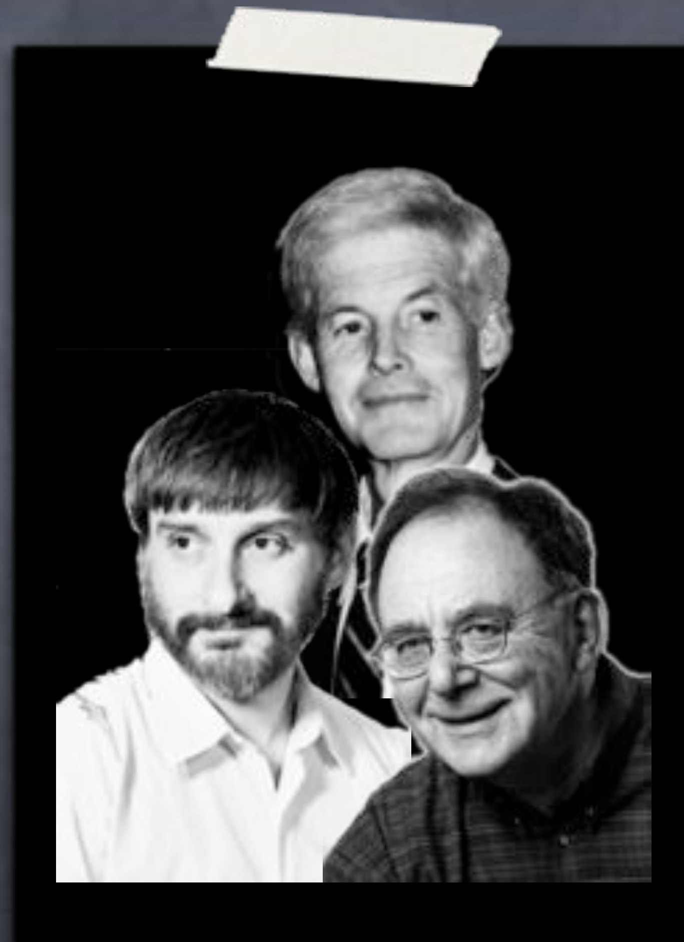

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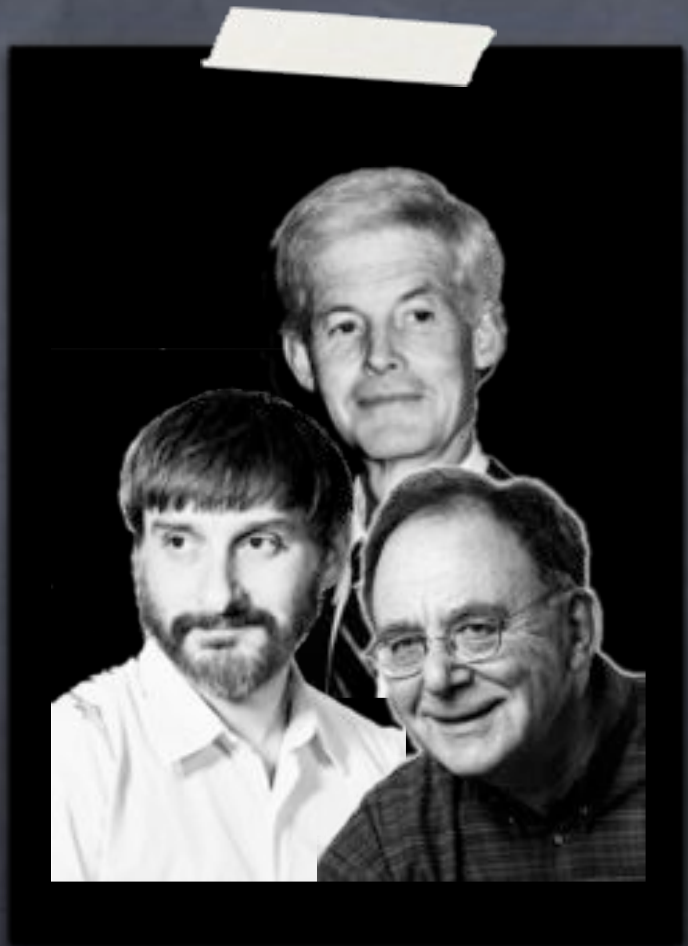
  - Parsimonious: Karp + number of witnesses doesn't change



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- A language L is NP-Hard if for all L′ in NP, L′ $\leq_p$ L
- A language L is NP-Complete if it is NP-Hard and is in NP
  - To efficiently solve all problems in NP, you need to efficiently solve L and nothing more

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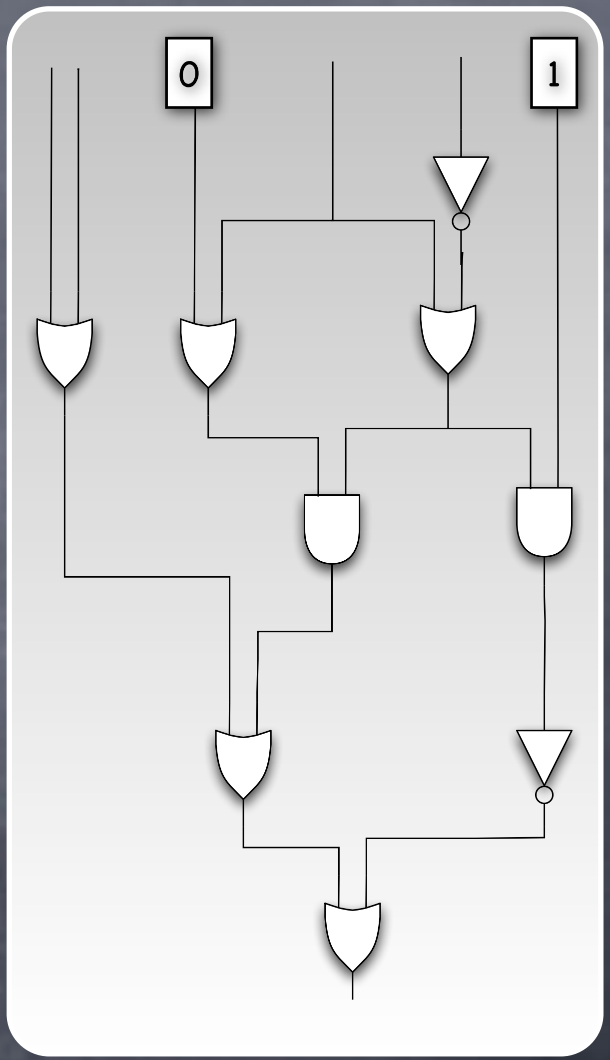
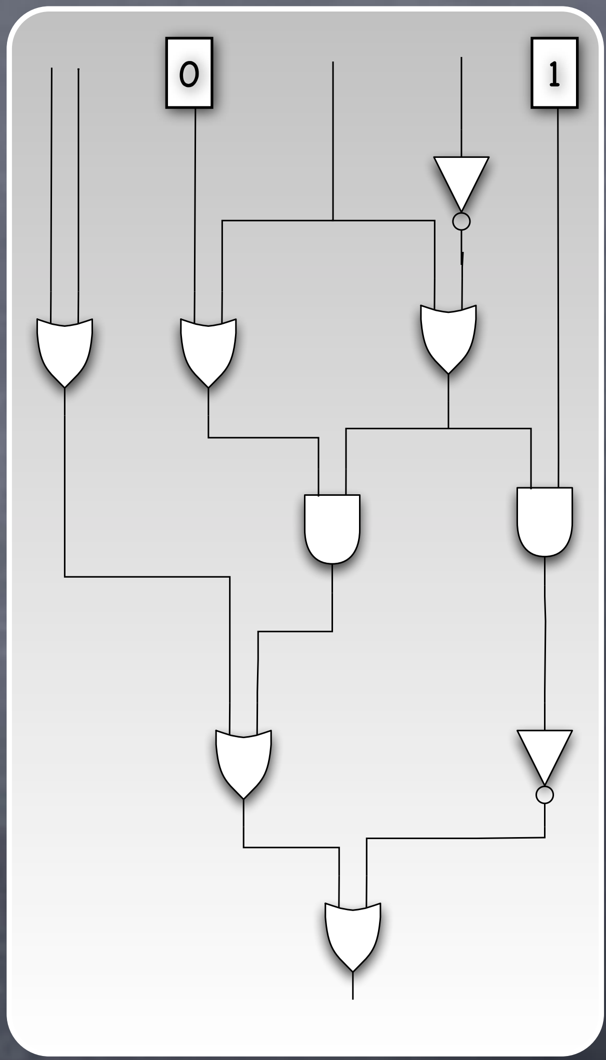
- Any "natural" NPC language?

Boolean Circuits



Boolean Circuits

- Boolean valued wires, AND, OR, NOT, CONST gates, inputs, output, directed acyclic graph



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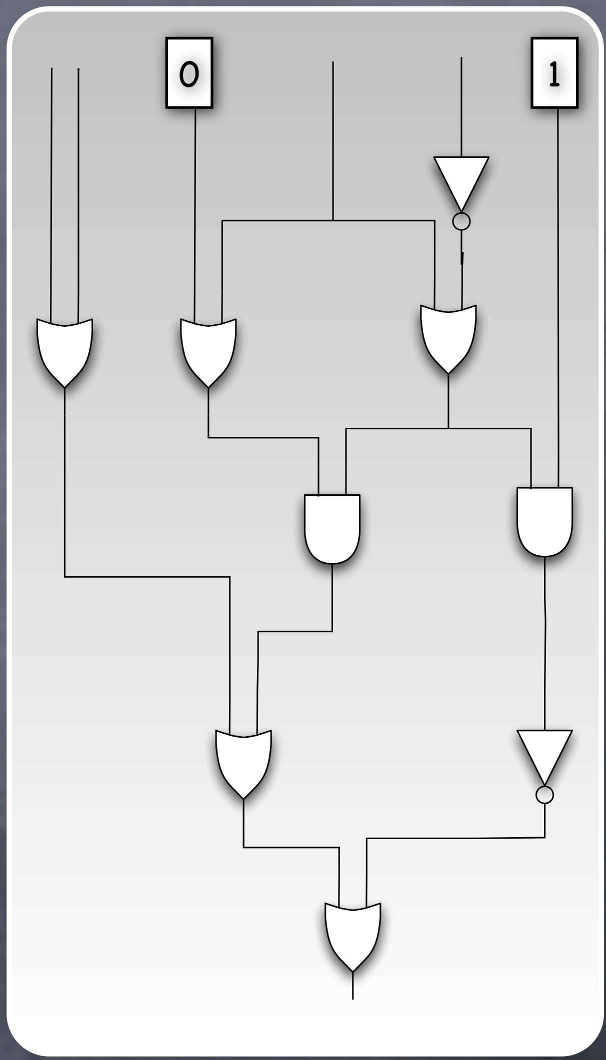

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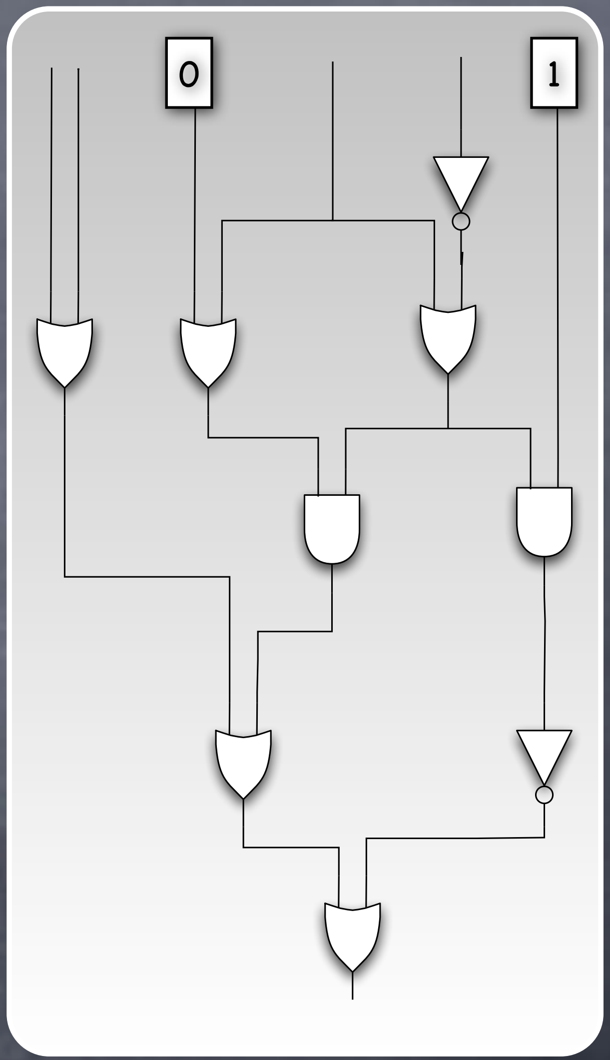

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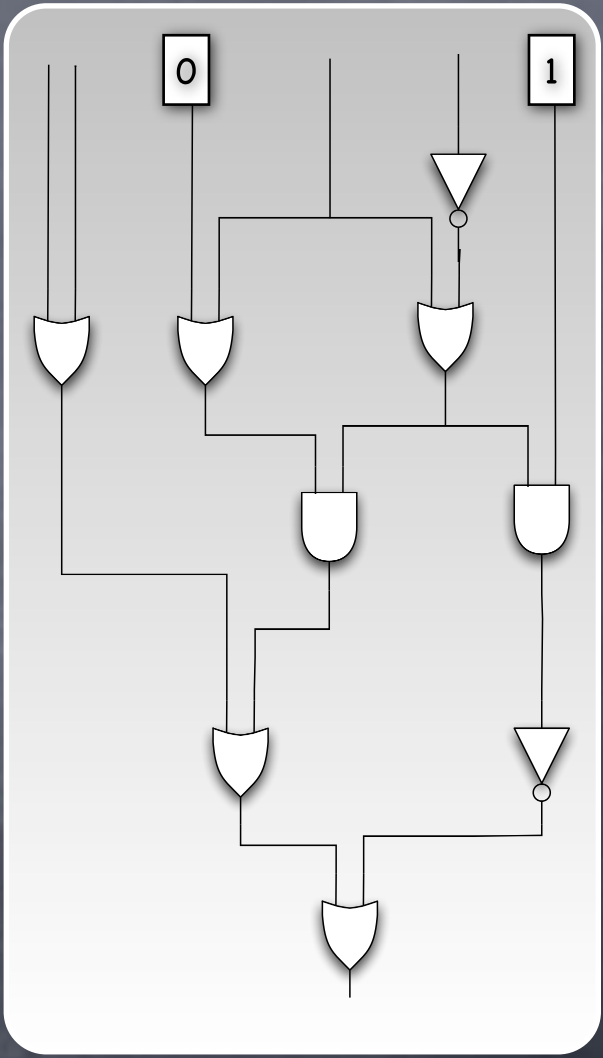
- CKT-SAT: given ckt, is there a "satisfying" input (output=1). In NP.



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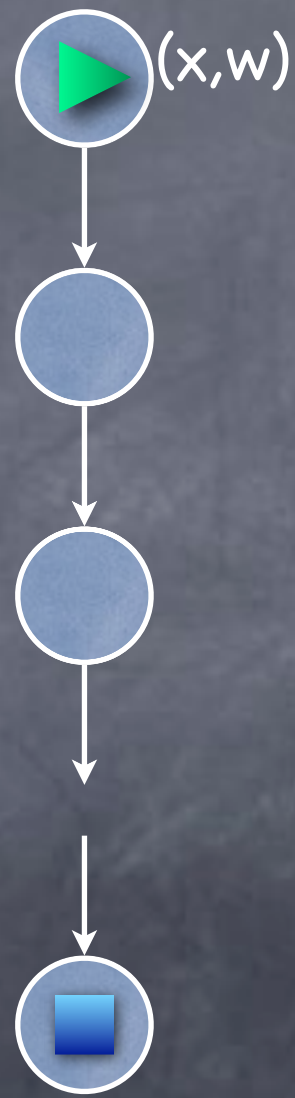
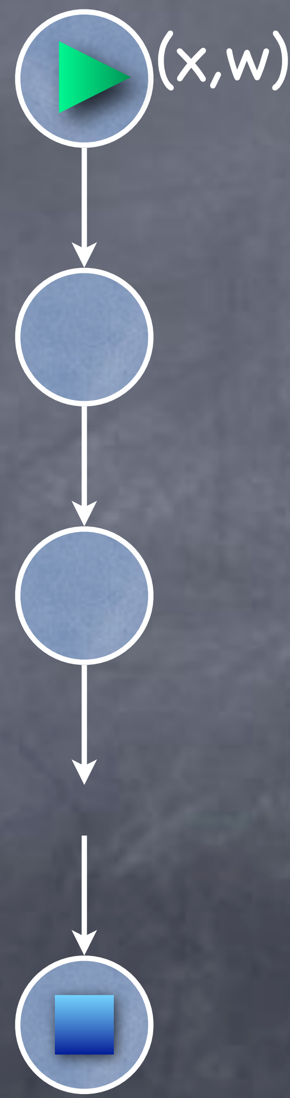
  - Ensure reduction is poly-time

TM to Circuit



TM to Circuit

- Wires for configurations: a bundle for each tape cell, encoding (content,state), where state is encoded in the cell with the head



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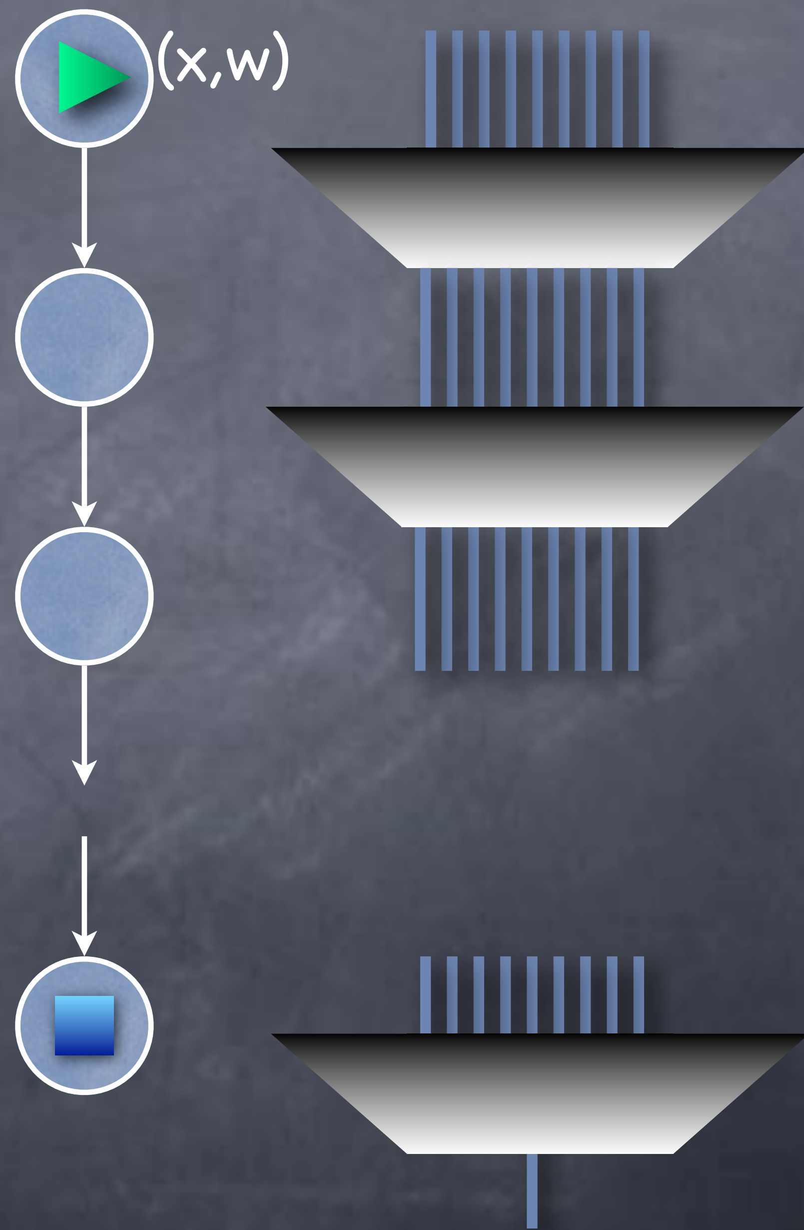

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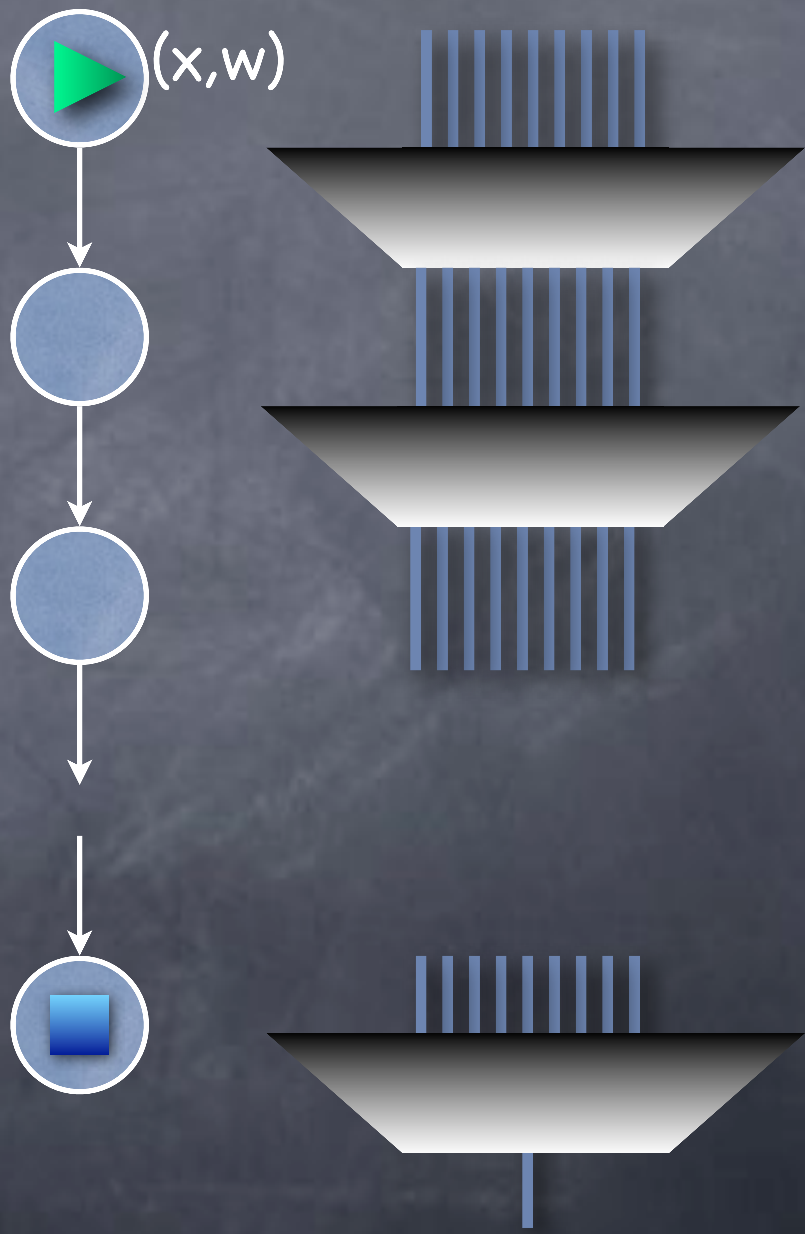

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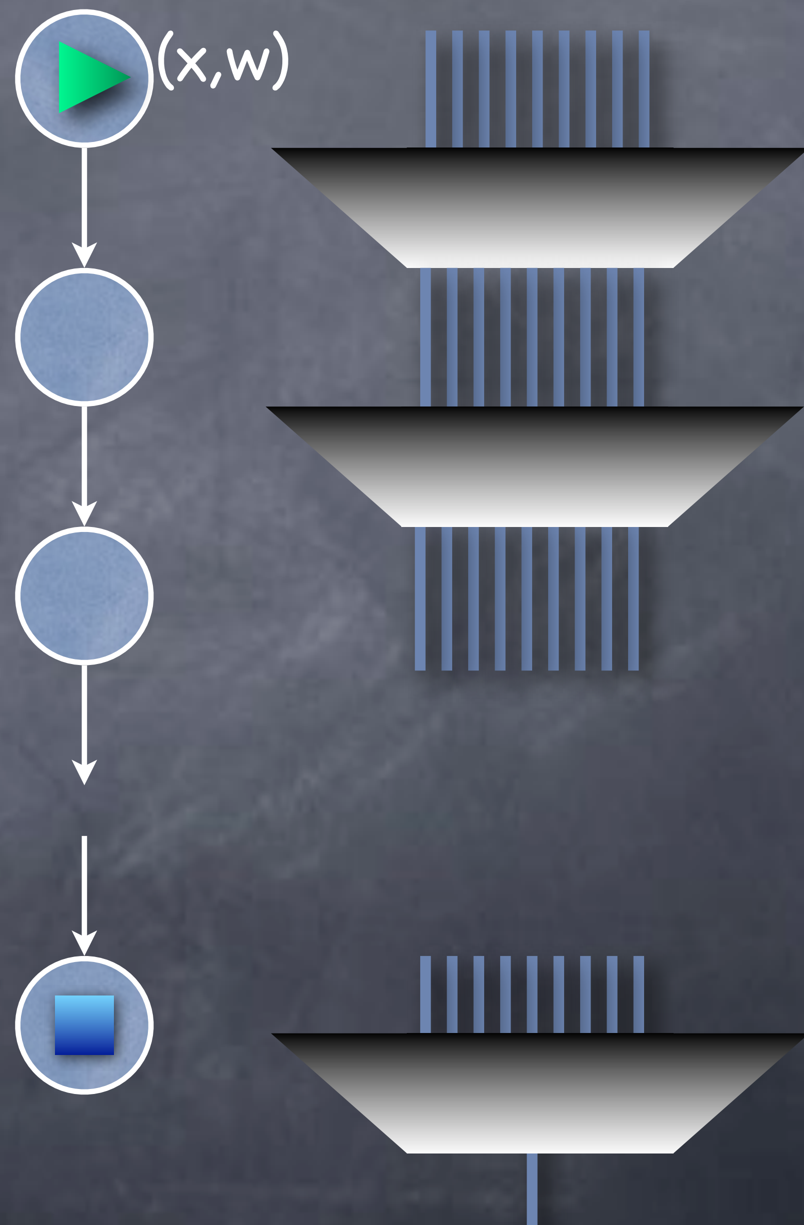

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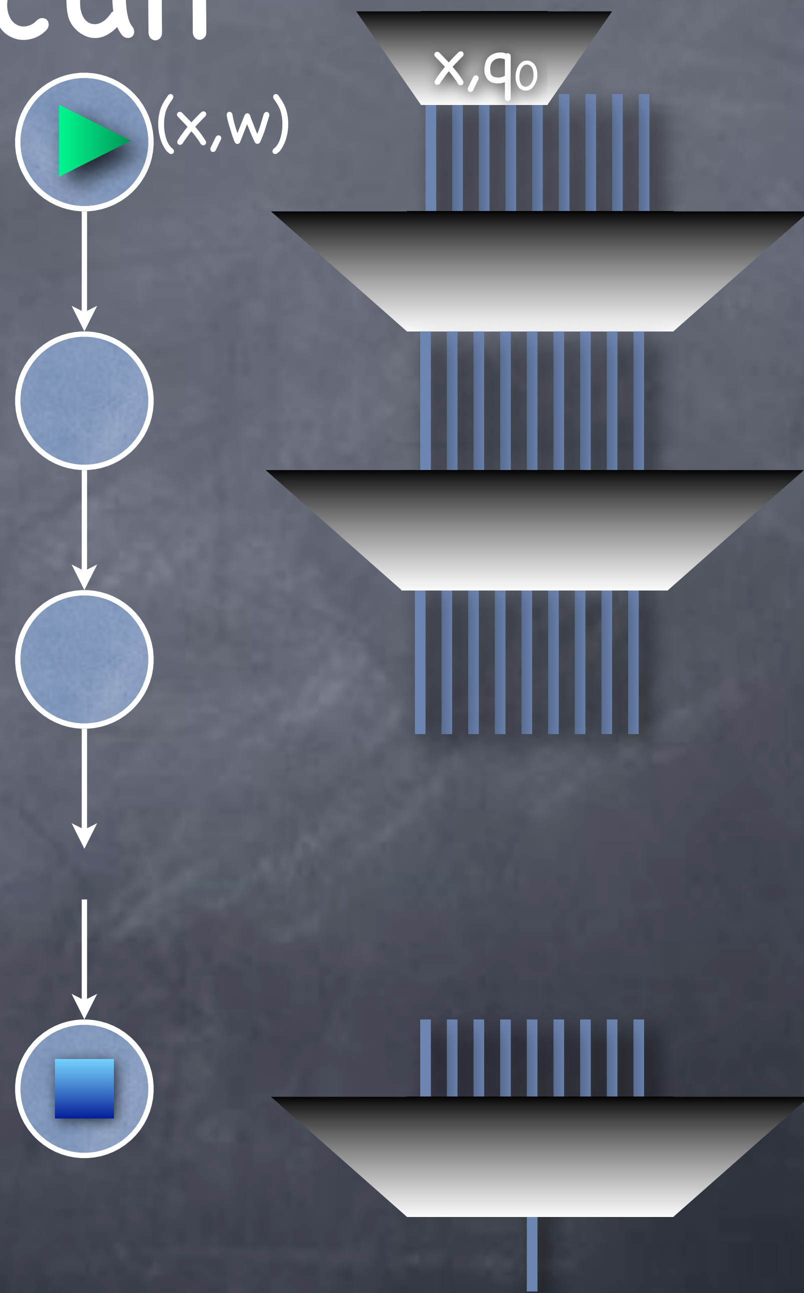

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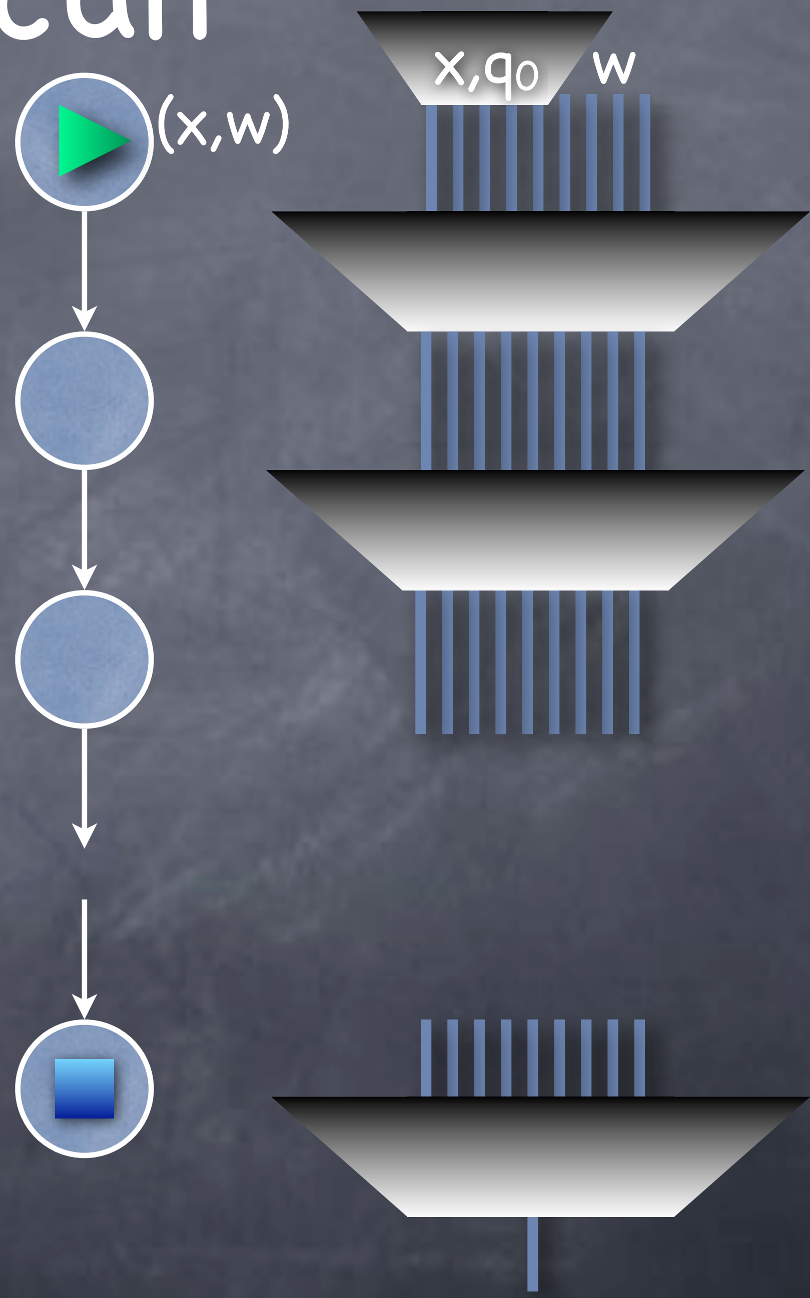

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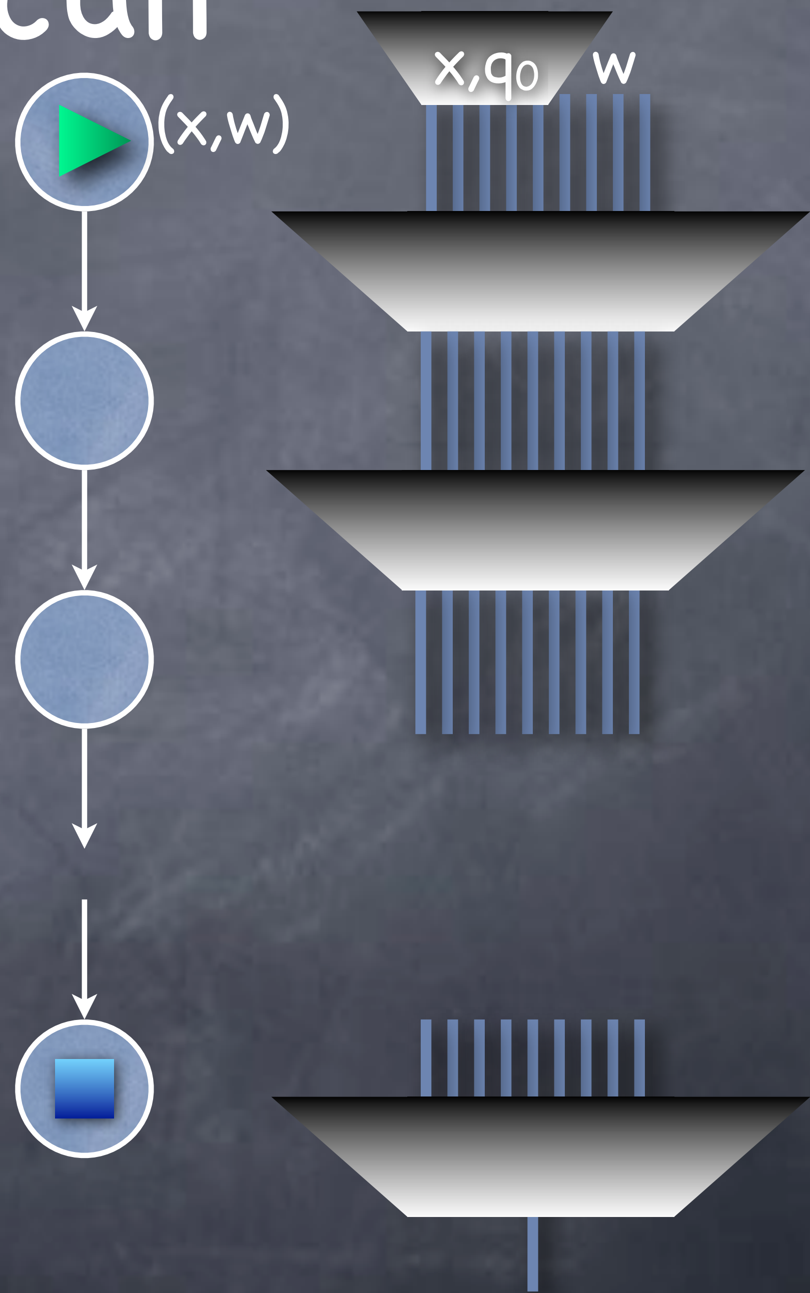

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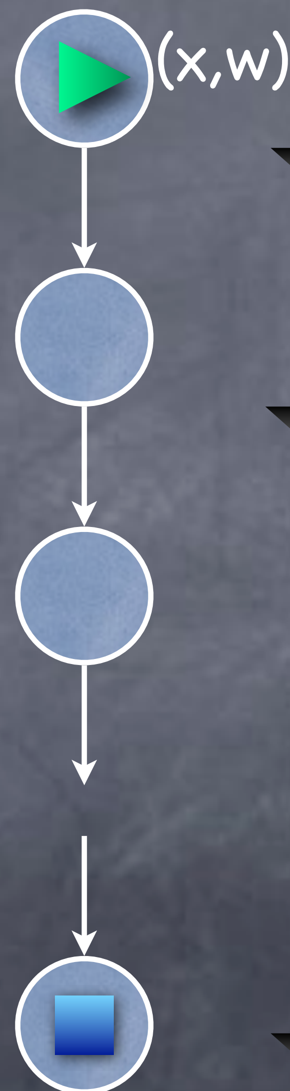
- Circuit size = $O(T^2)$



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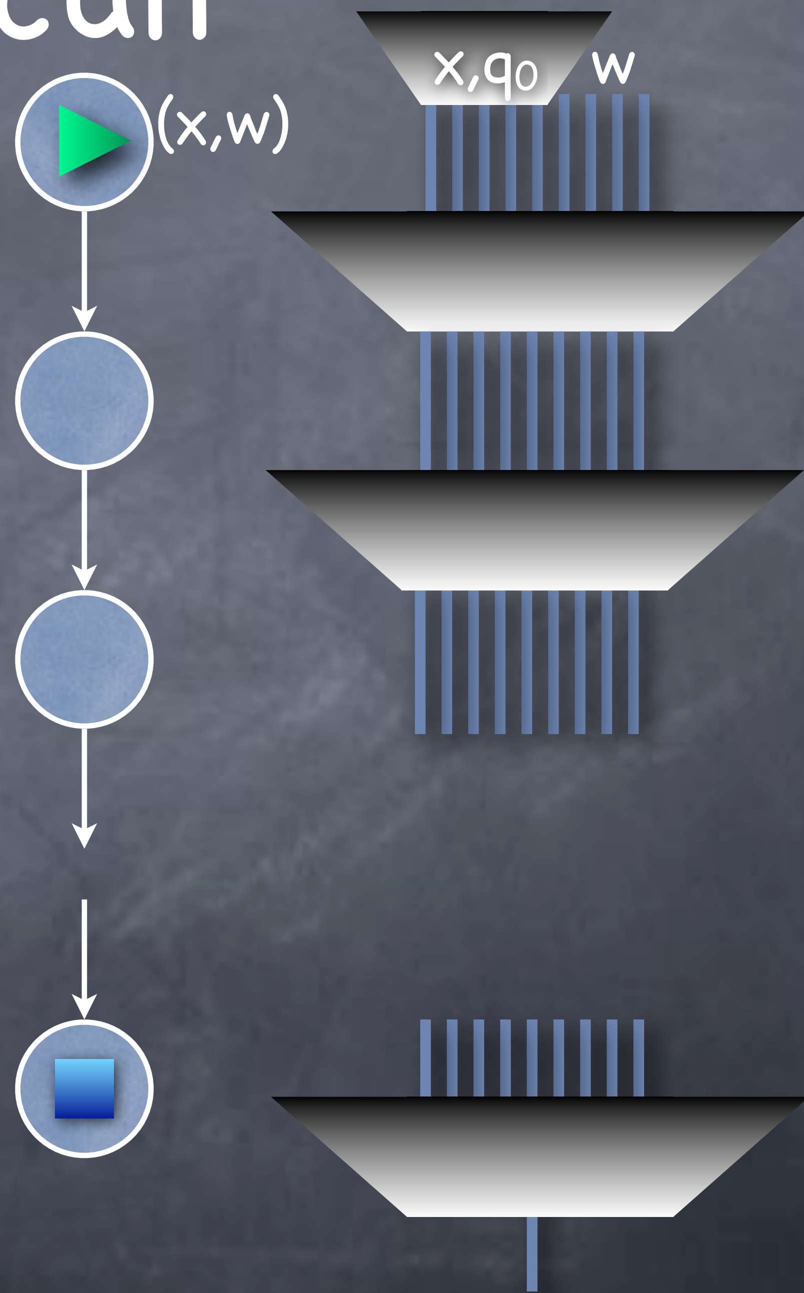

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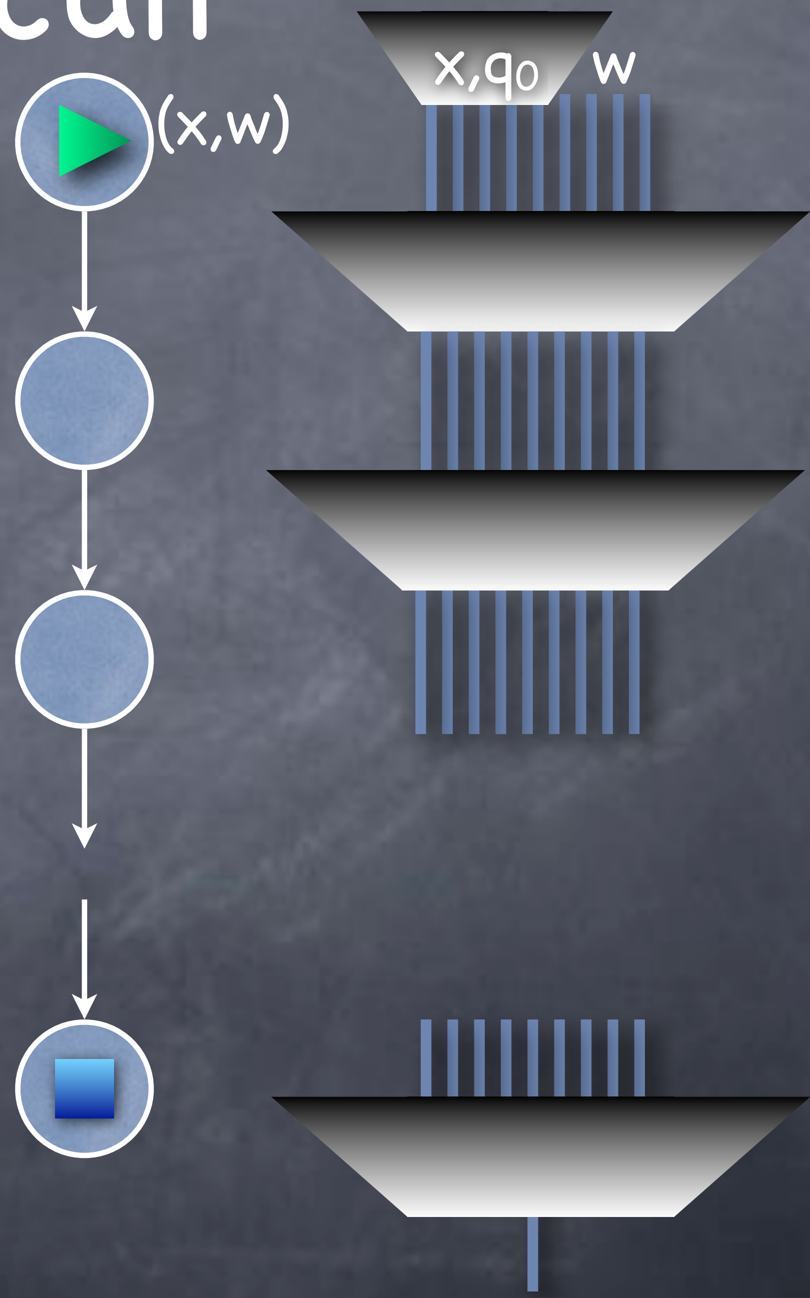

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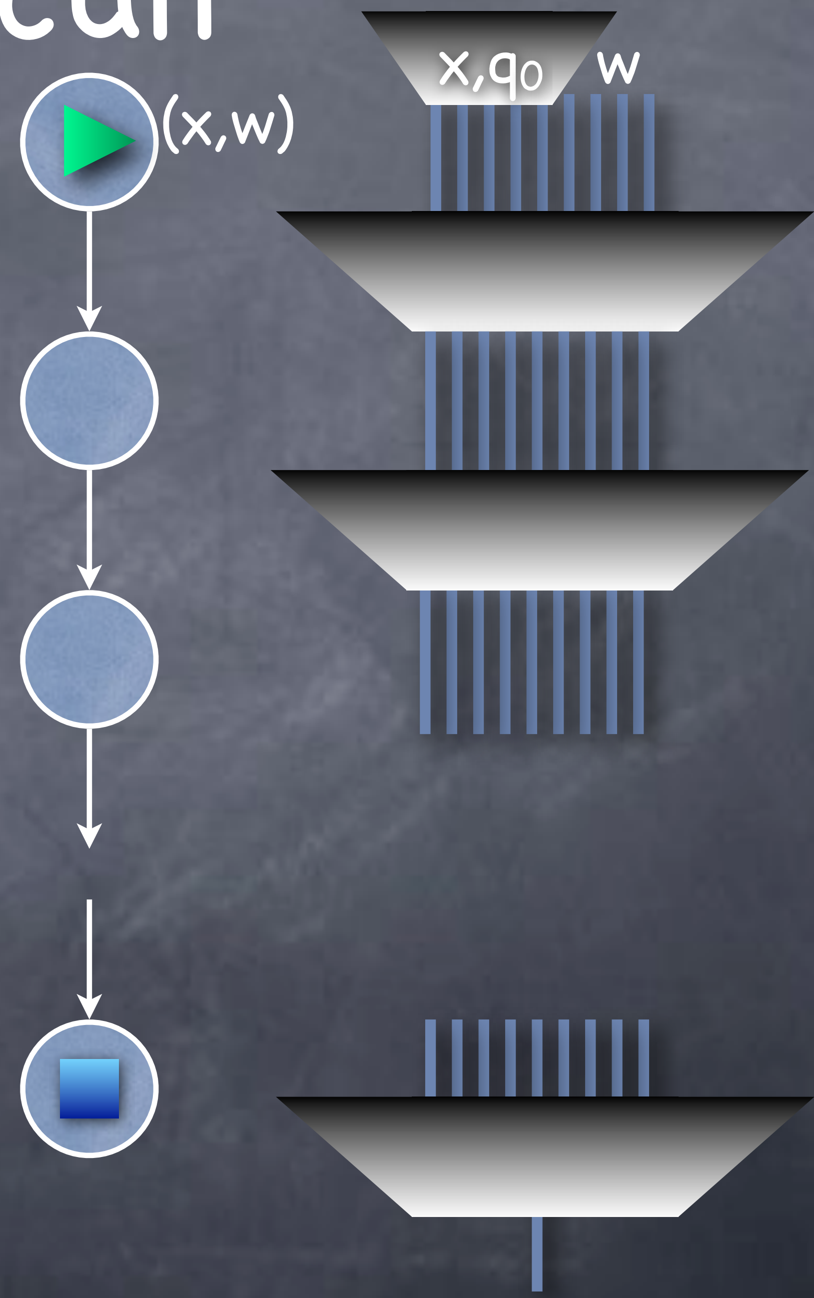

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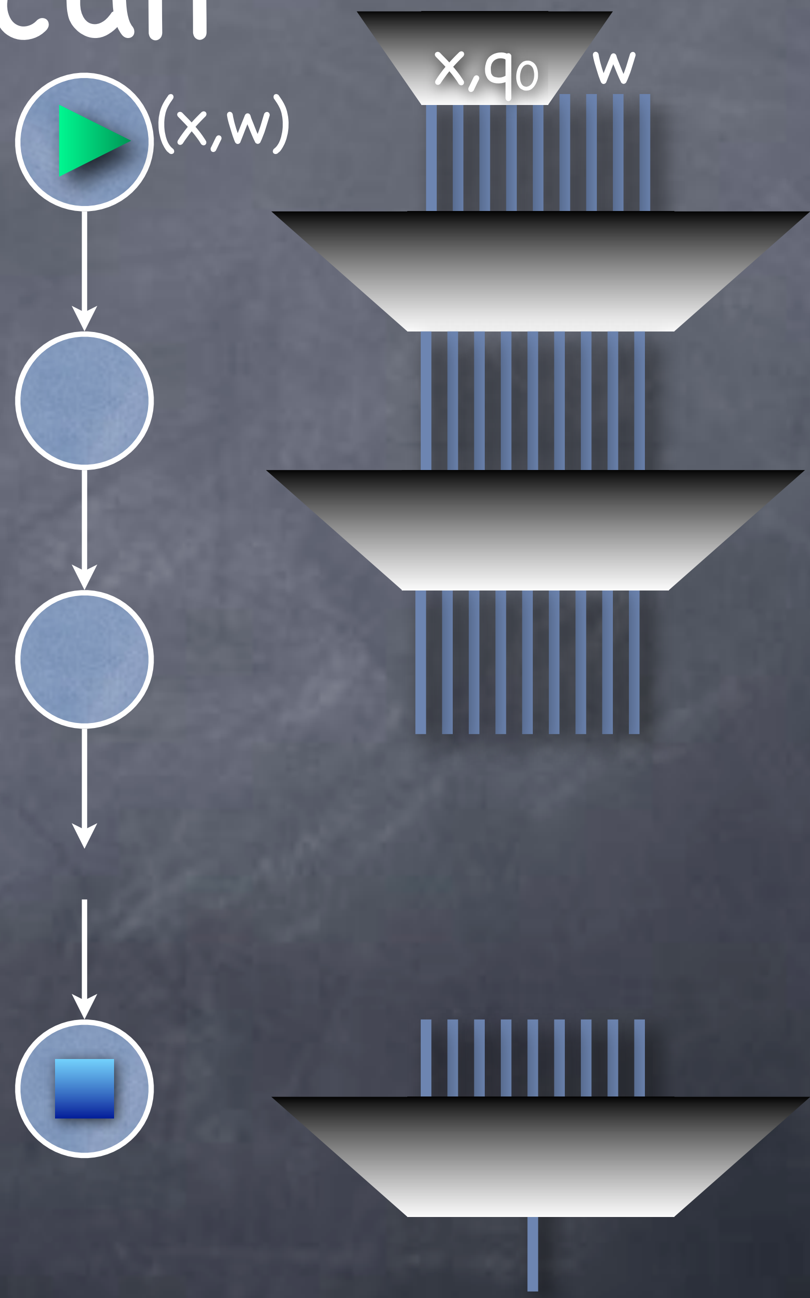

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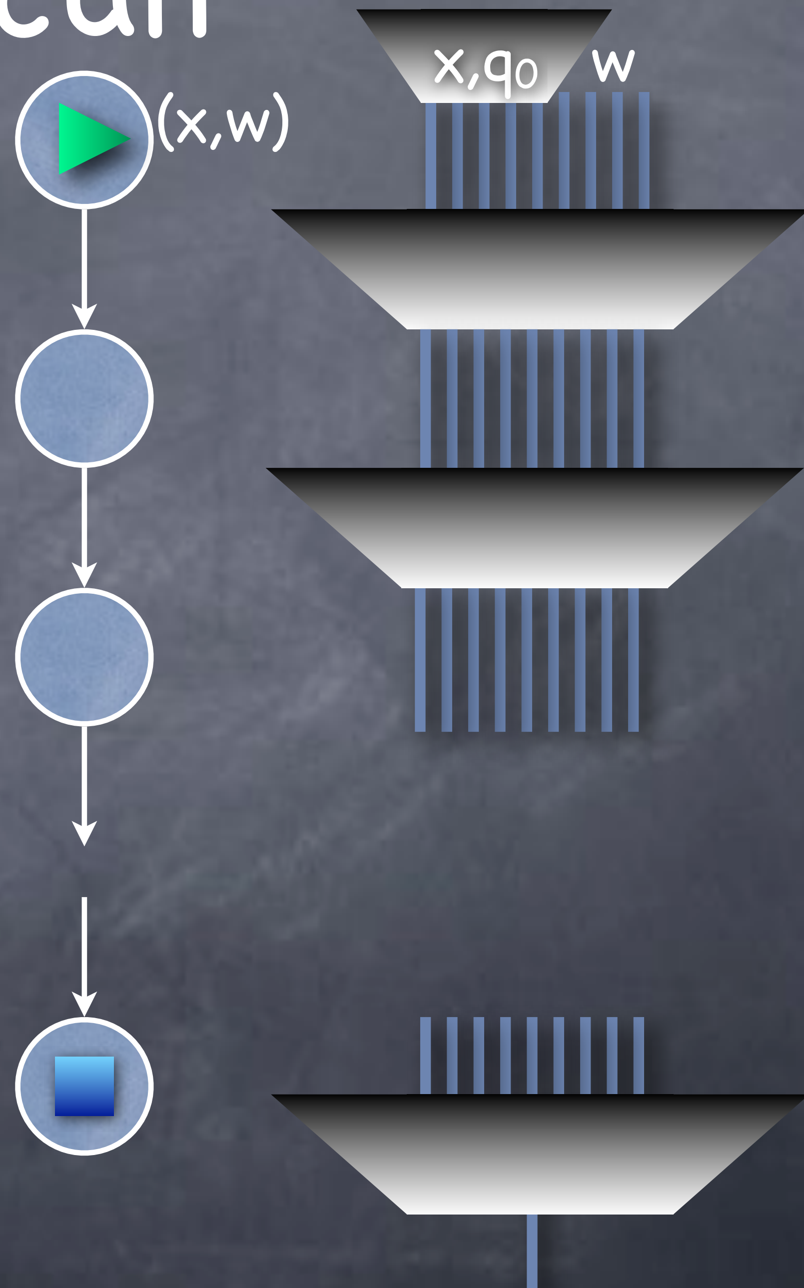
  - CKT-SAT is NP-complete



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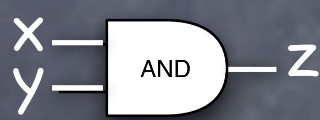
    - e.g. 

      : $(z \Rightarrow x)$, $(z \Rightarrow y)$, $(\neg z \Rightarrow \neg x \vee \neg y)$.

      i.e., $(\neg z \vee x)$, $(\neg z \vee y)$, $(z \vee \neg x \vee y)$.

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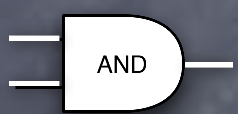
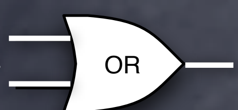
    - e.g. x, y 

 z: $(z \Rightarrow x)$, $(z \Rightarrow y)$, $(\neg z \Rightarrow \neg x \lor \neg y)$.

      i.e., $(\neg z \lor x)$, $(\neg z \lor y)$, $(z \lor \neg x \lor y)$.
    - and x, y 

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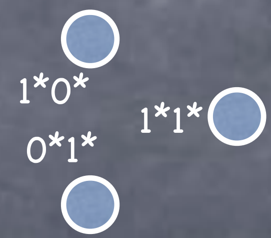

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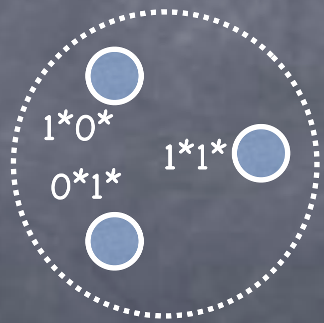

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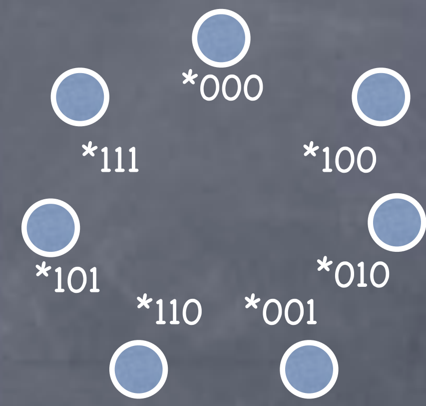

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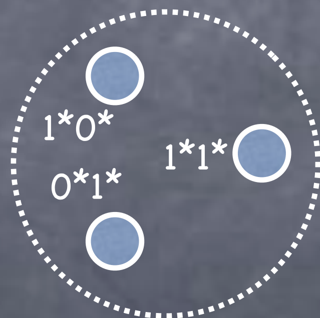

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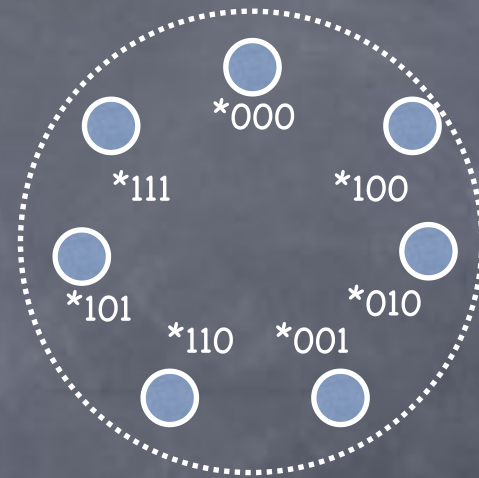

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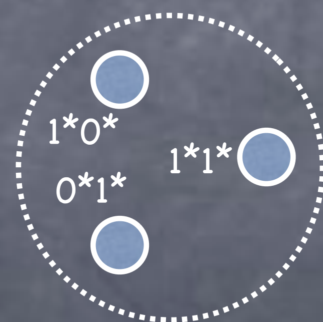

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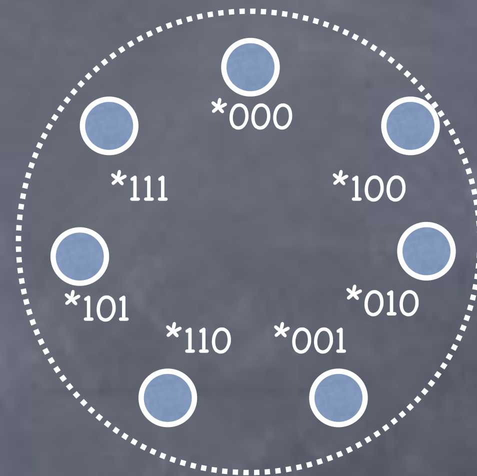

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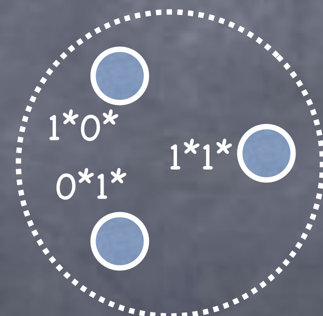

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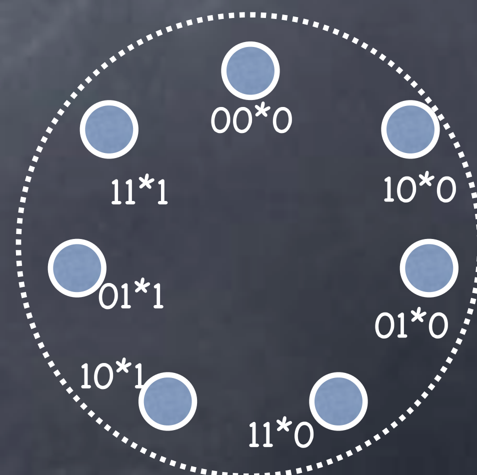

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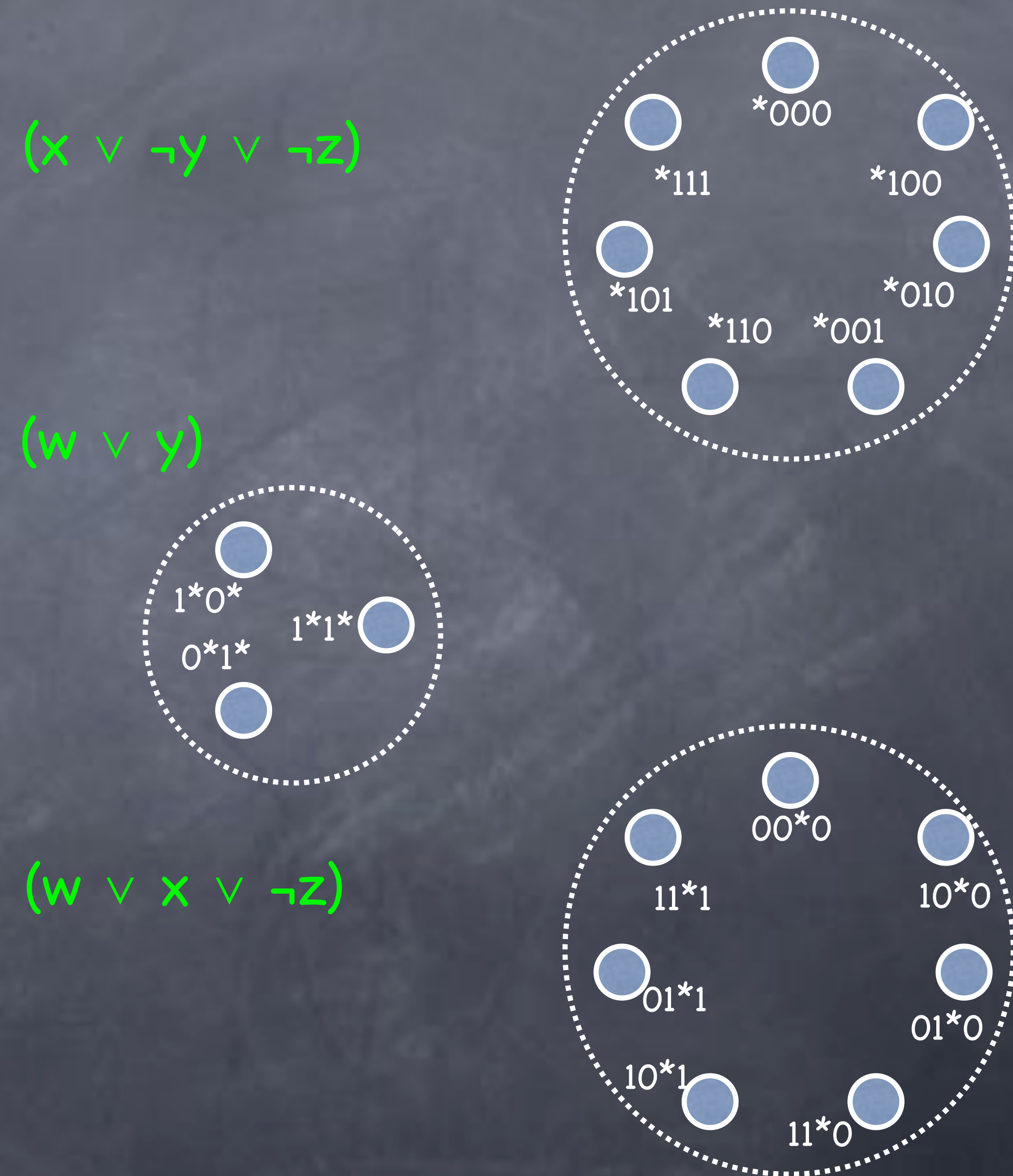

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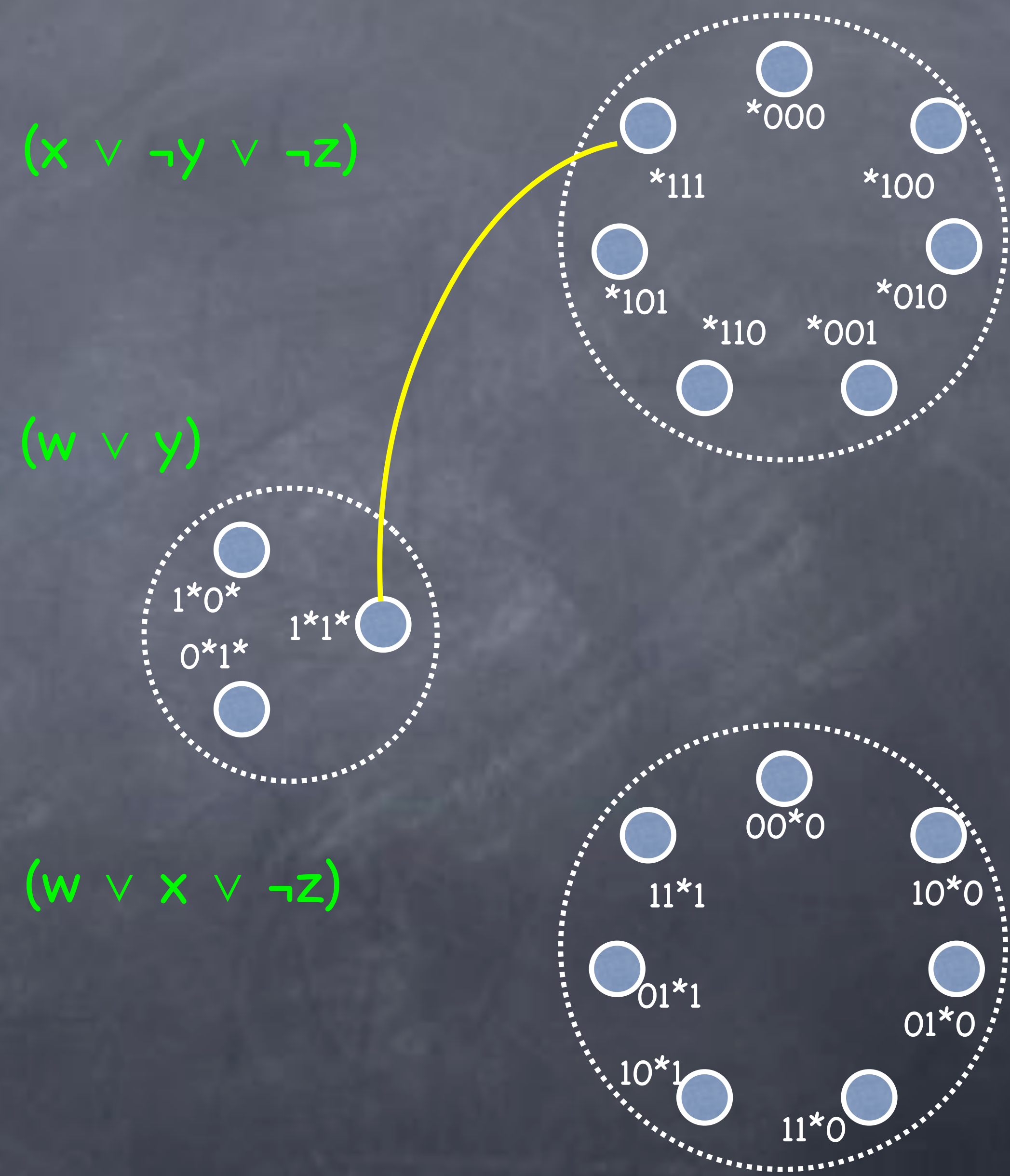

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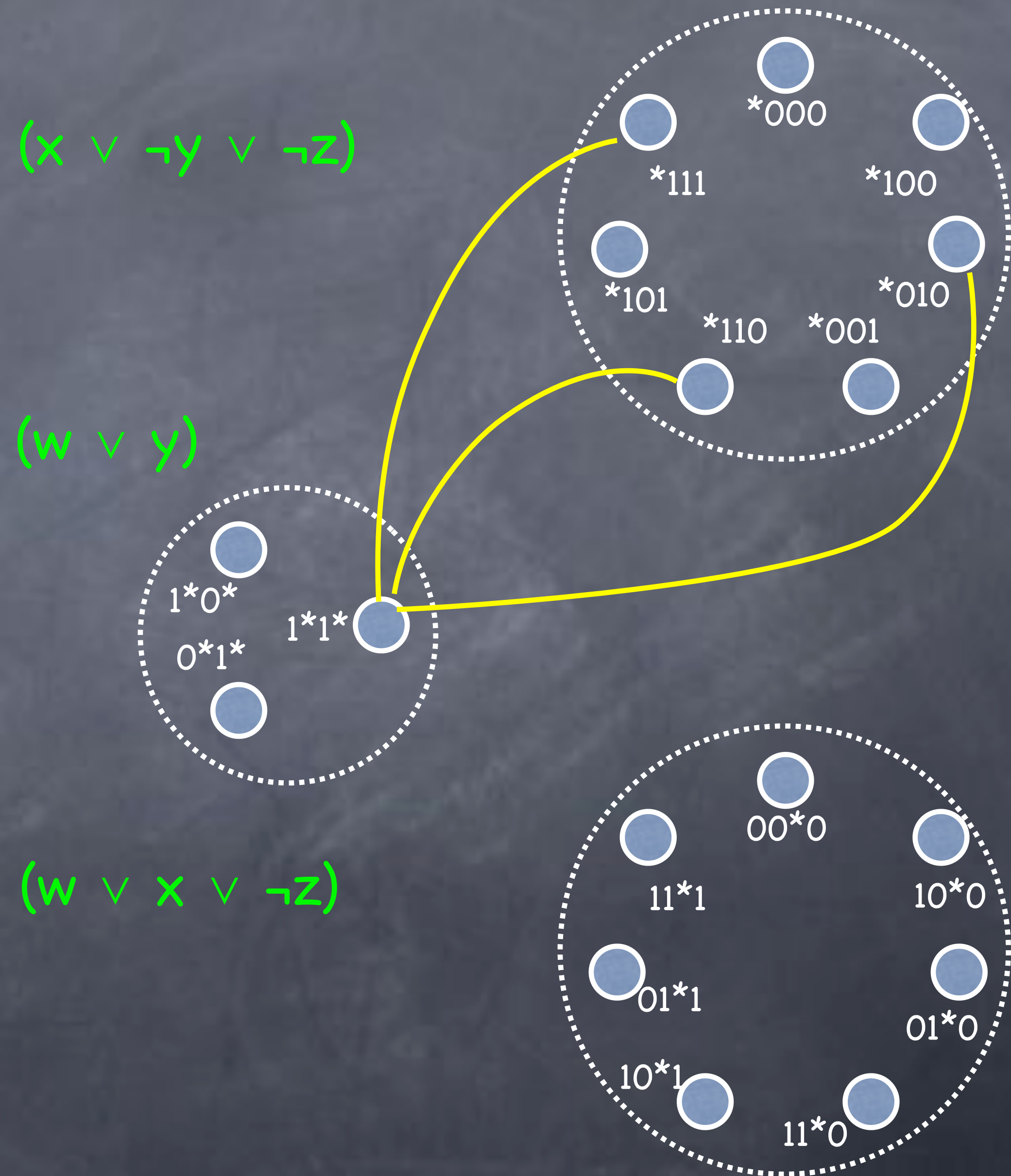

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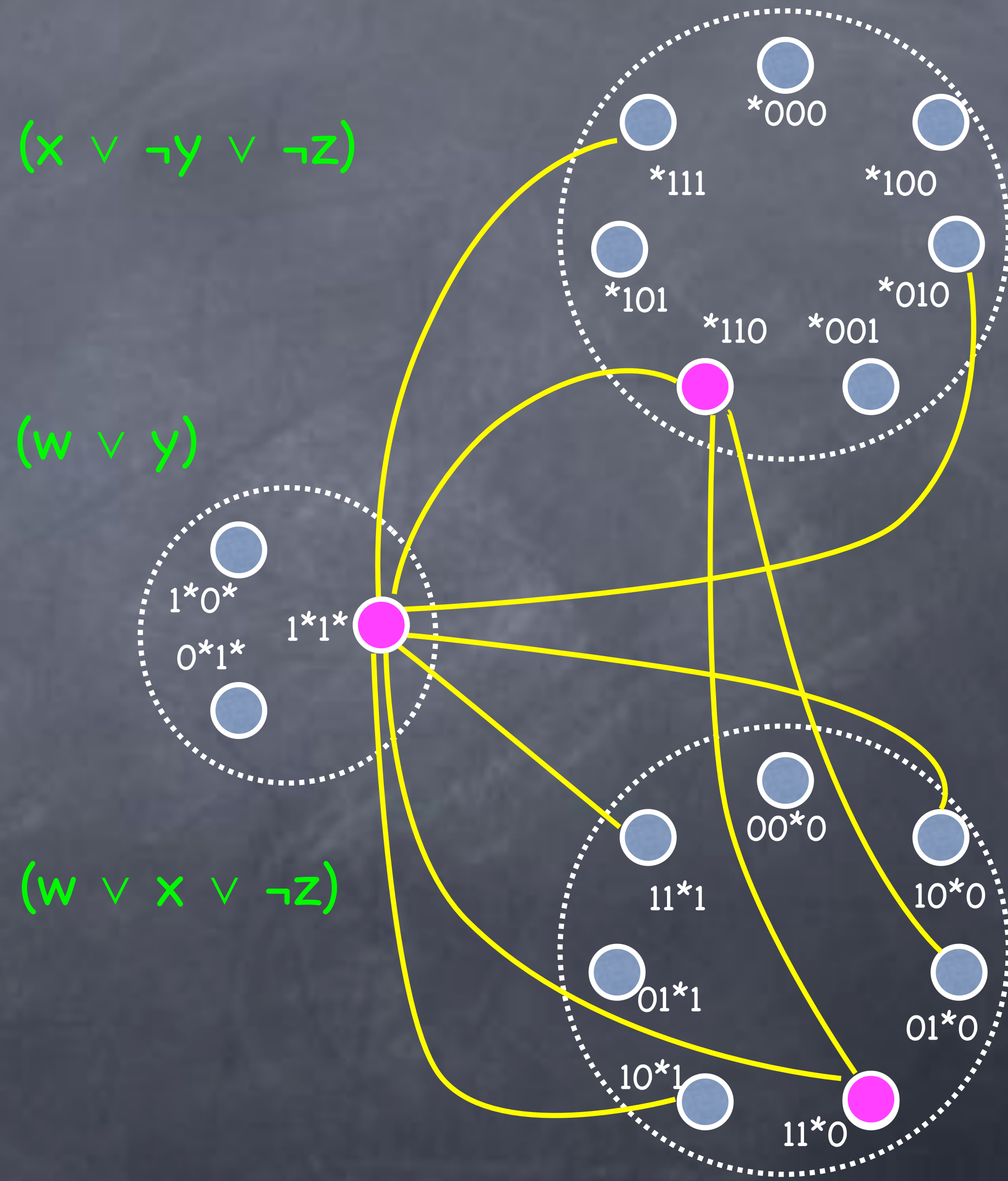

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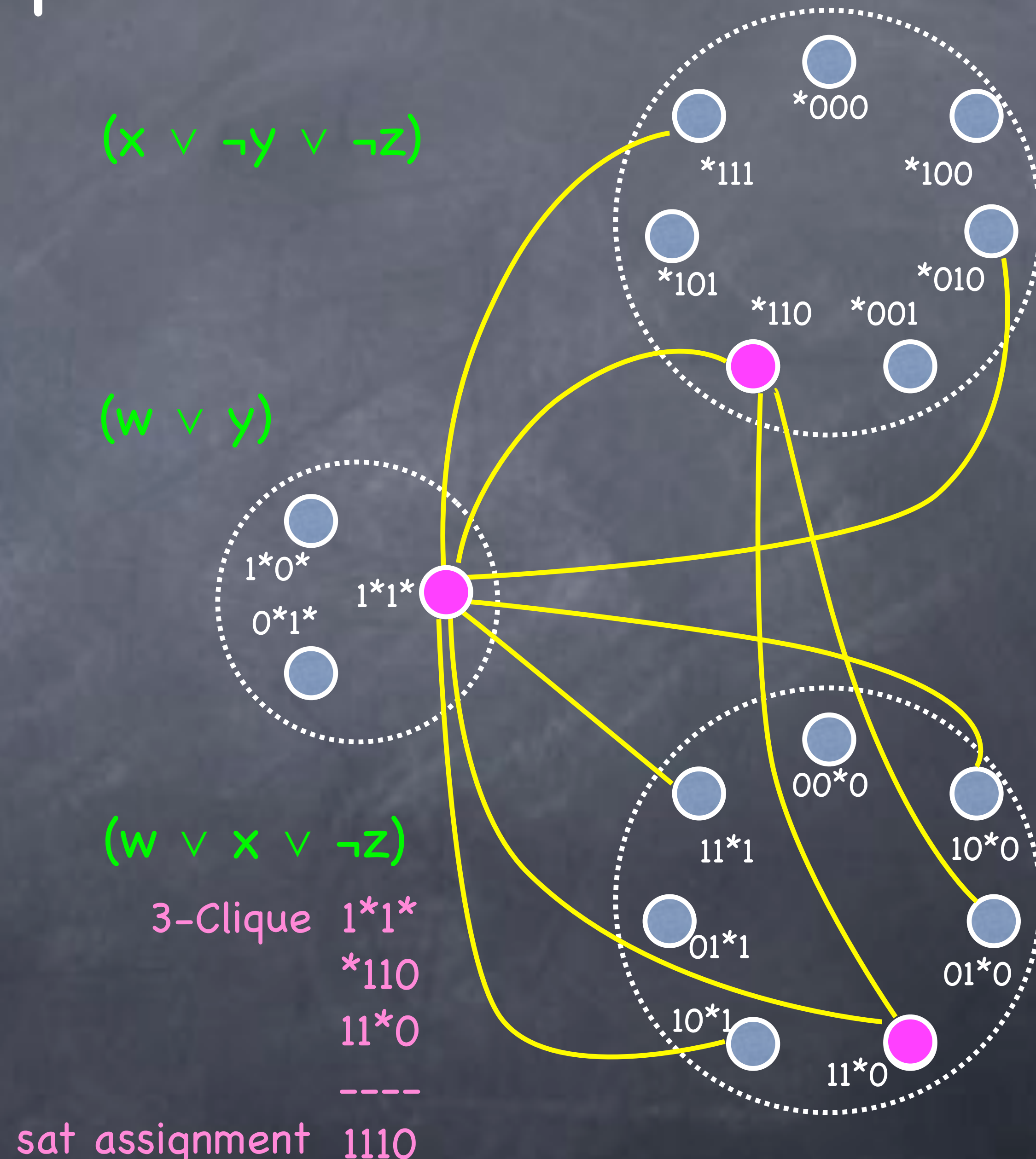

INDEP-SET and VERTEX-COVER

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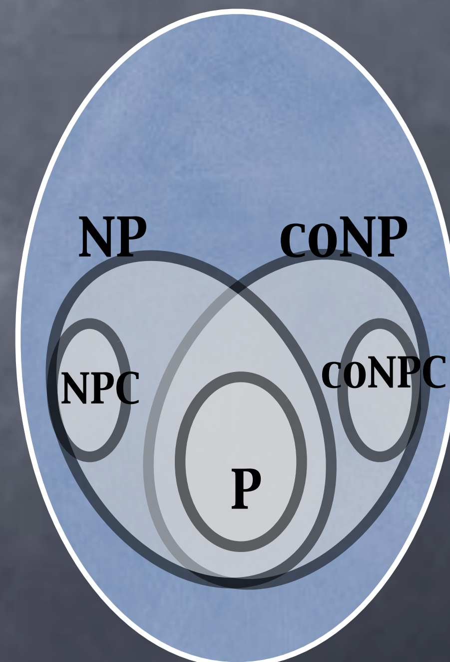
NP, P, co-NP and NPC



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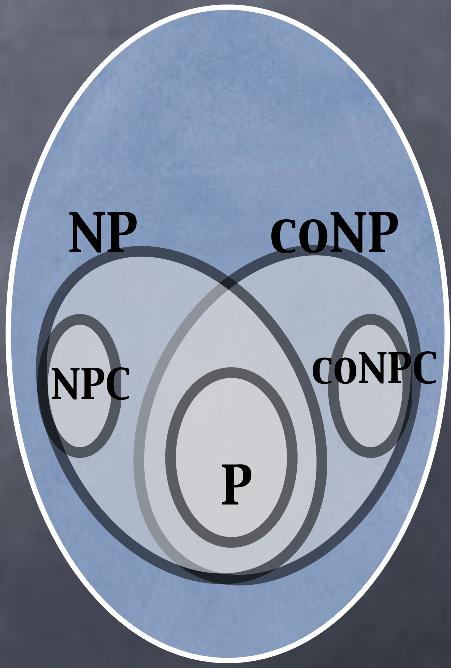

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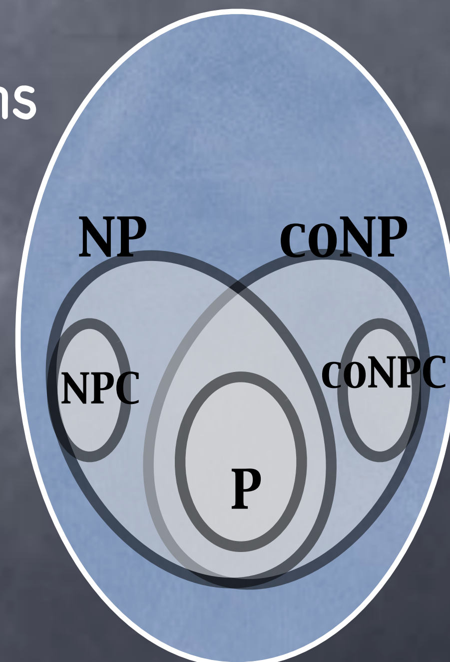

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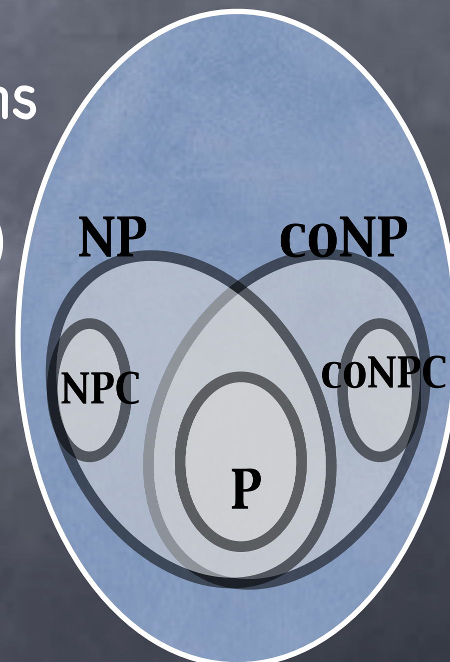

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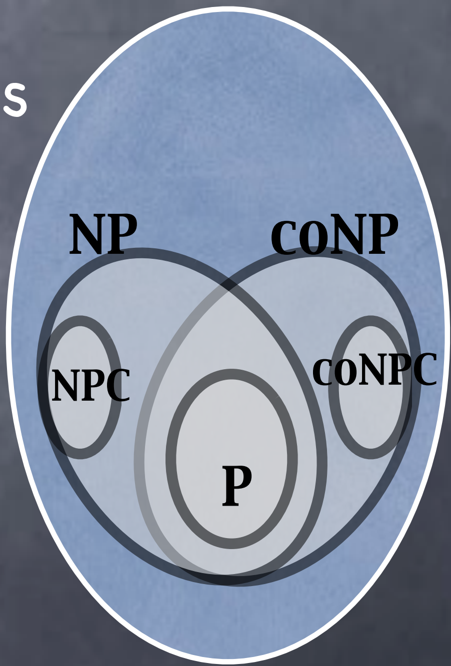

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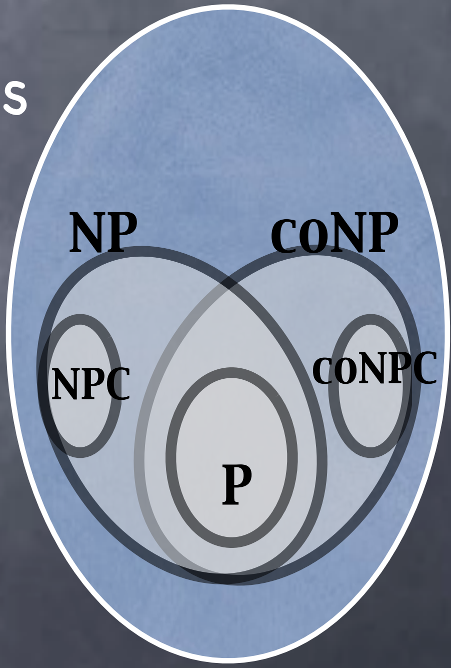

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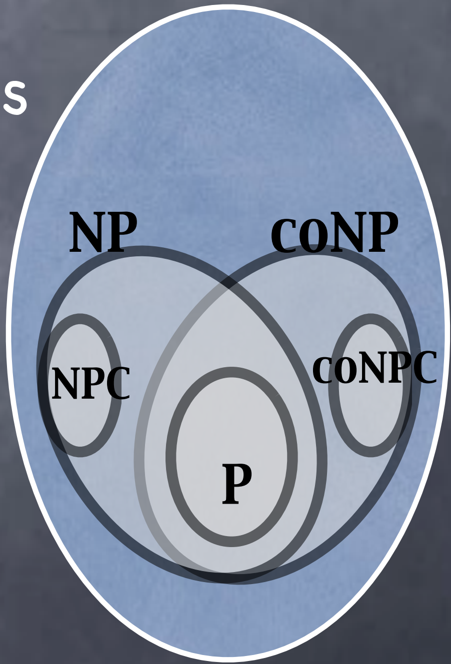
  - Note: if L in NPC, L^c is in co-NPC



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