CS 579: Computational Complexity
Class Test

Tuesday, March 17, 2009

You have 75 minutes for the following five problems. The first four problems carry 10 points each (though some are easier/shorter than the others). If you have time, you can attempt the final extra credit problem. Any points you earn in that problem can make up for any points you lose in the other problems.

Problem 1. Show that $\mathbf{NP} \cap \mathbf{co\text{-}NP} = \mathbf{P}^{\mathbf{NP} \cap \mathbf{co\text{-}NP}}$.

Problem 2. **US** (for Unique Satisfiability) is a class of languages defined similarly to **NP**, but with the requirement that there is *exactly one* witness for inputs in the language. (If there is zero or more than one witness, the input is not in the language.) That is, **US** contains a language L iff there is a language $F \in \mathbf{P}$ such that

$$x \in L \iff \exists! w, |w| \leq \mathrm{R}(|x|), (x, w) \in F.$$

($\exists!$ stands for "there exists unique").

Show that $\mathbf{co\text{-}NP} \subseteq \mathbf{US}$

Problem 3. Show that $\mathbf{DTIME}(T) \subsetneq \mathbf{DSPACE}(T)$. You can use the following result: a deterministic TM running in time $T(n)$ (n being input size) can be simulated by a deterministic TM using space $T(n)/\log T(n)$.

Problem 4. Choose the correct statement below and prove it.

1. $\mathbf{P}^{\mathbf{E}} = \mathbf{E}$
2. $\mathbf{E} \subsetneq \mathbf{P}^{\mathbf{E}} \subsetneq \mathbf{EXP}$
3. $\mathbf{P}^{\mathbf{E}} = \mathbf{EXP}$
4. $\mathbf{EXP} \subsetneq \mathbf{P}^{\mathbf{E}}$

Problem 5 (Extra Credit). Show that $\mathbf{BPP} \subseteq \mathbf{ZPP}^{\mathbf{NP}}$.

(Hint: Follow the argument used in showing $\mathbf{BPP} \subseteq \mathbf{NP}^{\mathbf{NP}}$*.)*