# Computational Complexity

Lecture 1
in which we talk about
Time Complexity, P, NP and coNP

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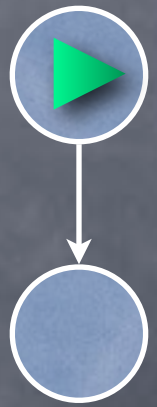

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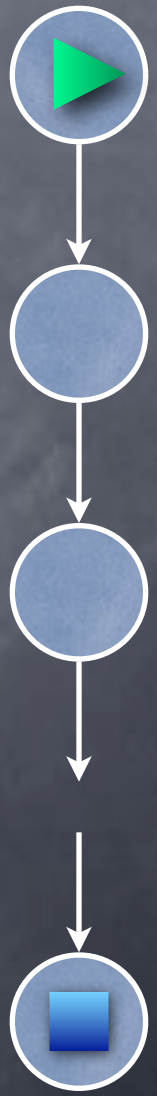

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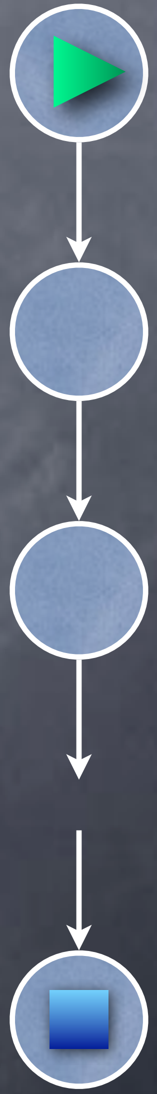
  - output explicitly encoded in the final configuration (say, in the control-state)



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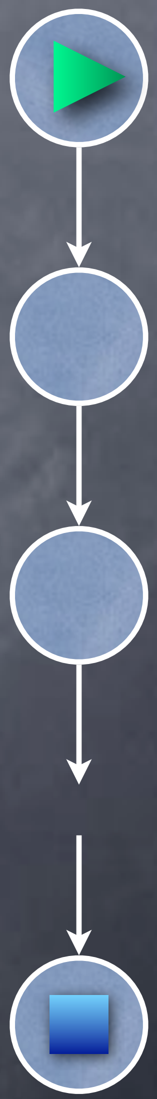

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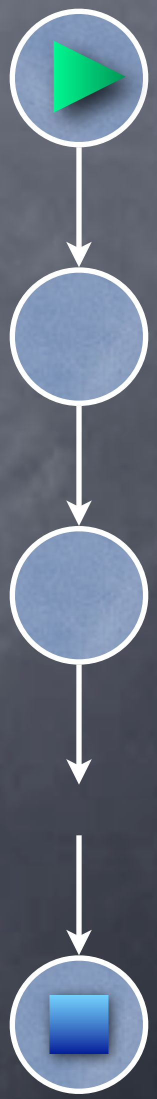

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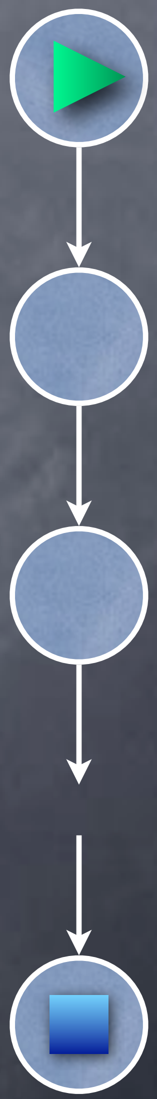

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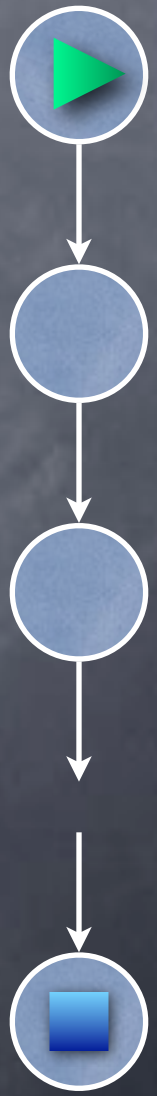

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  - (Note: complexity T is a function of n)

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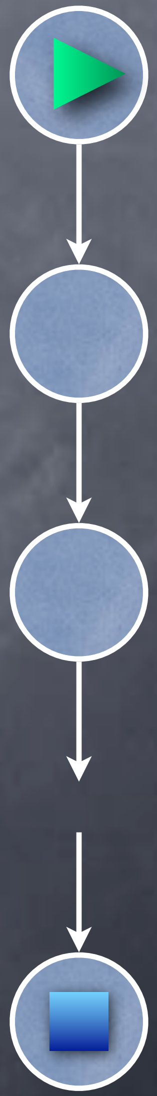

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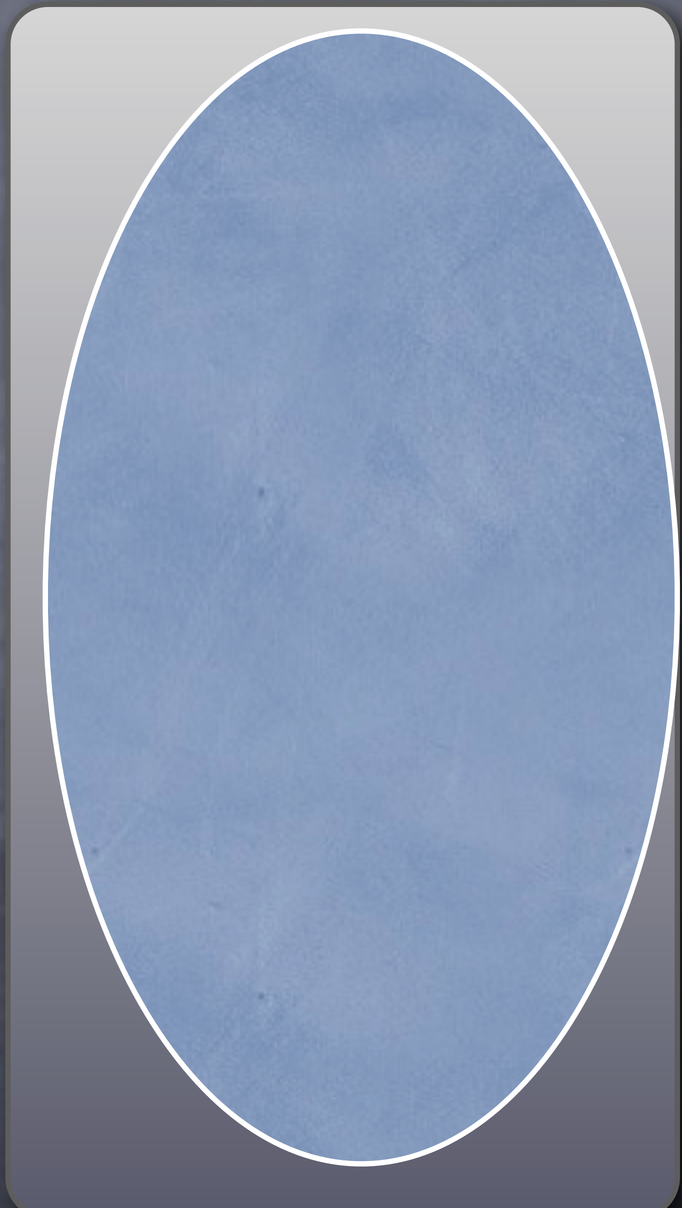

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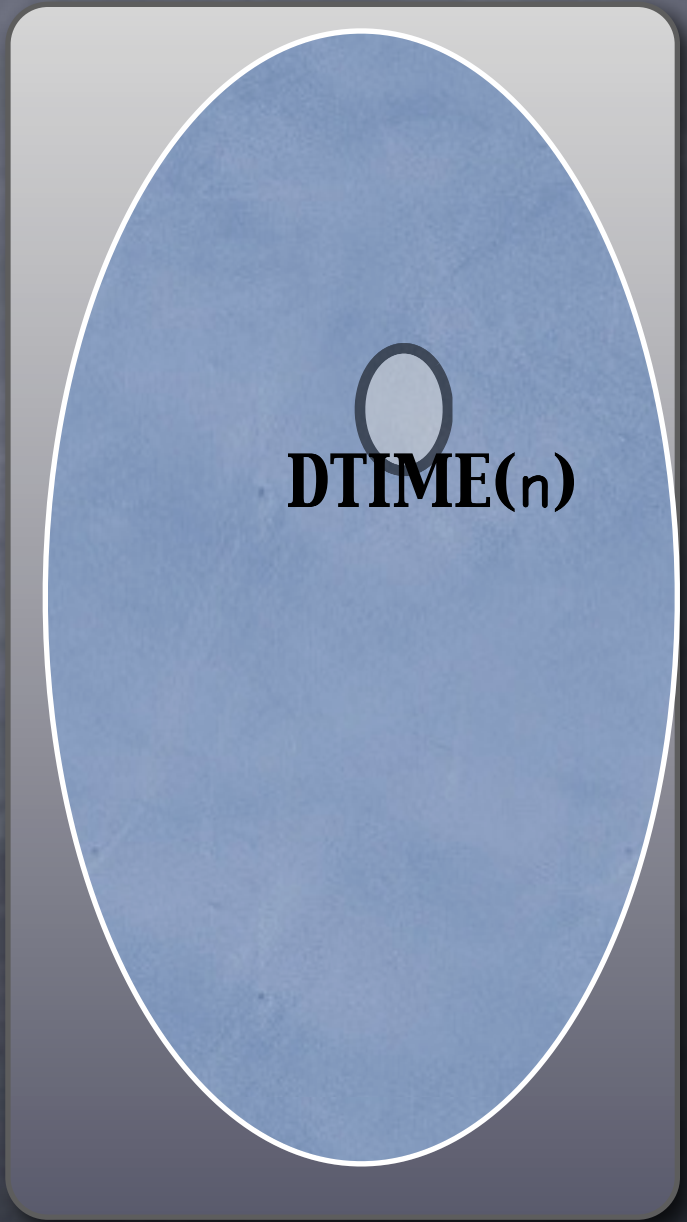

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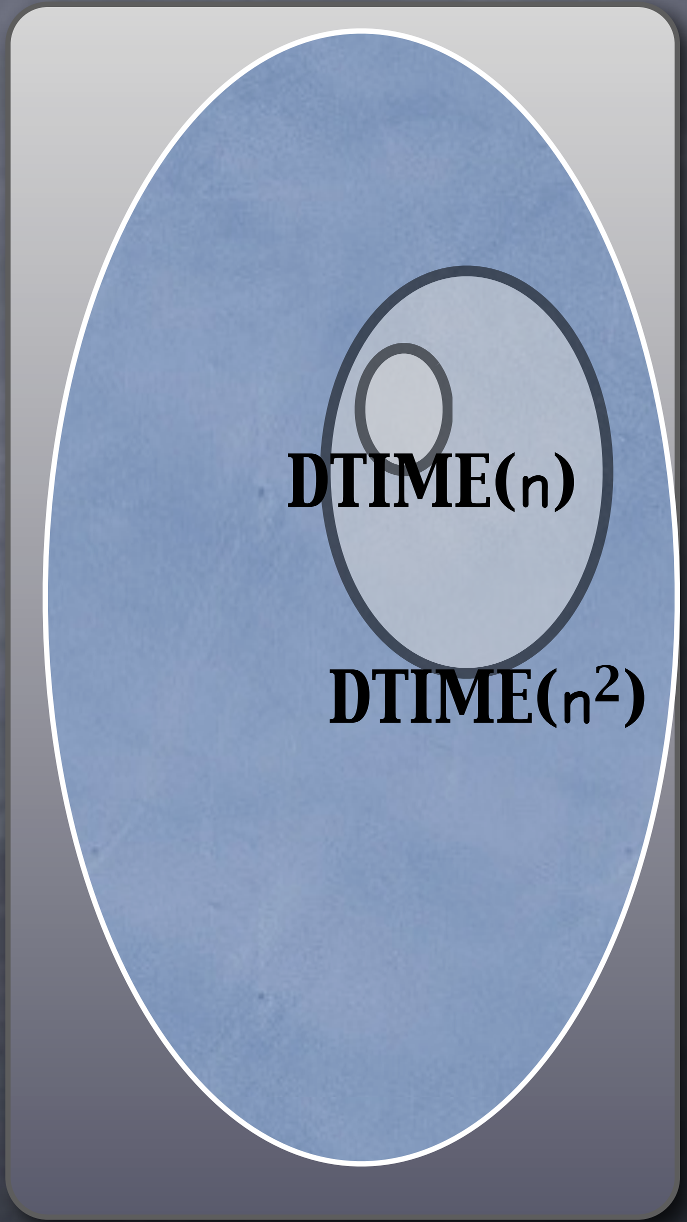

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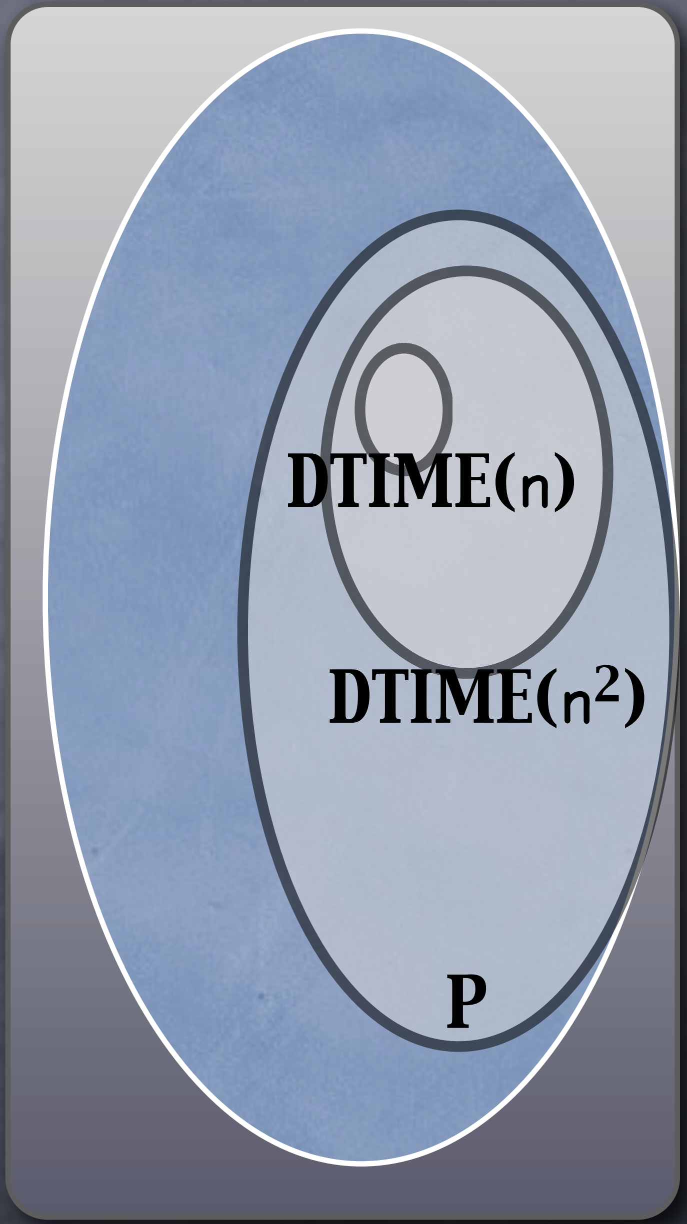

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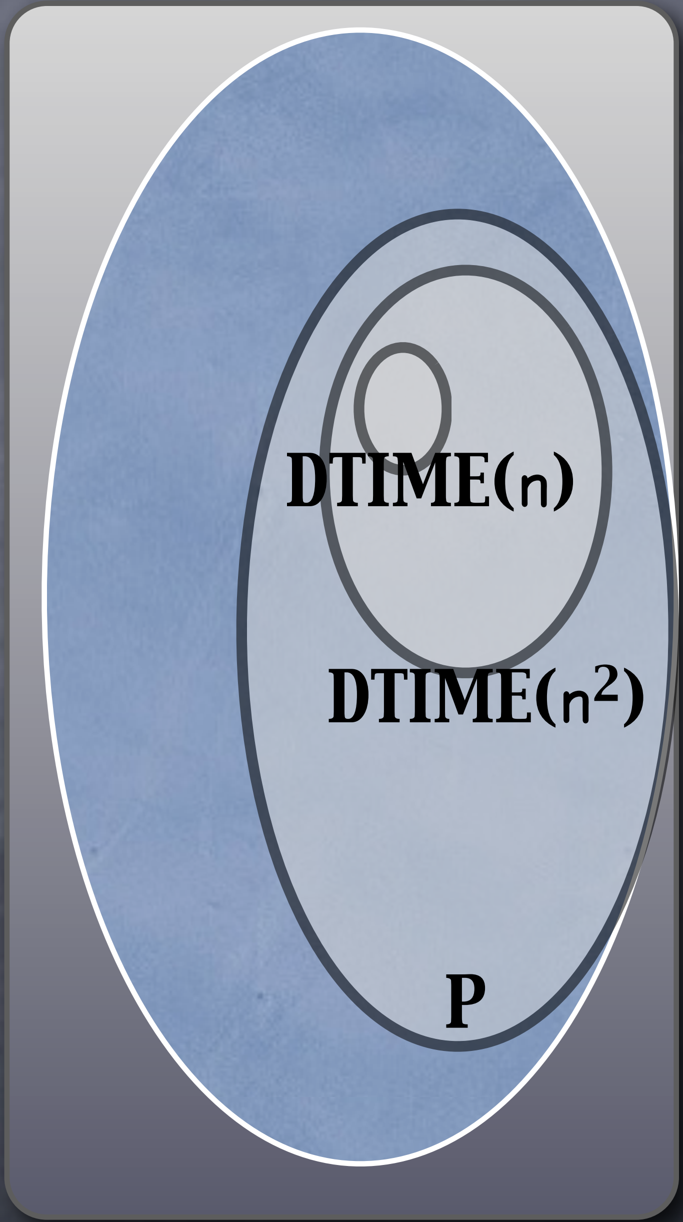

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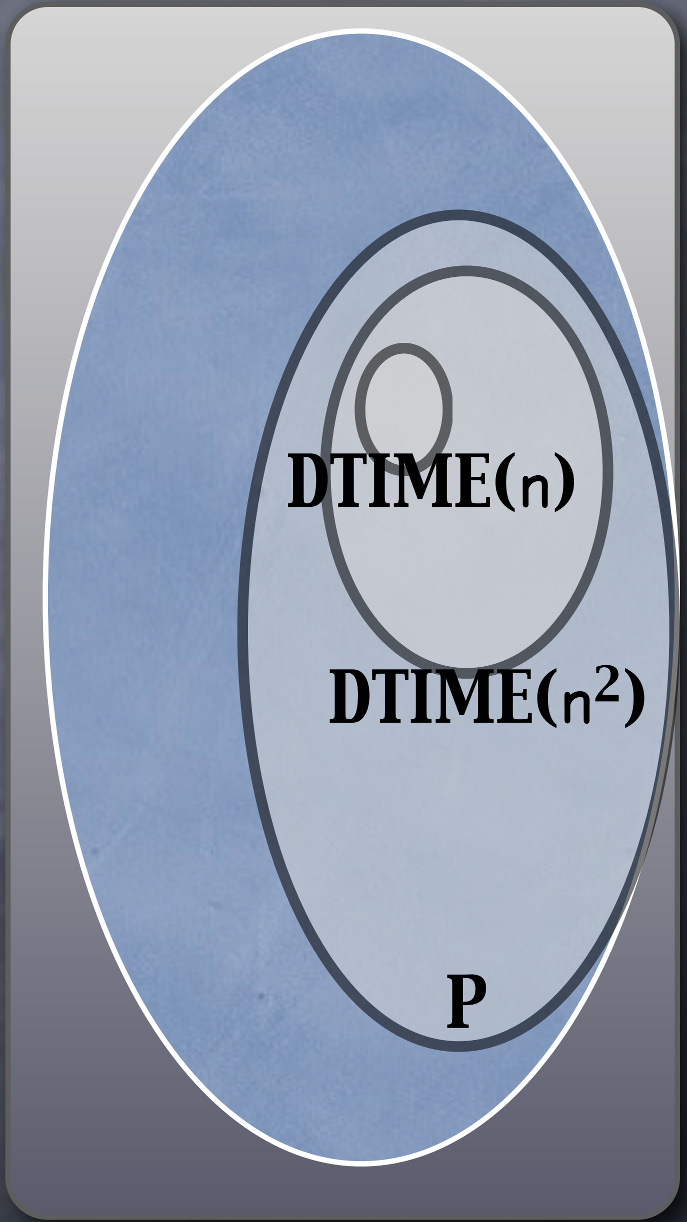
- DTIME(T) depends on the specifics of the TM model (no. of tapes, alphabet size)
- But P is robust: Models can simulate each other with only "polynomial slow down"



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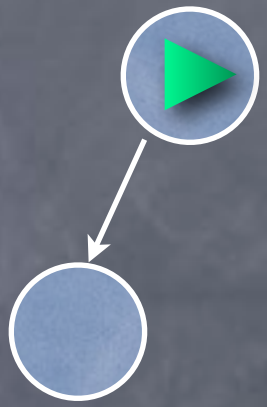

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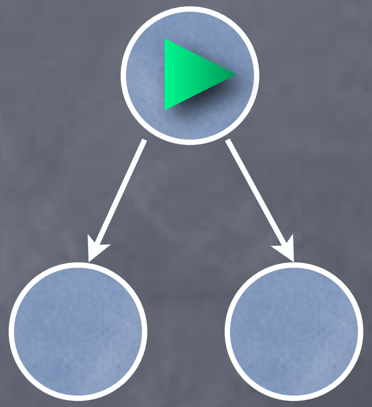

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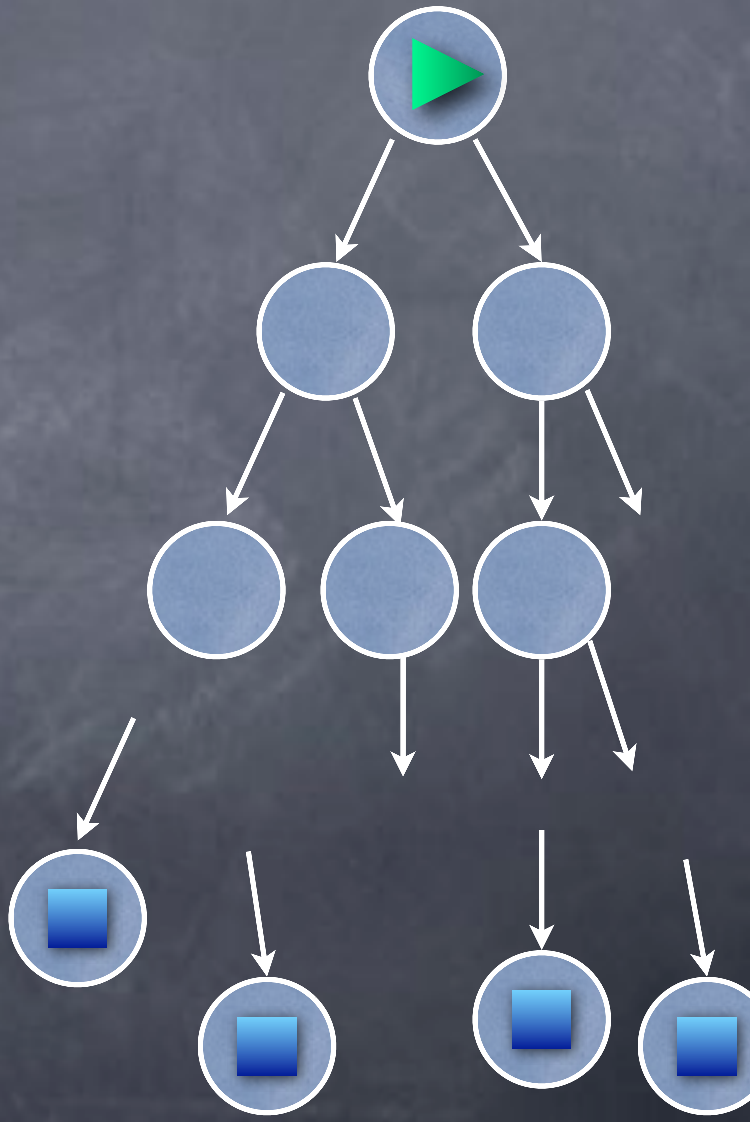

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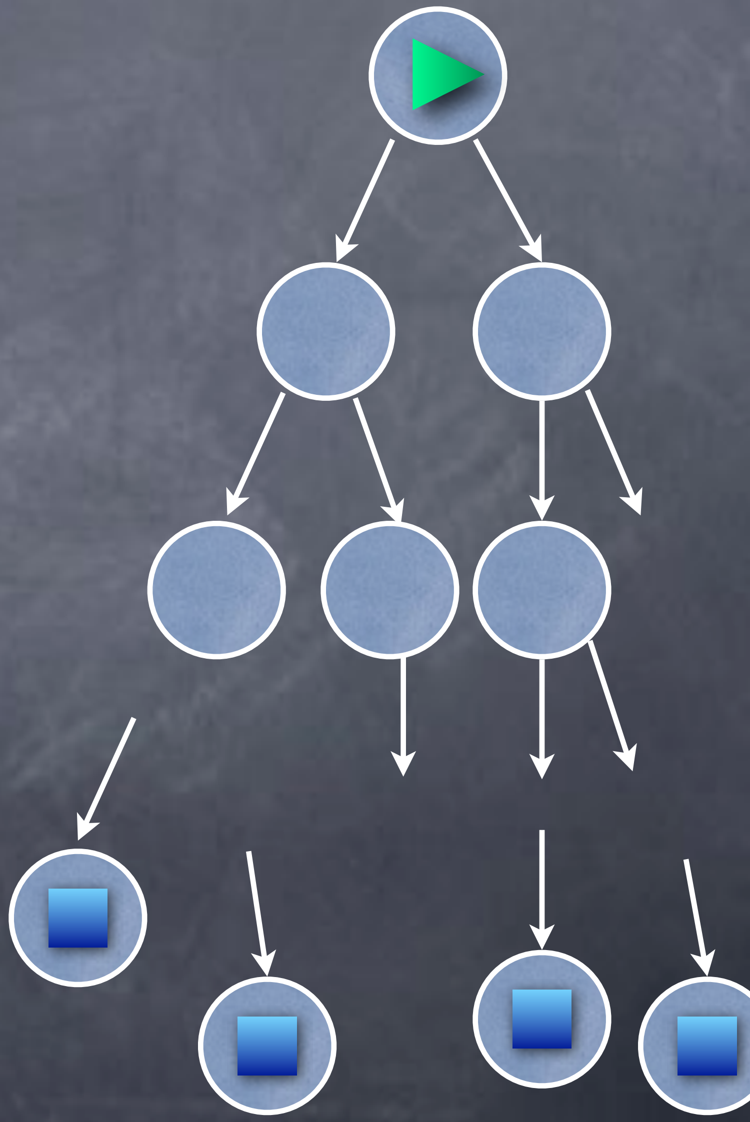

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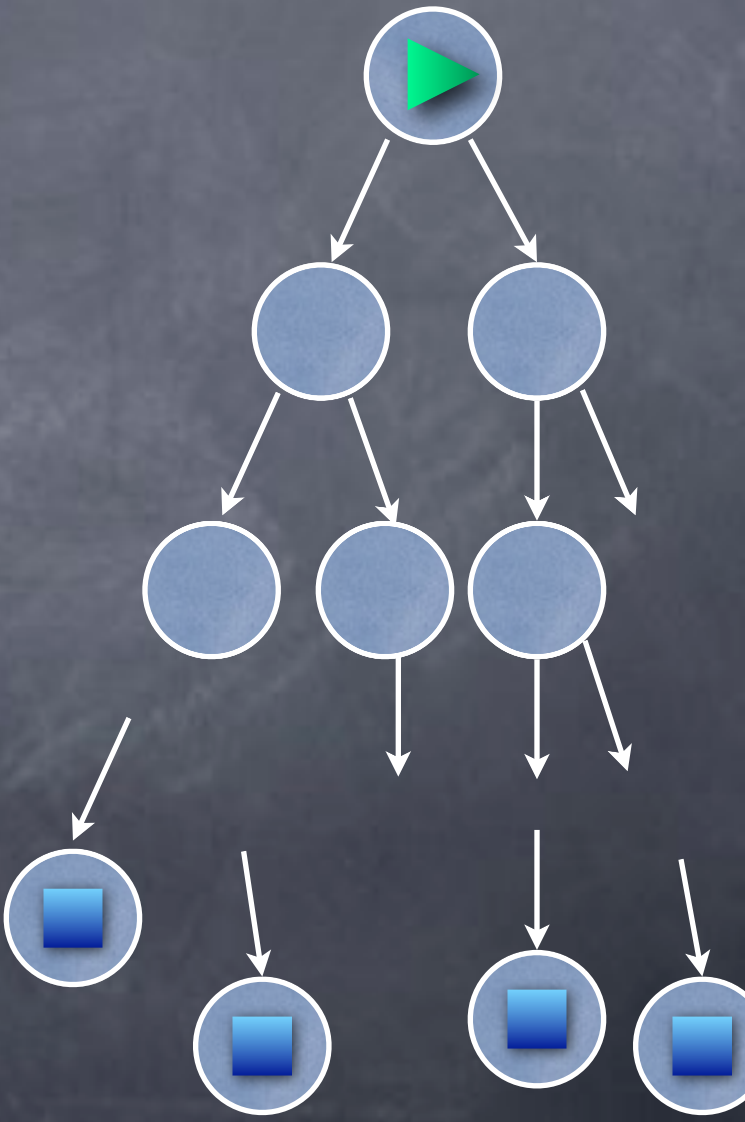

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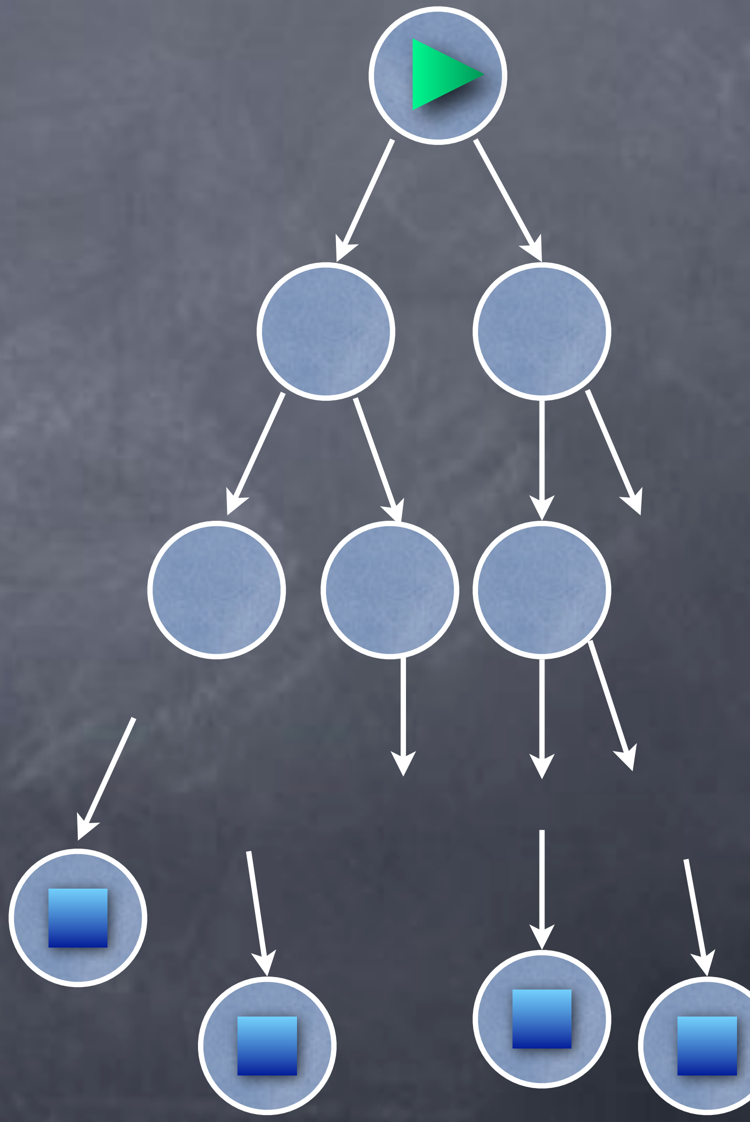
- L $\in$ NTIME(T): an NTM decides L in time at most T



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  - Finding a certificate (or even finding if there exists a certificate) may take longer

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- Note: Completeness and soundness

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- All problems in P (empty certificate)

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  - For next bit query $L_1$-oracle with (x,00) and (x,01)

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- Solve all sorts of optimization problems efficiently!

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      - No! L' in DTIME($2^{poly(|x|)}$)

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    - $L = \{x \mid \exists w, |w| = O(2^{poly(|x|)}) \text{ s.t. } (x,w) \in L'\}$, and L' in EXP?
      - No! L' in $DTIME(2^{poly(|x|)})$
        - i.e., L' in P

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no counter-example

  - flip accept/reject states and flip "there exists" and "for all" in the acceptance criterion (NTM $\leftrightarrow$ "co-NTM")
  - $L^c = \{ x \mid \nexists w \text{ s.t. } (x,w) \in L' \} = \{ x \mid \forall w\ (x,w) \in L'^c \}$

# P, NP and co-NP

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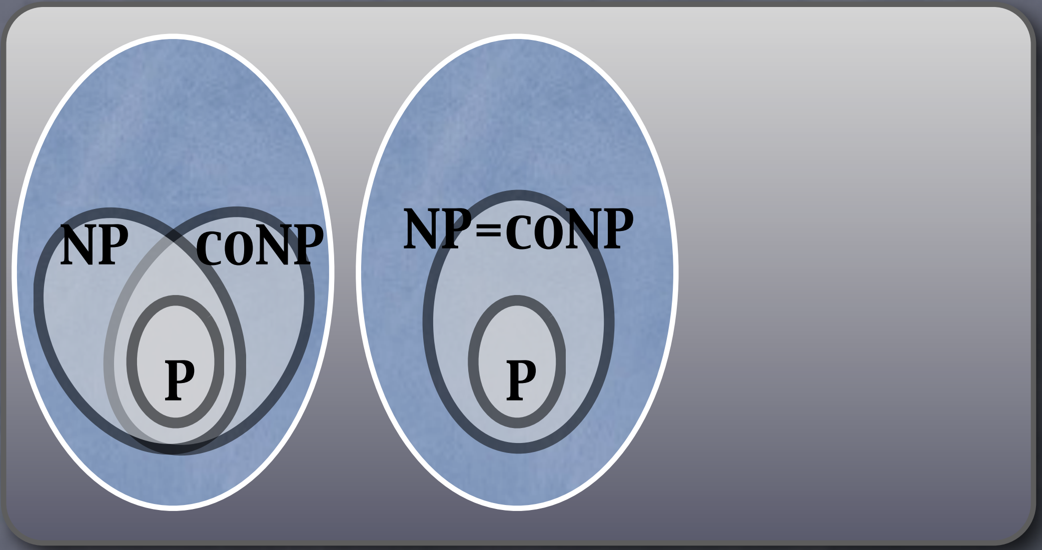

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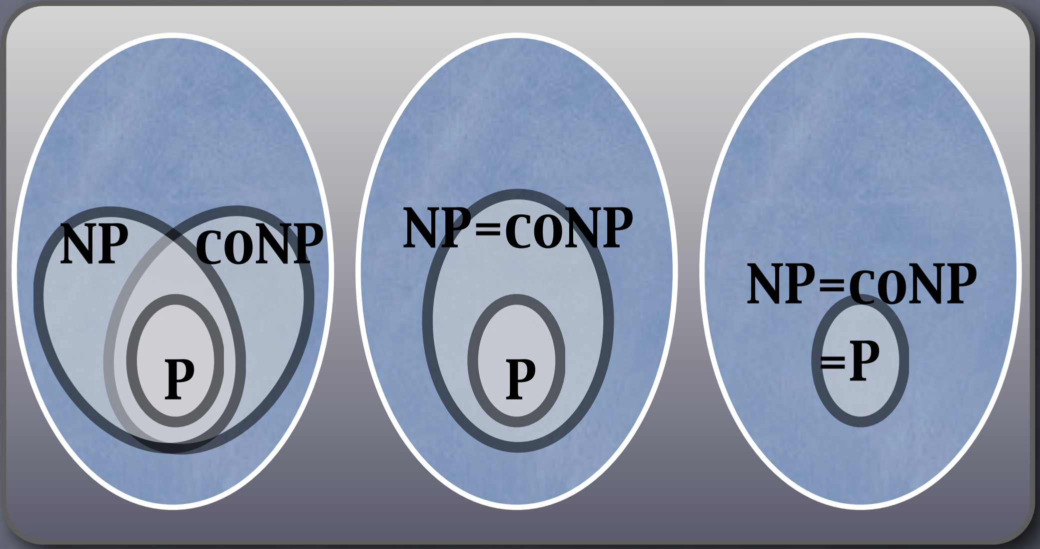
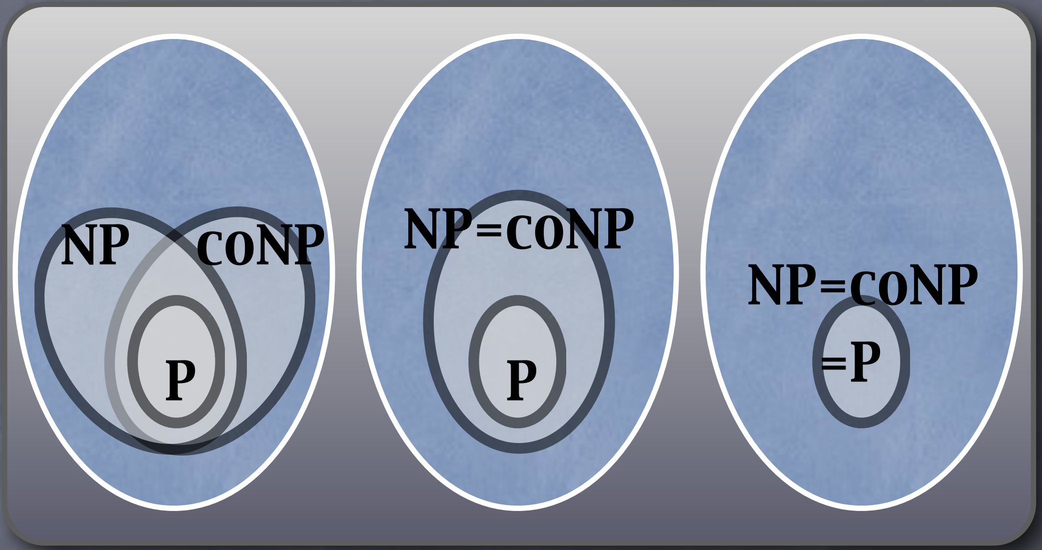


# P, NP and co-NP

- Different possibilities
- If P=NP, then



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- Different possibilities
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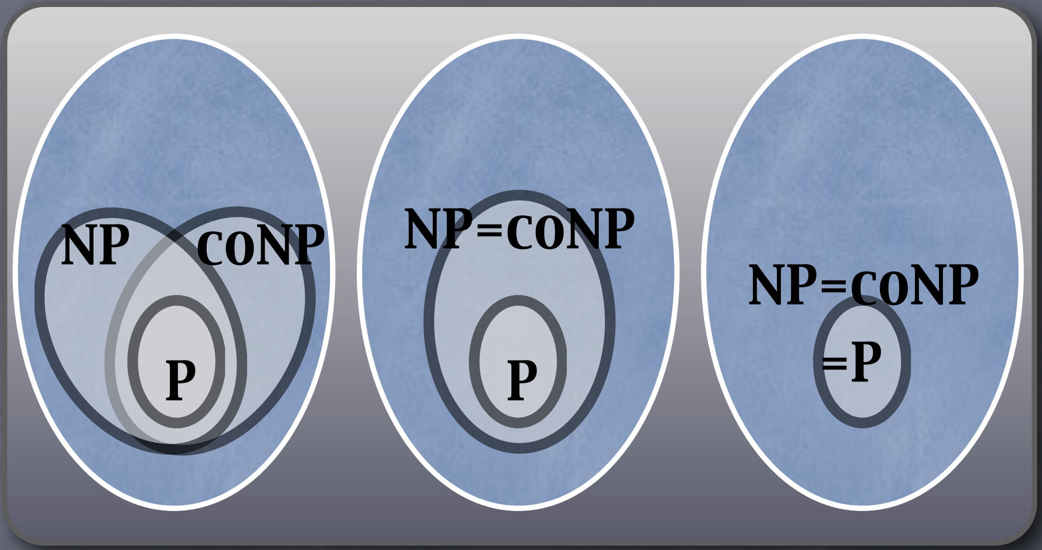

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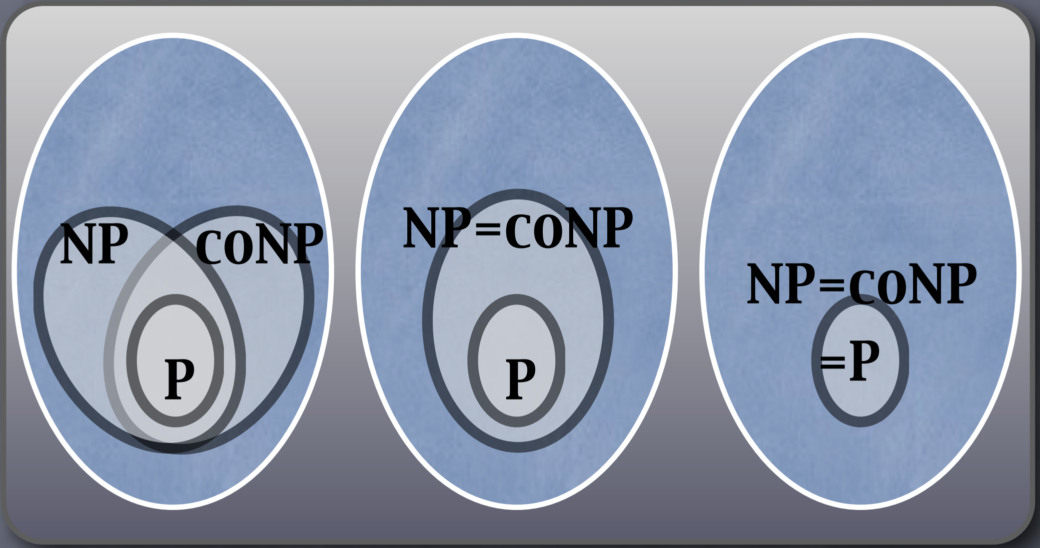


- Different possibilities
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  - Also, EXP = NEXP [Exercise]

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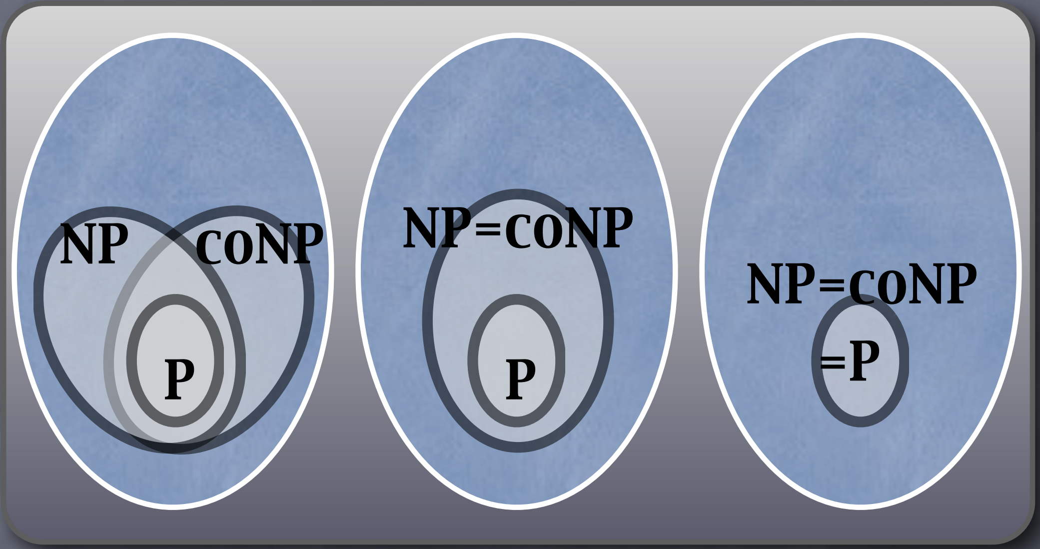


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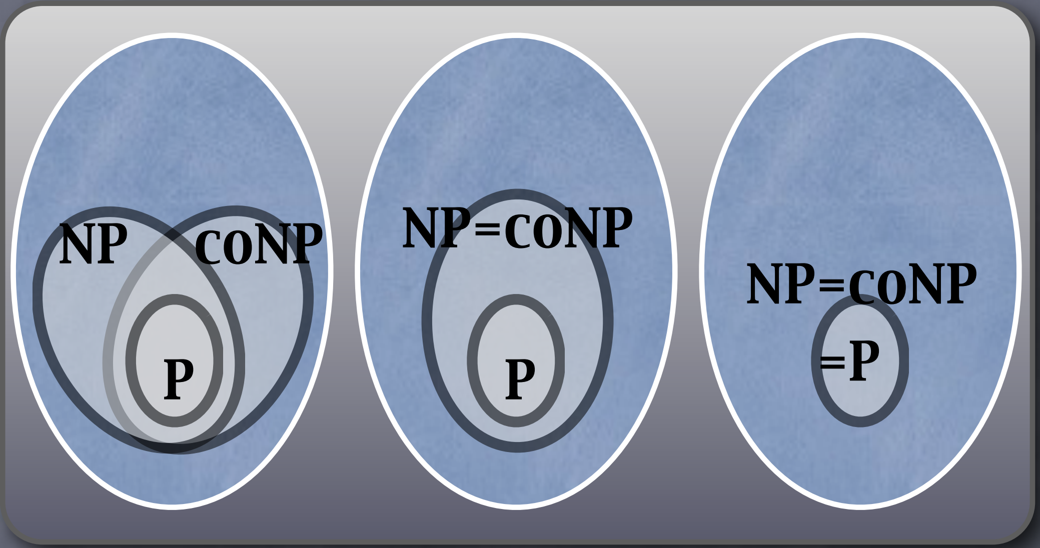


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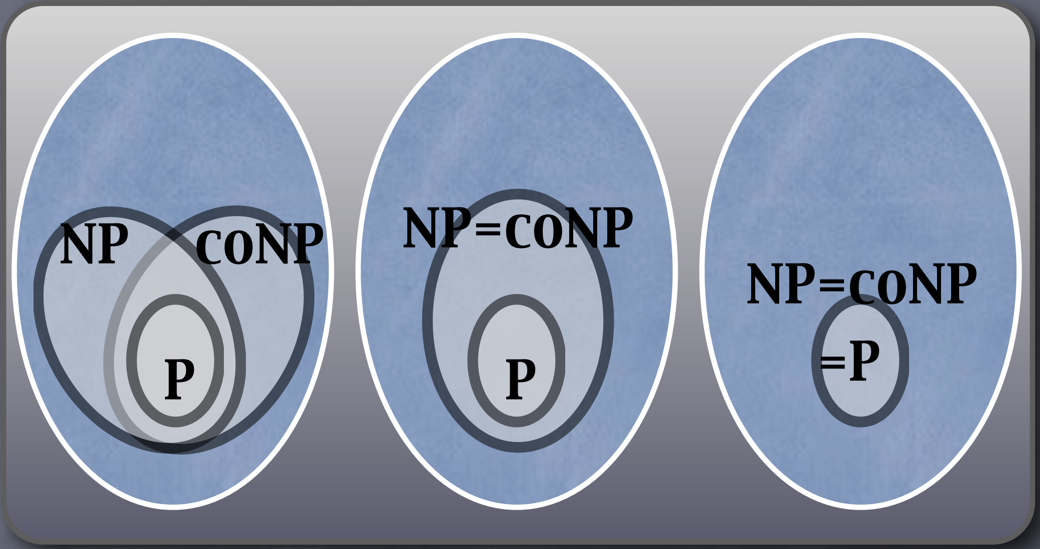


- Different possibilities
- If P=NP, then
  - coNP = coP = P = NP
  - Also, EXP = NEXP [Exercise]
    - padding to scale up both classes
    - x → (x,pad), so that Exp(|x|) = Poly(|x,pad|)
- If P=NP, then the complexity landscape would get greatly simplified than believed (more later)

# Today

# Today

- DTIME

# Today

- DTIME
  - P, EXP

# Today

- DTIME
  - P, EXP
- NTIME

# Today

- DTIME
  - P, EXP
- NTIME
  - Two views: non-determinism and certificate

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  - P, EXP
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- co-NTIME

# Today

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  - Two views: non-determinism and certificate
  - NP, NEXP
- co-NTIME
  - Two views: co-NTM and "no counter-example"

# Next ~~Class~~ Lecture

# Next ~~Class~~ Lecture

- NP completeness

# Next ~~Class~~ Lecture

- NP completeness
  - As hard as it gets inside NP

# Next ~~Class~~ Lecture

- NP completeness
  - As hard as it gets inside NP
    - a la reductions (of course)