Computational Complexity

Lecture 0

Computation

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- A paradigm of modern science

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- Theory of computation/computational complexity is to computer science what theoretical physics is to electronics

Computational Complexity

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  - Algorithms to solve them
    - in various models of computation
- Complexity of a problem (in a comp. model)
  - How much “resource” is sufficient/necessary

Computational Complexity

of
Problems

in
Models of
computation

w.r.t
Complexity
measures

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    - A <u>Boolean</u> function evaluation (TRUE/FALSE)

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  - Decide if input is in L

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Flying Spaghetti Monster?

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Eq reduces to Mo

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- Non-uniform computation: circuit families

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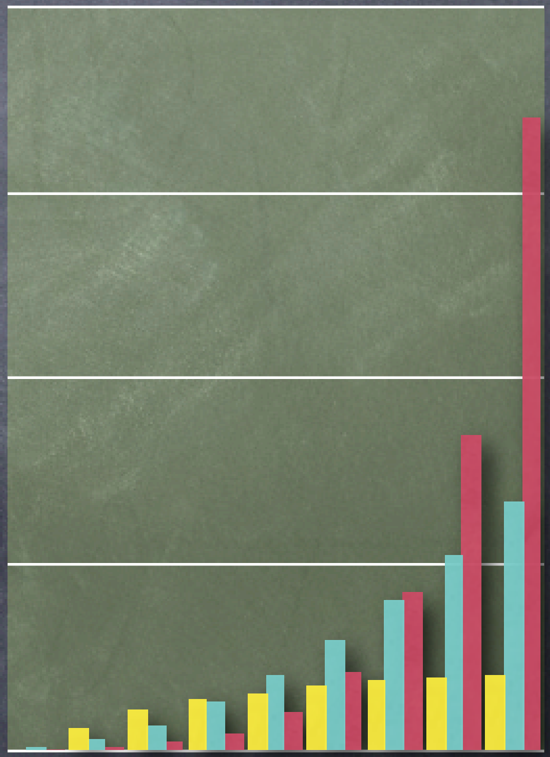

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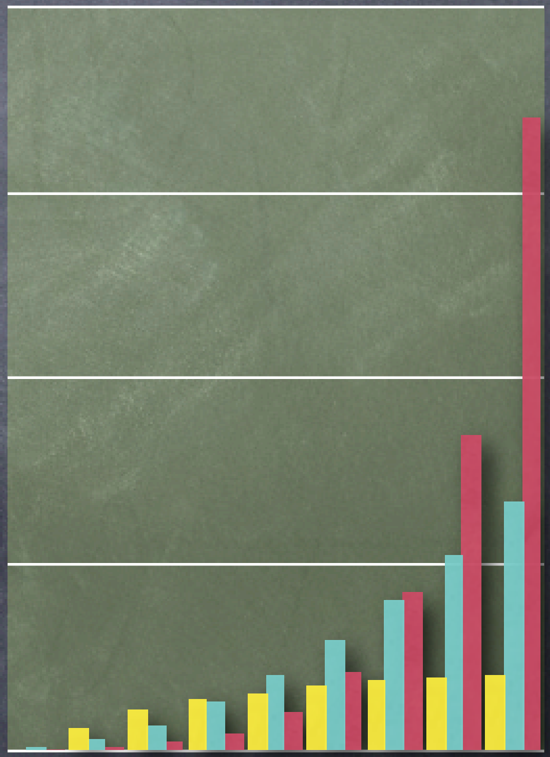

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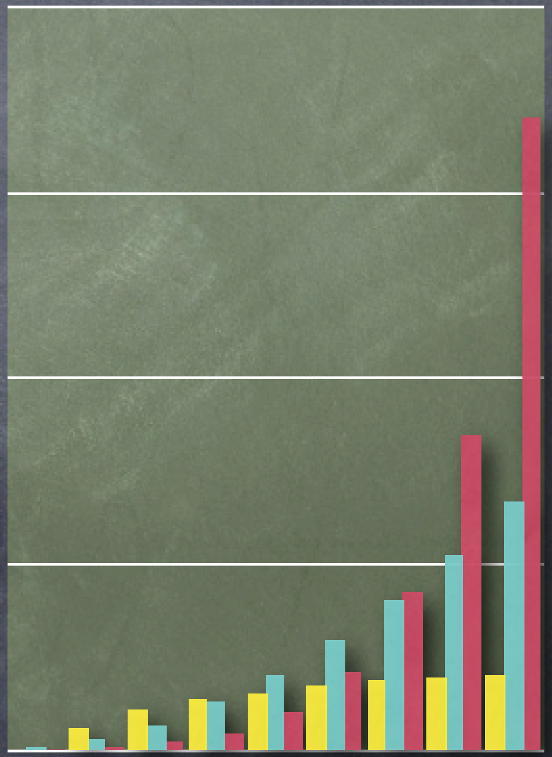

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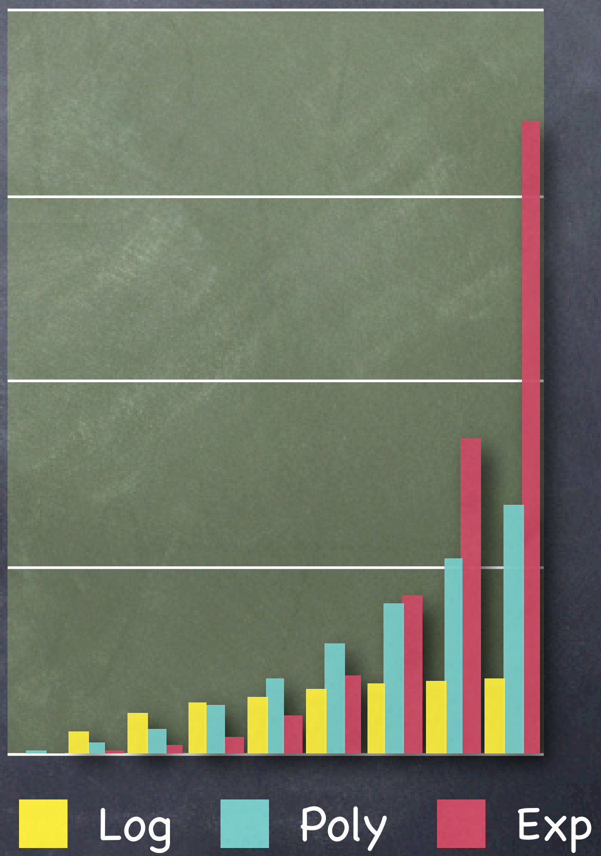

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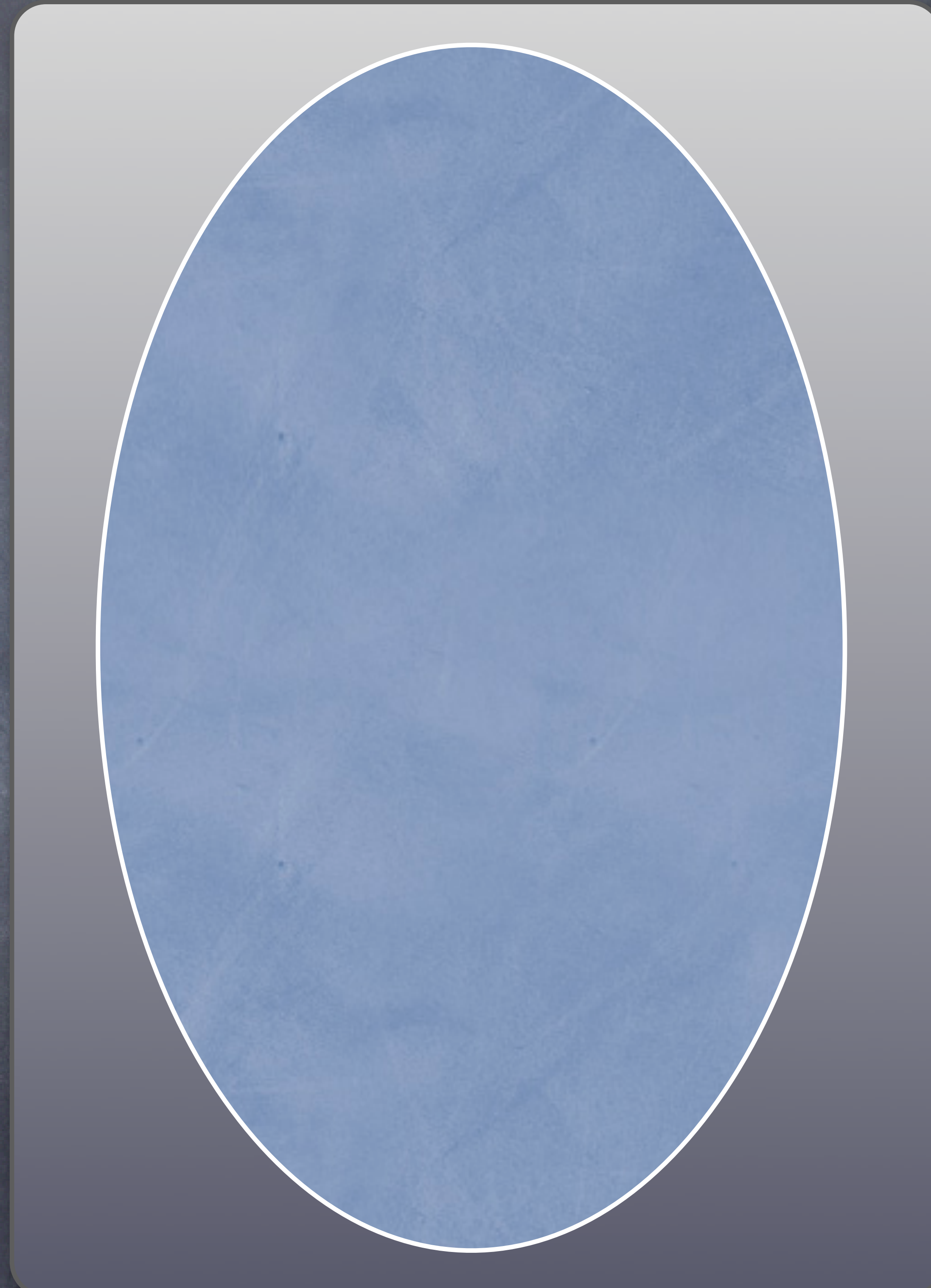

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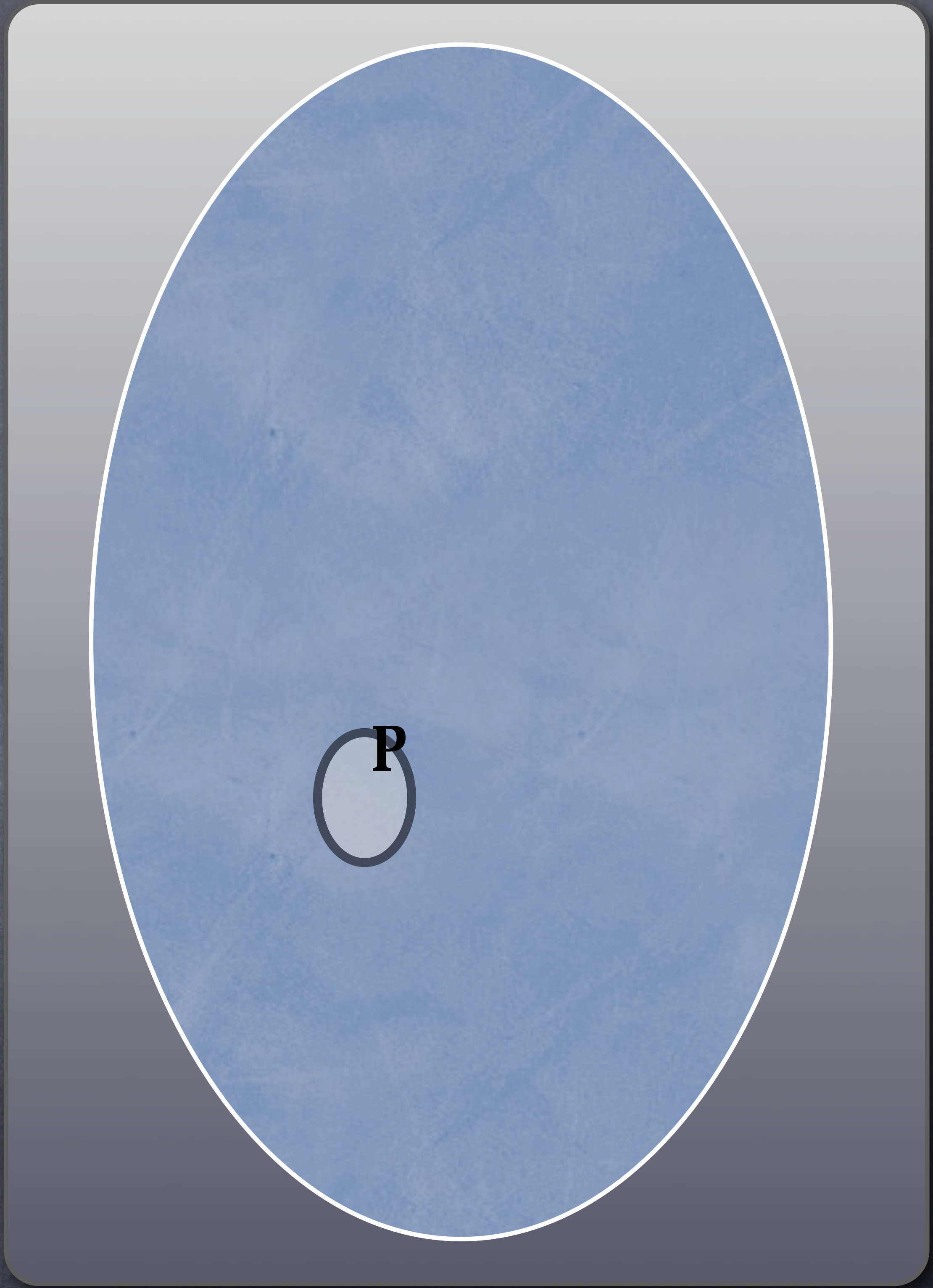

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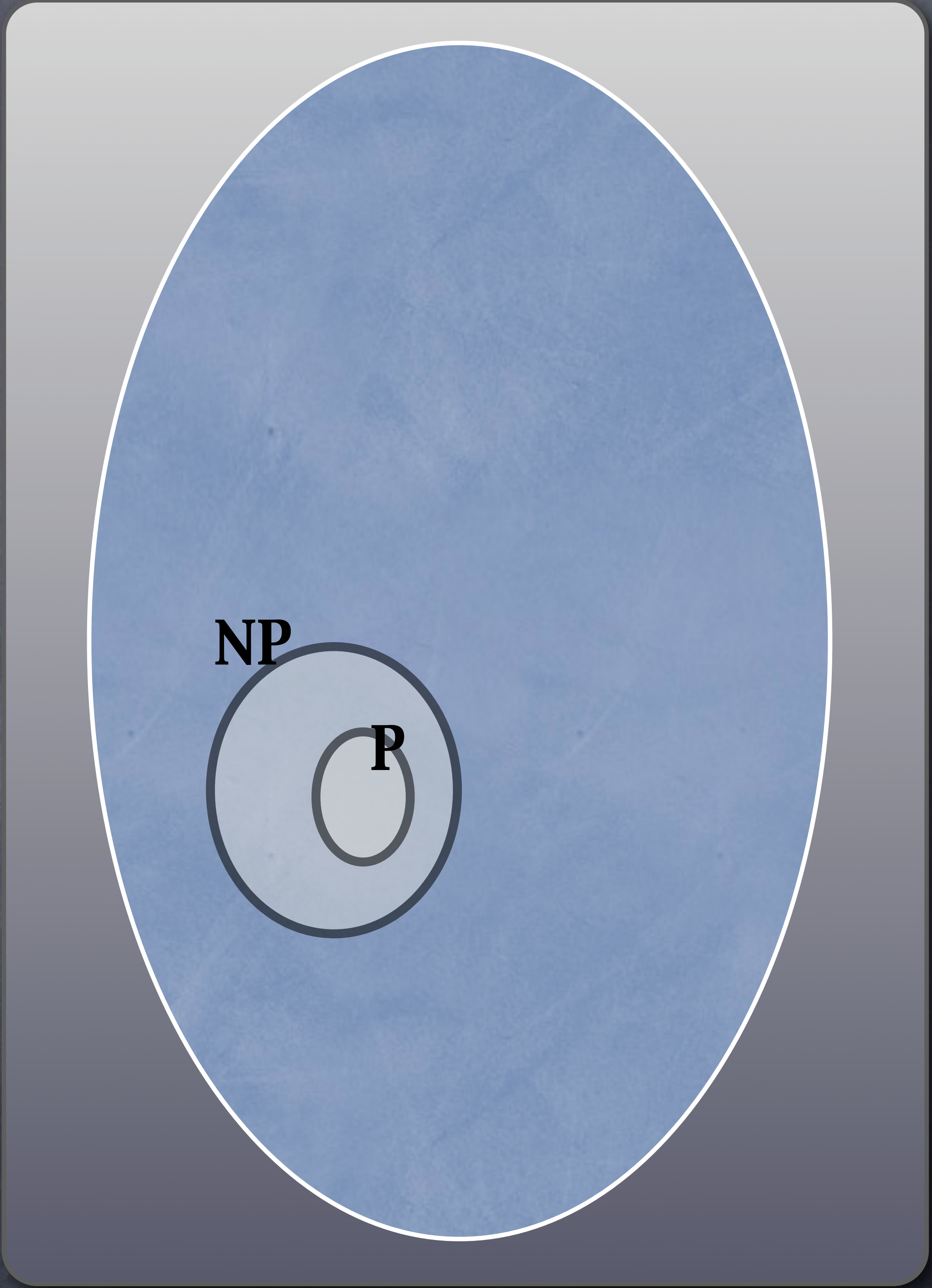

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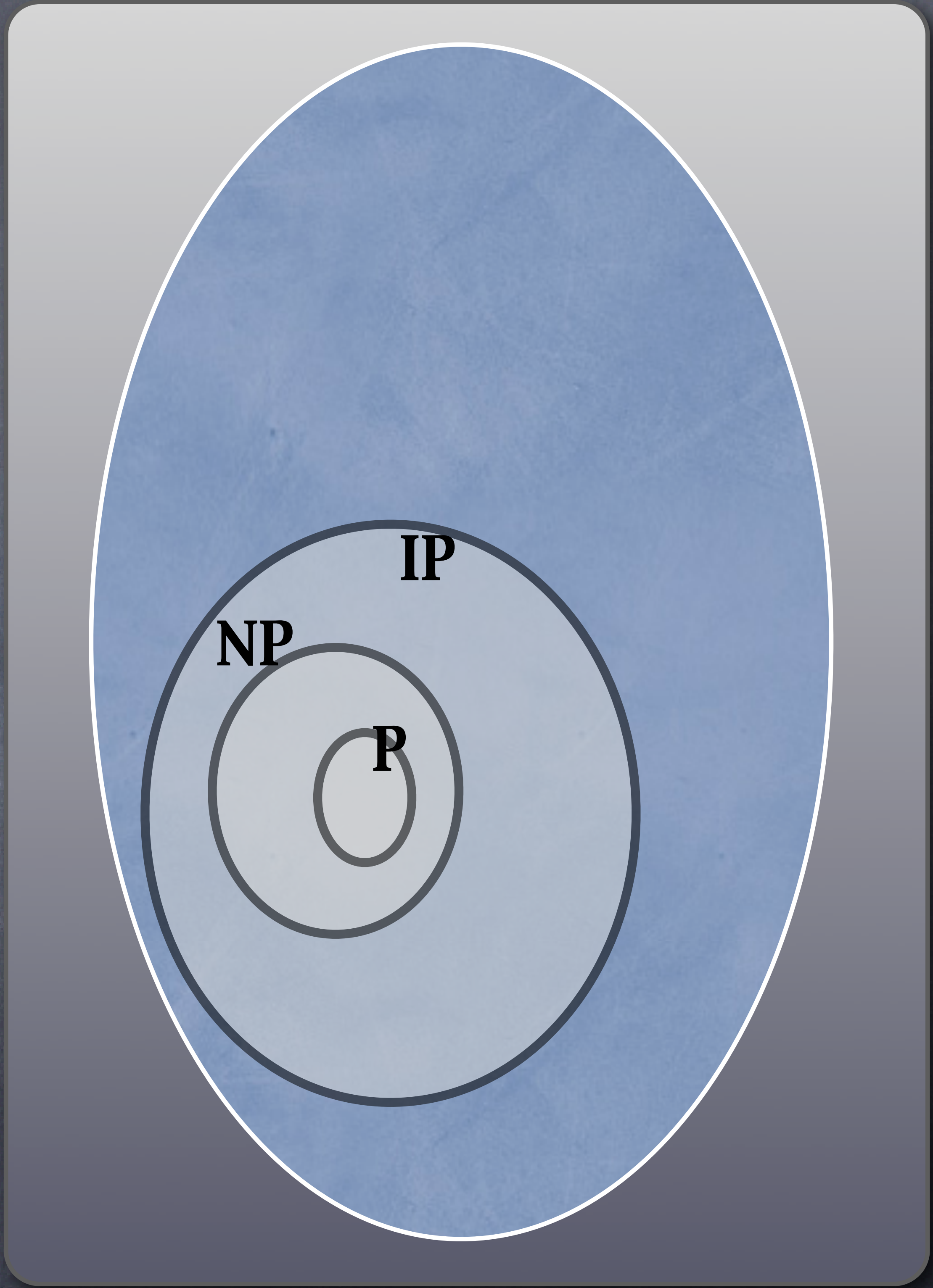

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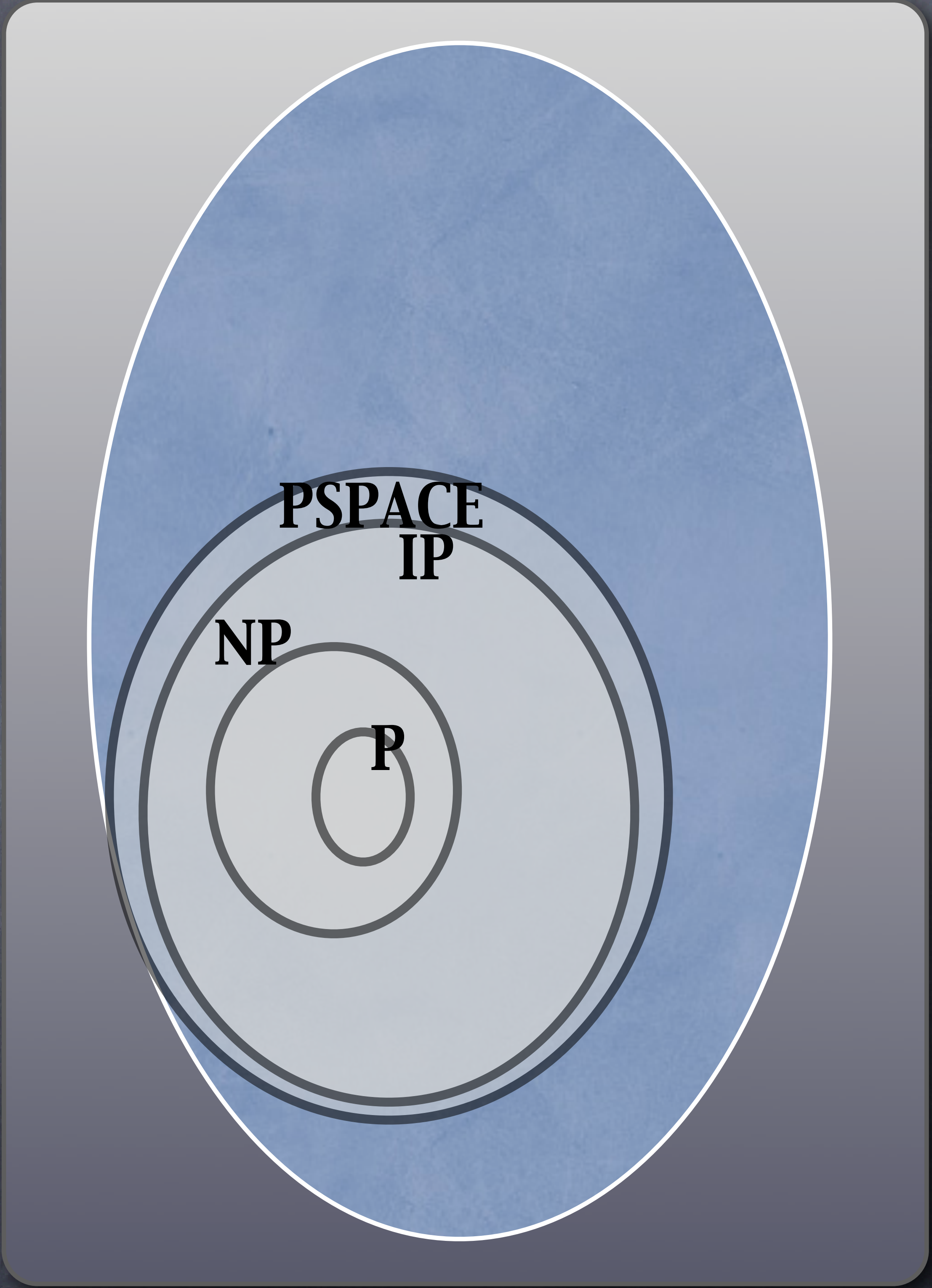

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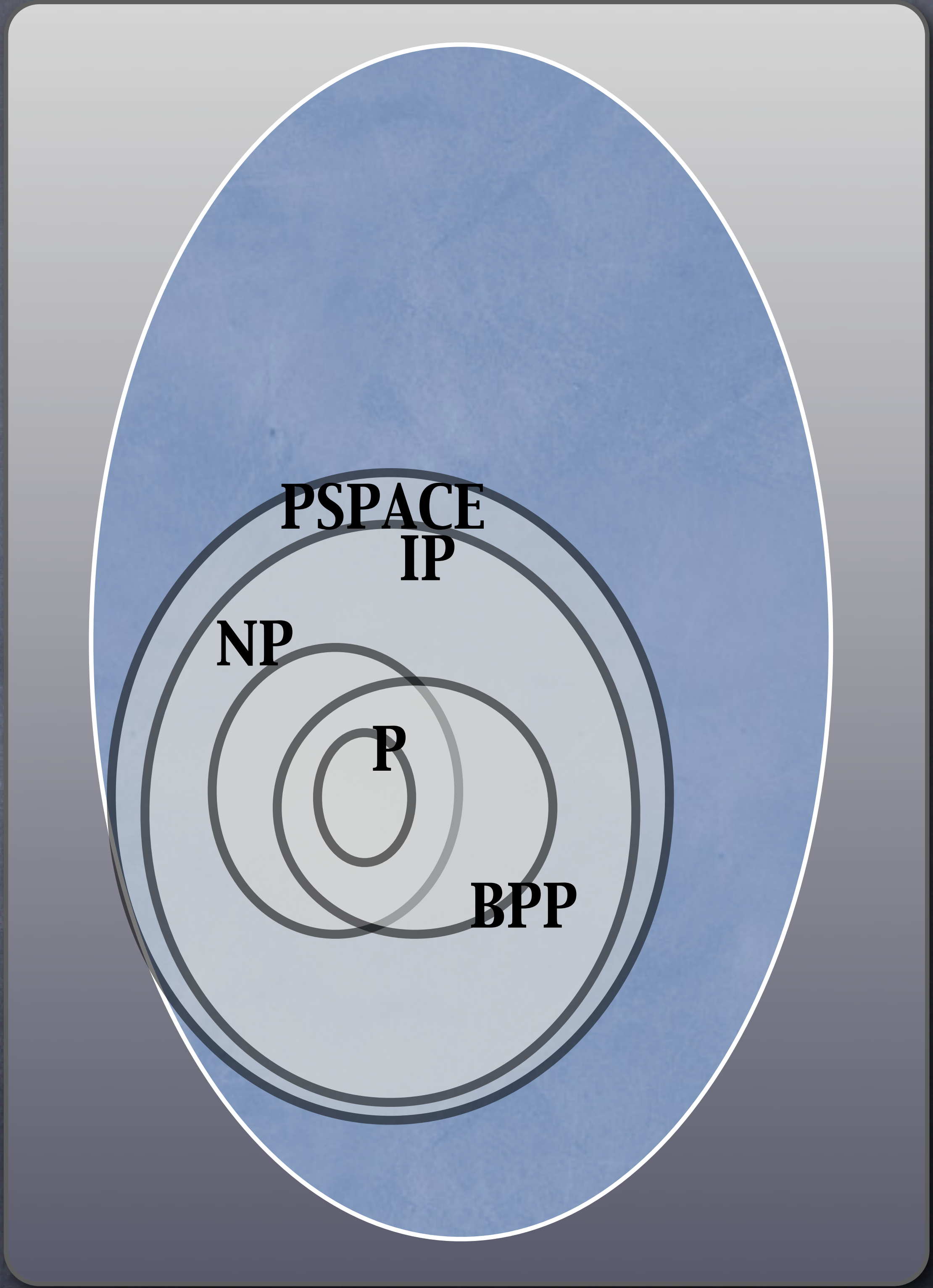

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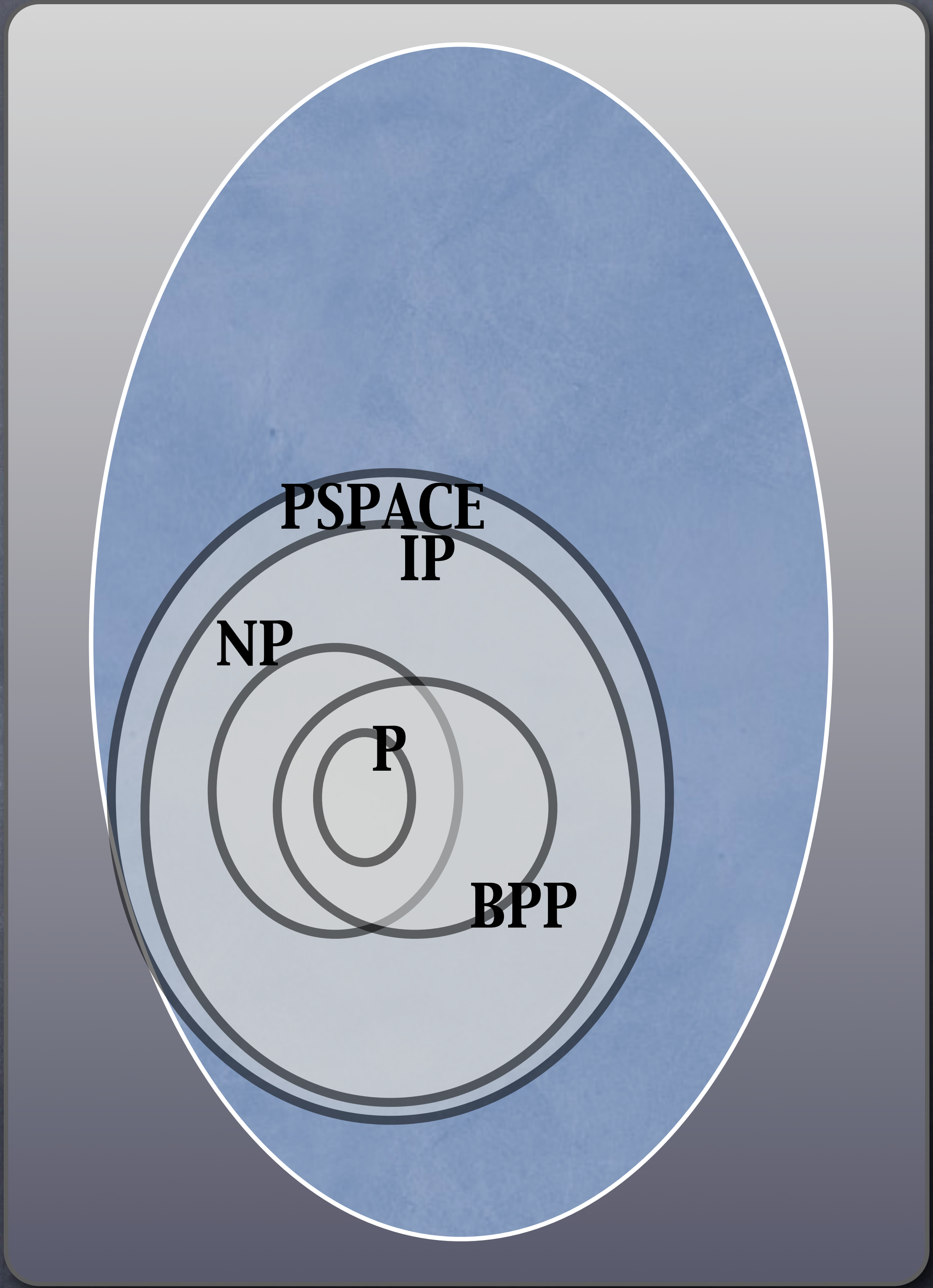

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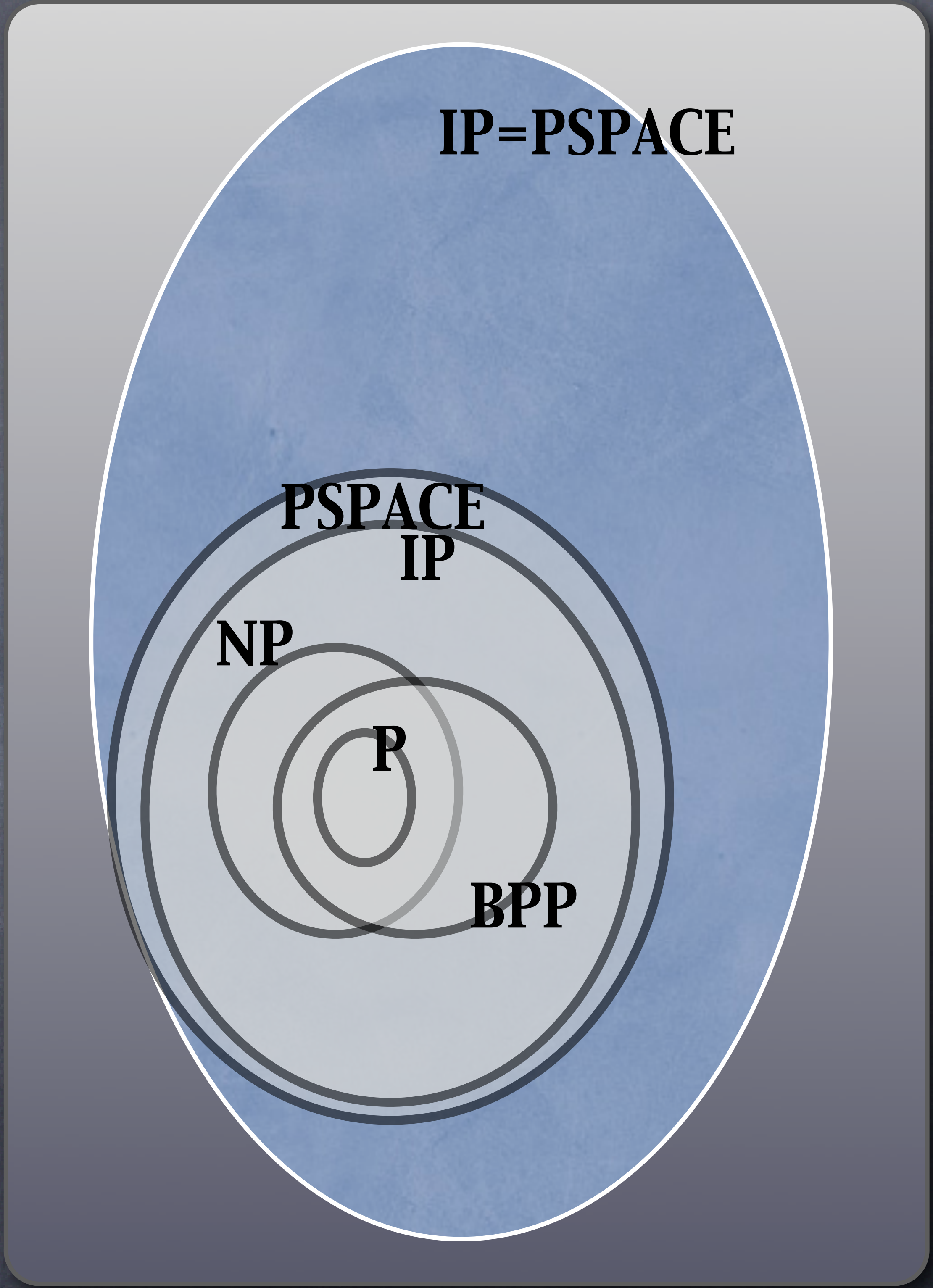

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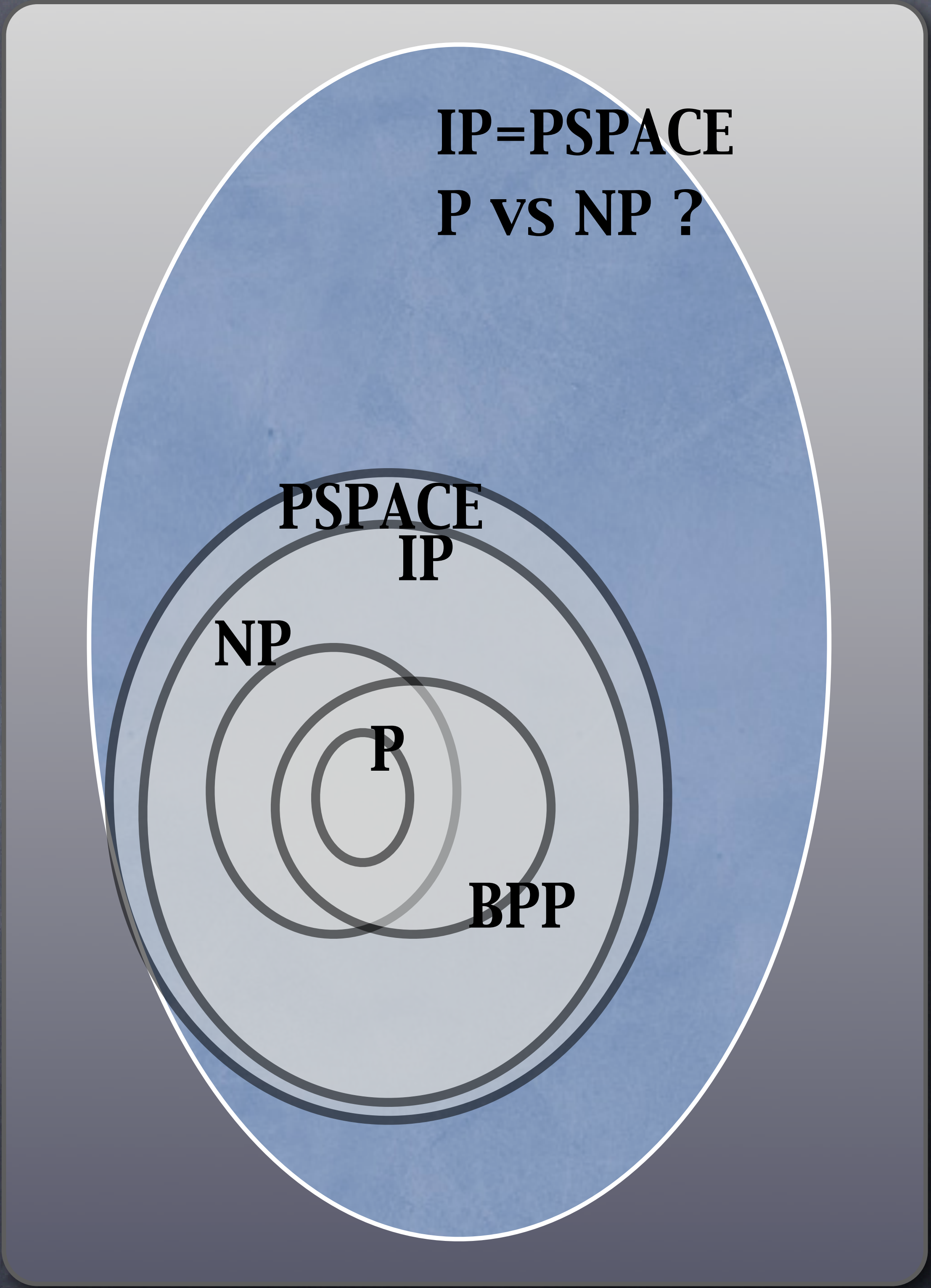

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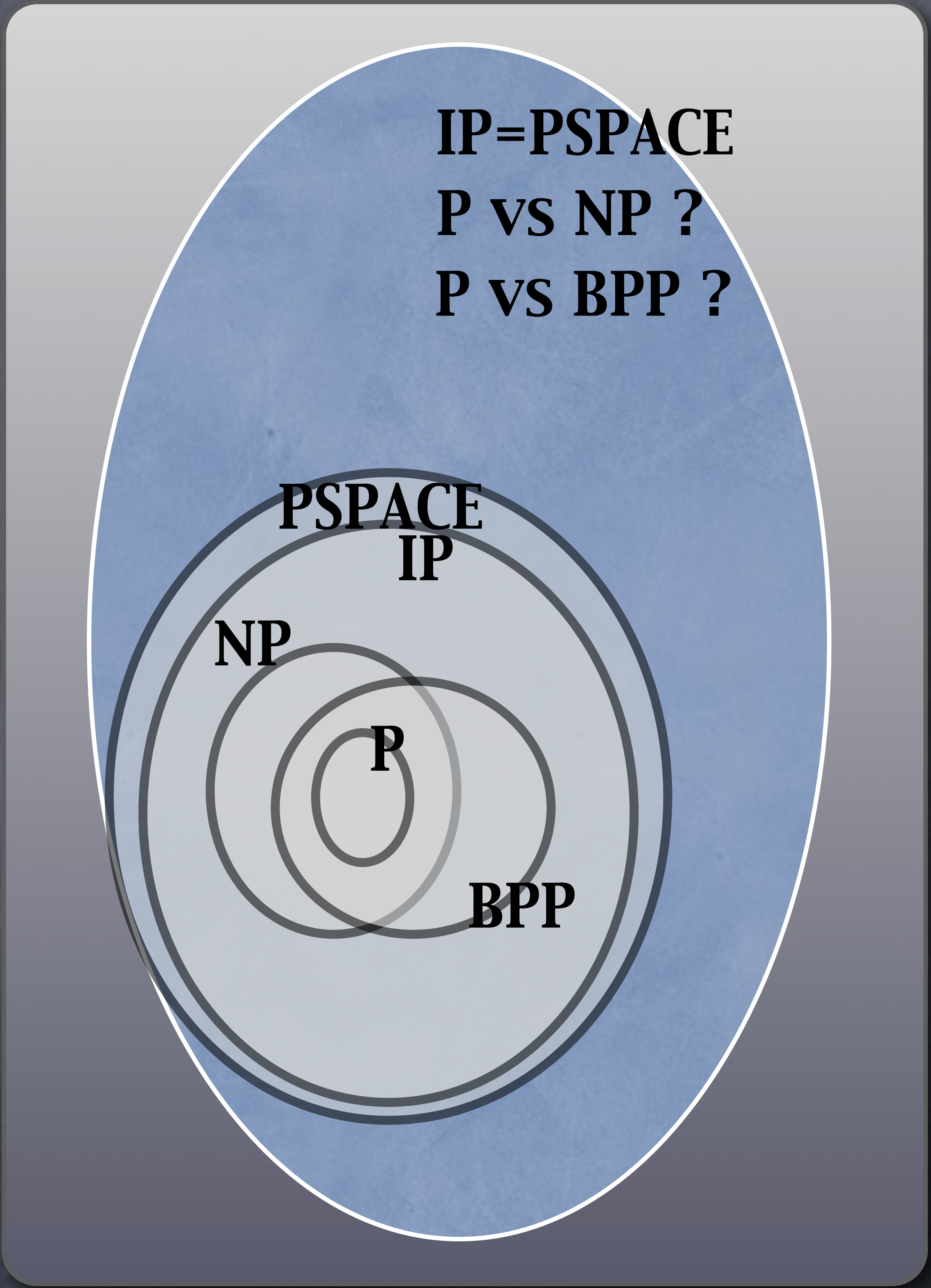

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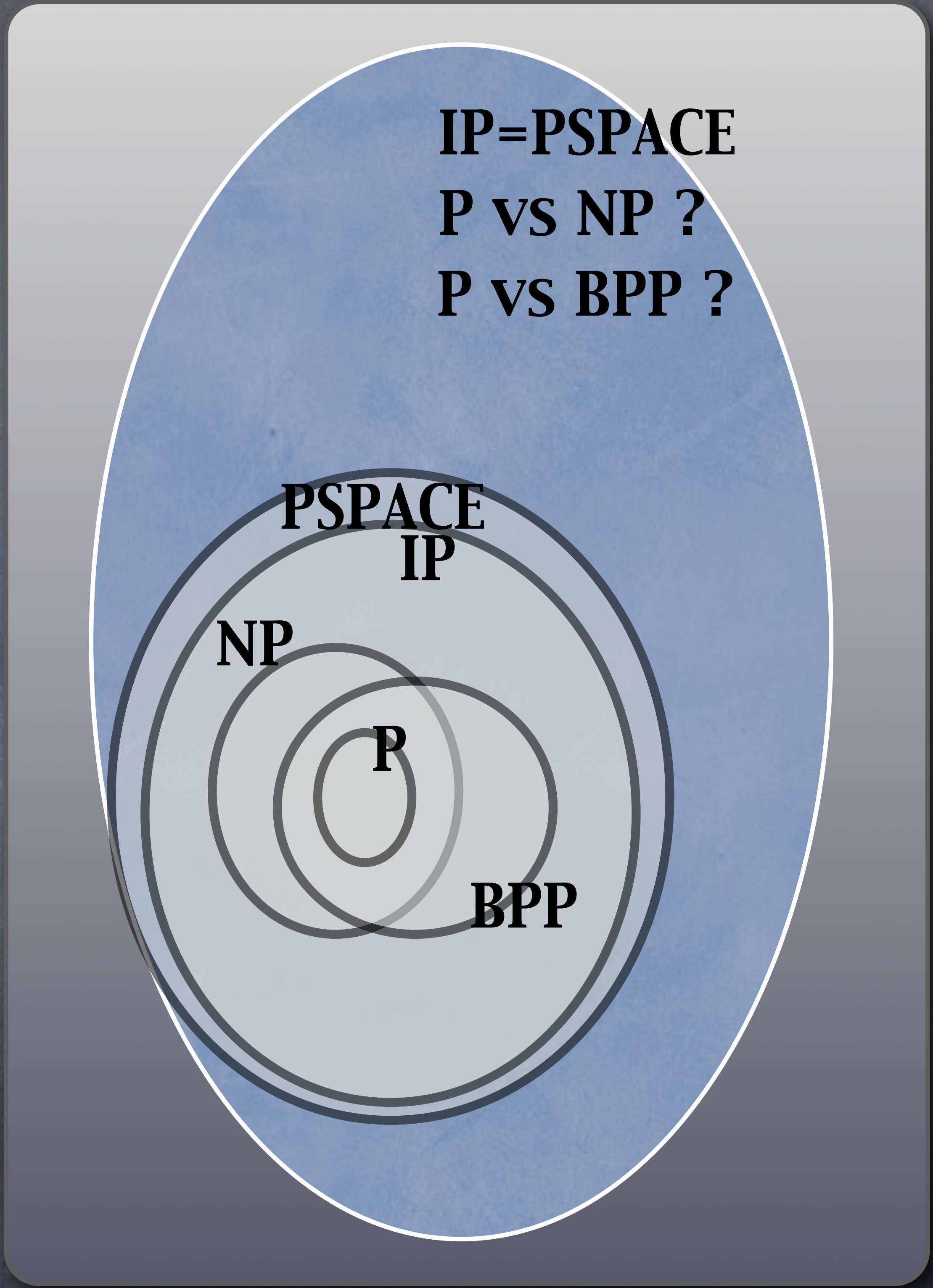

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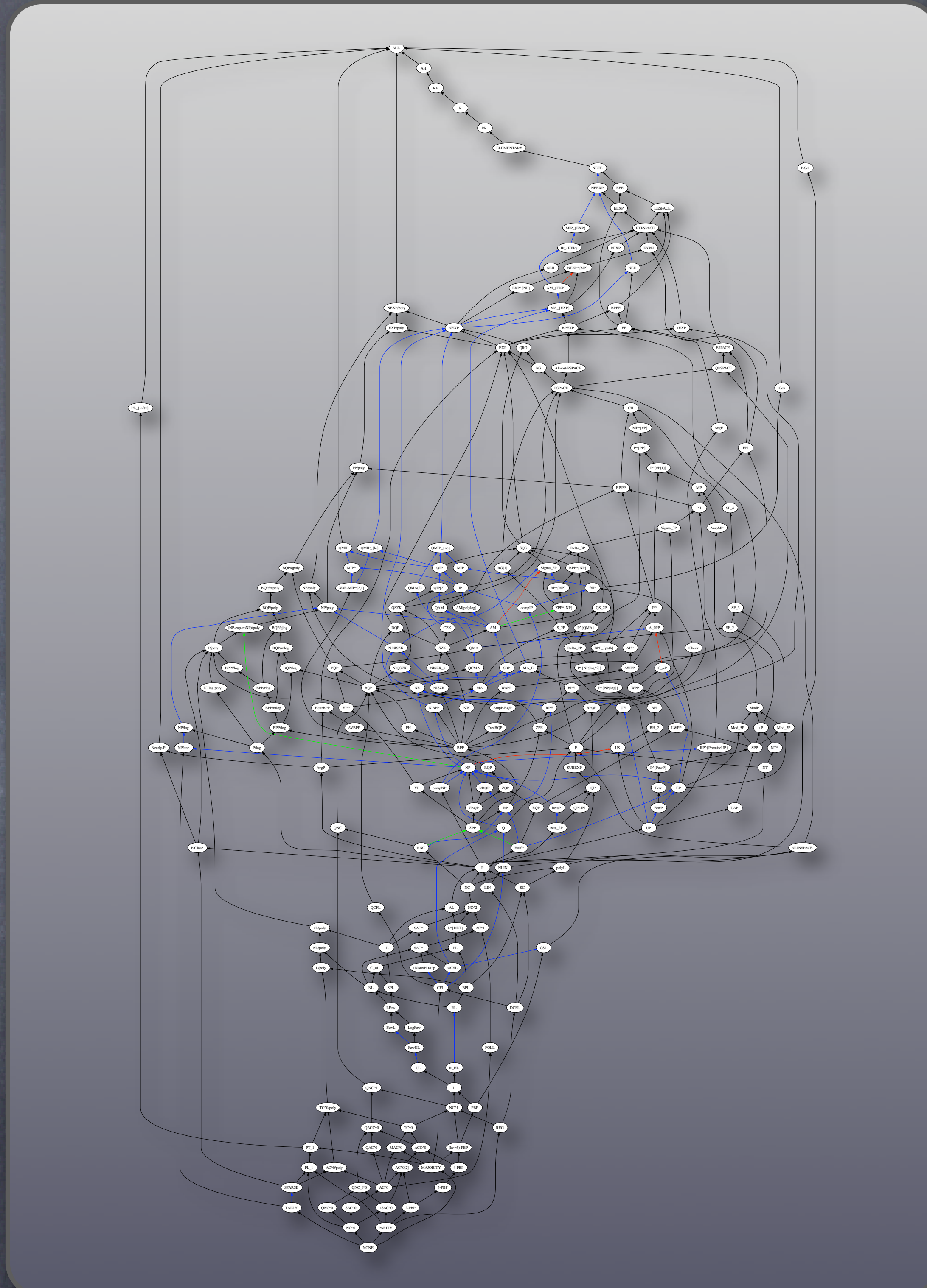

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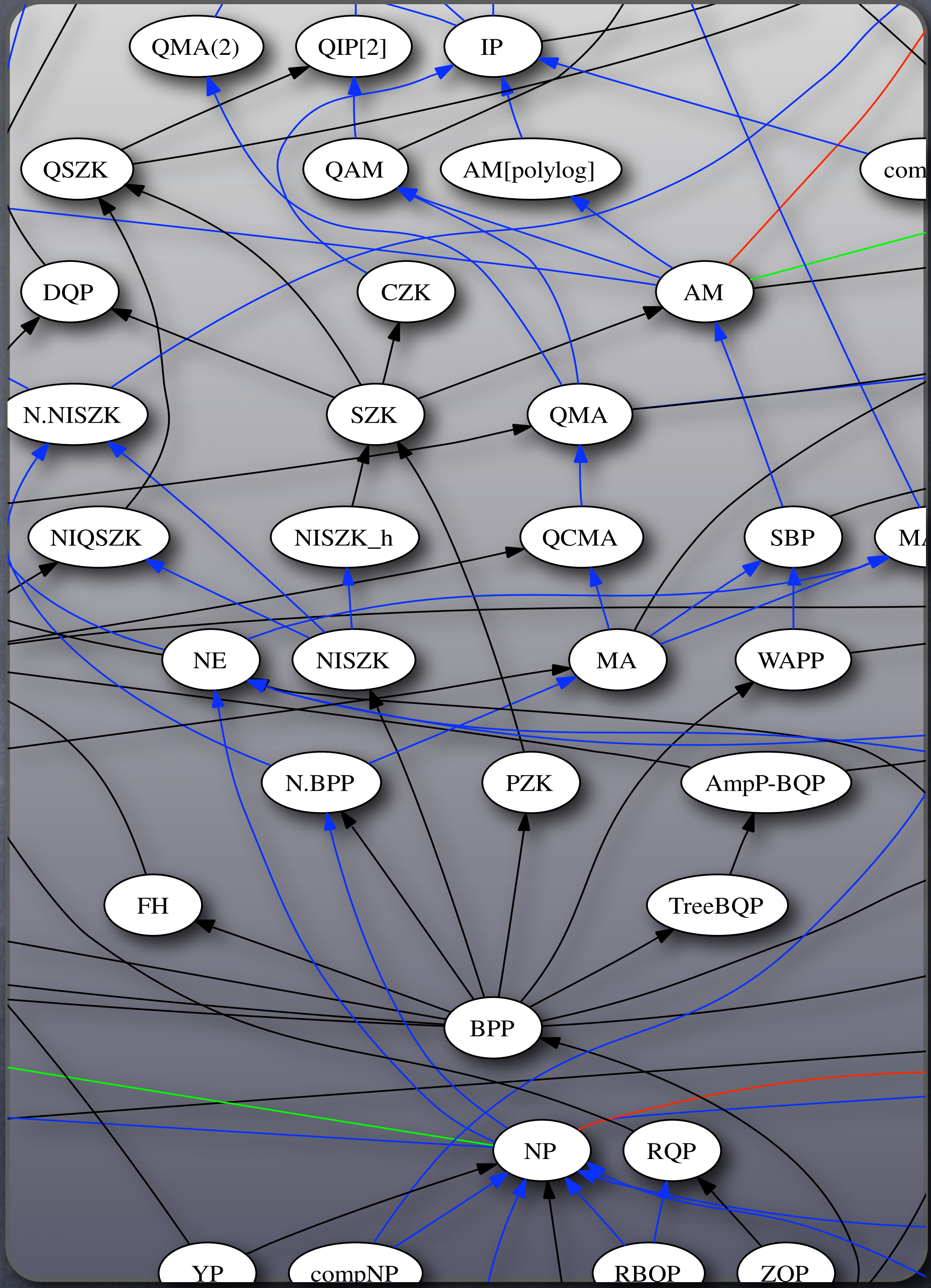

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- Various connections: time vs. space, randomness vs. hardness

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    - For most keys scheme should be secure

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