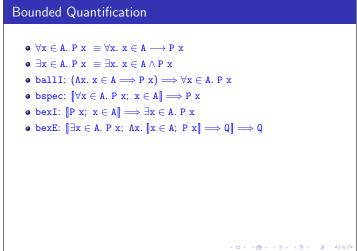


Proofs about Sets Natural deduction proof rules: • equalityI: $||A \subseteq B; B \subseteq A|| \implies A = B$ • equalityE: $||A \subseteq B; B \subseteq A|| \implies A \subseteq B$ • subsetI: $||A \cap B|| \implies A \subseteq B$ • sutsetD: $||A \cap B|| \implies C \in B$ • IntI: $||C \cap A|| \implies C \in B$ • IntD1: $||C \cap A|| \implies C \in B$ • set_eqI: $||A \cap B|| \implies C \in A$ • IntD2: $||C \cap A|| \implies C \in B$ • set_eqI: $||A \cap C|| \implies C \in B$ • set_eqI: $||A \cap C|| \implies C \in B$ • collect_mem_eq: $||A \cap C|| \implies C \in B$ • ... (see Tutorial)



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Demo: Some Set Theory

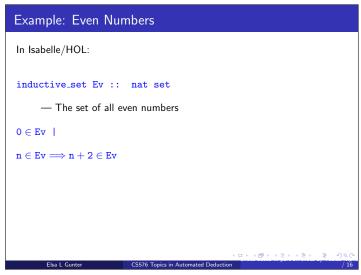
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Informally The empty set is finite Adding an element to a finite set yields a finite set These are the only finite sets

Example: Finite Sets In Isabelle/HOL: inductive_set Finites :: 'a set set - The set of all finite sets { } ∈ Finites | A ∈ Finites ⇒ insert a A ∈ Finites Elsa L Gunter CSS76 Topics in Automated Deduction / 16

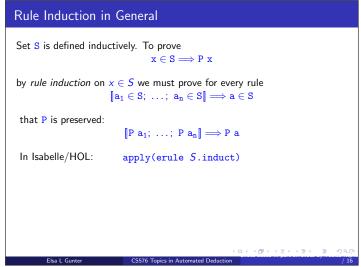
Informally • 0 is even • If n is even, then so is n+2• These are the only even numbers

Example: Even Numbers



Proving Properties of Even Numbers Easy: $4 \in Ev$ $0 \in Ev \Longrightarrow 2 \in Ev \Longrightarrow 4 \in Ev$ Trickier: $m \in Ev \Longrightarrow m+m \in Ev$ Idea: induct on the length of the derivation of $m \in EV$ Better: induct on the *structure* of the derivation

To prove $n \in Ev \Longrightarrow P \ n$ by rule induction on $n \in Ev$ we must prove $\bullet \ P \ 0$ $\bullet \ P \ n \Longrightarrow P(n+2)$ Uses rule Ev : induct: $\|n \in Ev; \ P \ 0; \ \Lambda n. \ P \ n \Longrightarrow P(n+2)\| \Longrightarrow P \ n$ An elimination rule



Demo: Inductive Set Definition

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Demo: Evens are infinite