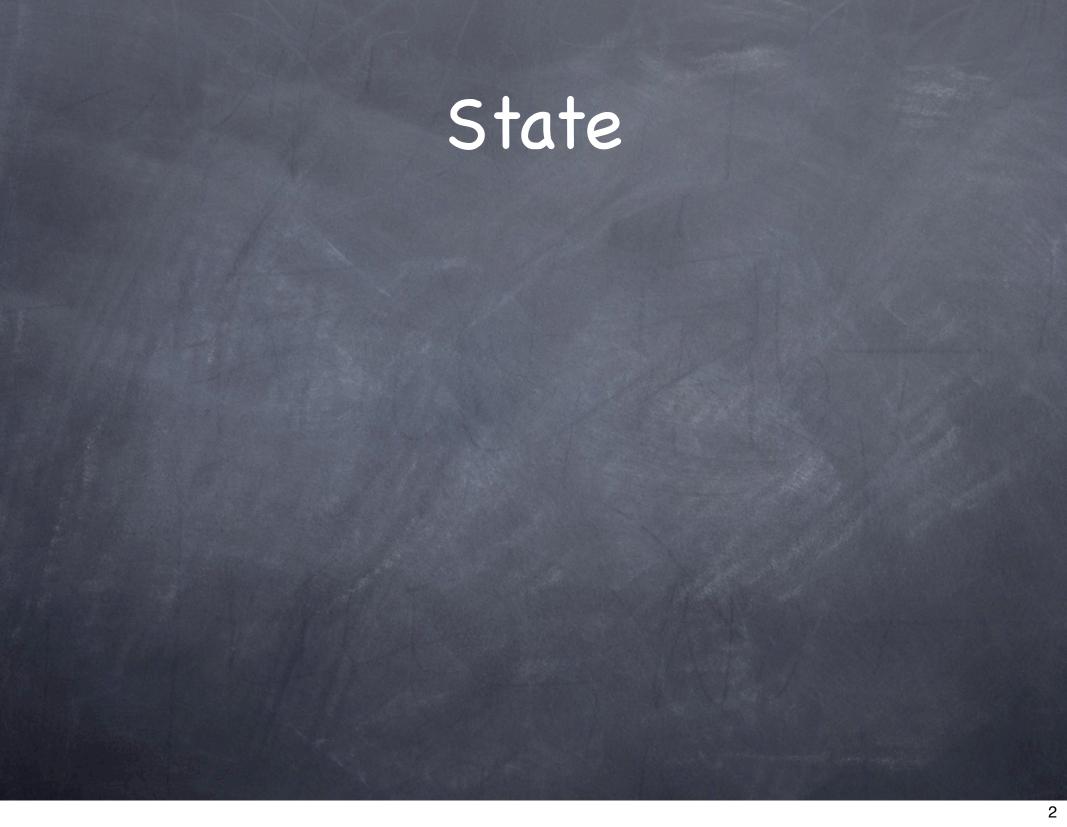
# Quantum Computation

Lecture 27 And that's all we got time for!



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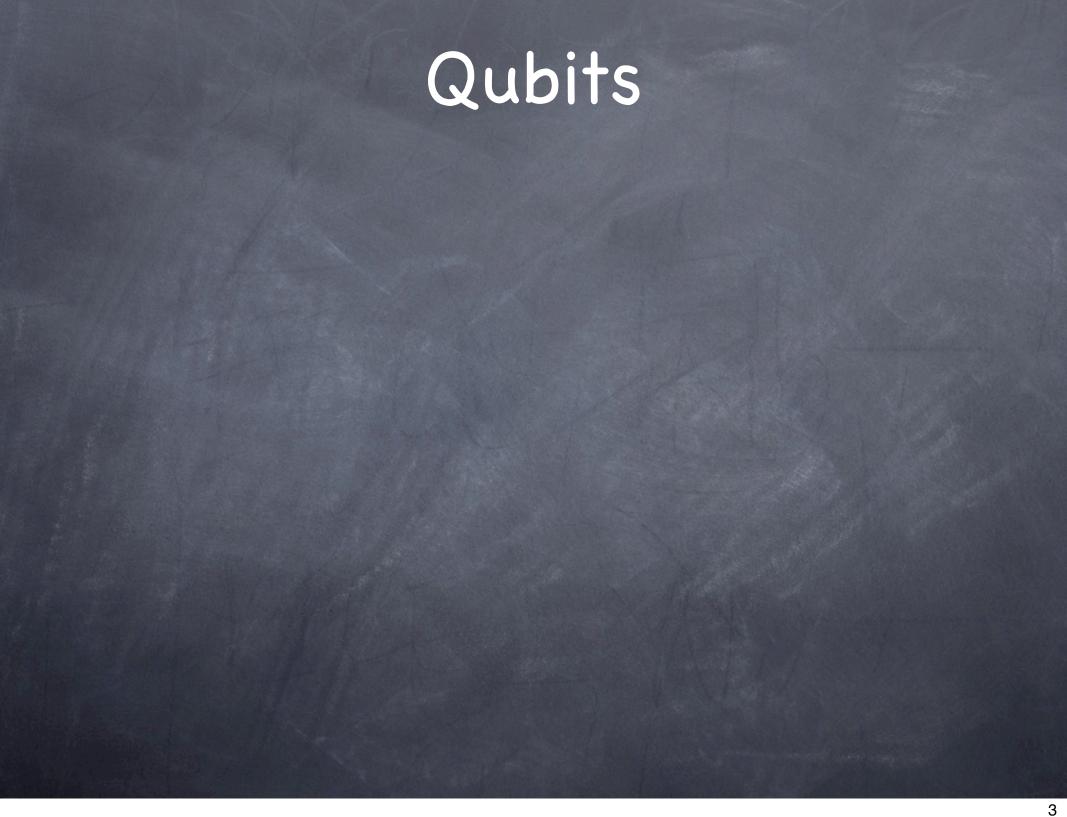
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- (Also, state can be "mixed": a probability distribution over amplitude vectors. Doesn't change power of quantum computing)

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- Can choose "non-standard" bases for measurement. But again, can do without it

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 $\circ$  e.g.:  $\sqrt{1/2}$   $\sqrt{1/2}$  (on one qubit),

0 1 0 0 (on 2 qubits)

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    - Contrast with classical case: probabilities can only add

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  - Toffoli gate has a classical analog (on 3 bits) that can be described as  $T(a,b,c) = (a,b,c \oplus a \land b)$

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  - "Copy" the output to unused qubits, and run the reverse computation to return the rest to original state

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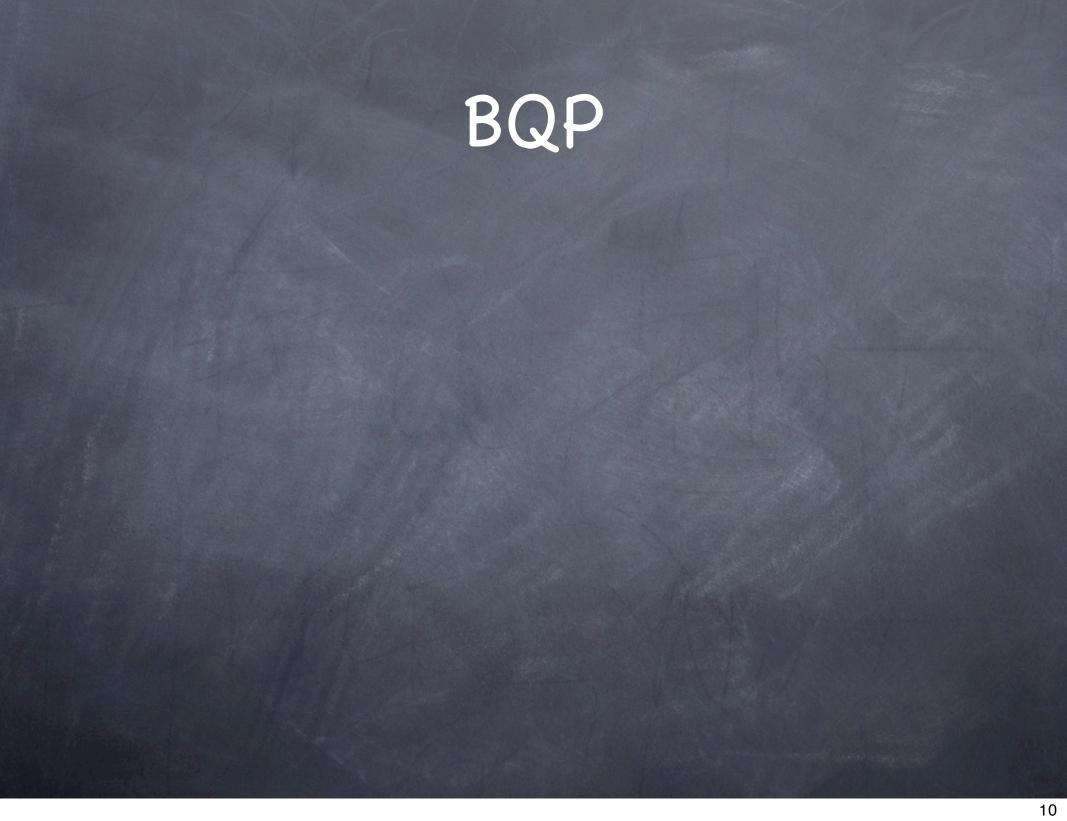
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  - $\otimes$   $x \in L \Rightarrow C_n(|x0^m\rangle) = 1 \text{ w.p. } > 2/3; x \notin L \Rightarrow C_n(|x0^m\rangle) = 1 \text{ w.p. } < 1/3$



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- How about BQP and NP?

# Two Quantum Algorithms

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#### Grover's Search

- Quadratic speedup for NP-complete problems (over the best known classical algorithms)
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- Polynomial sized quantum circuit for factoring
- Exponential speedup over the best known classical algorithms

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  - In  $O(2^{n/2})$  iterations, amplitude of  $|z\rangle$  becomes large (i.e., constant)

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