Topics in Automated Deduction (CS 576)

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General Recursive Functions: `fun`

Example:

```
fun fib :: "nat ⇒ nat" where
      "fib 0 = 0" |
      "fib 1 = 1" |
      "fib (Suc(Suc x)) = (fib x + fib (Suc x))"
```

Not primitive recursive because of `fib(Suc(Suc x))` on left, and because of `fib(Suc x)` on right.

`fun`: Rules of Use

Compared to `primrec`, very few restrictions:

- Can be used to define functions over any type
- Clauses in `fun` must be equations
- Left-hand side is function being defined applied to terms built from data constructors, distinct variables and wildcards
- Right-hand side is a expression made from the function being defined, the variables in the argument on the left, and previously defined terms
- If clauses overlap, first takes precedence.
- Calculates a measure from lexicographic ordering of some collection of arguments

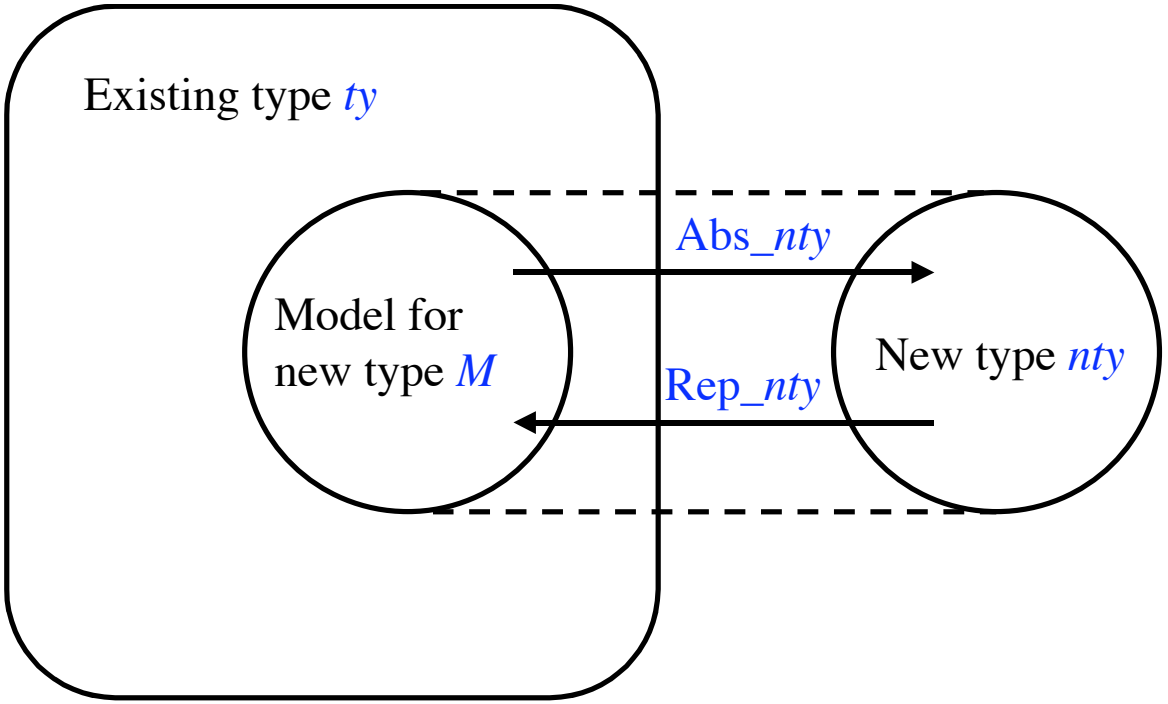
Example: `sep`

Define a function for putting a separator between all adjacent elements in a list:

```
fun sep ::  "'a * 'a list => 'a list" where
 "sep(a, []) = []" |
 "sep(a, [x]) = [x]" |
 "sep(a, x#y#zs) = x # a # sep(a,y#zs)"
```

Demo: General Recursive Functions

Introducing new types



Properties of a Defined Type

Syntax for `typedef`:

$$\texttt{typedef}\ \mathit{nty} = \text{"}\mathit{modeling\_set}\text{"}$$
$$\mathit{Proof\ of}\ \exists x.x \in \mathit{modeling\_set}.$$

introduces a new type named *nty*, and functions

$$\texttt{Abs\_}\mathit{nty} :: \mathit{ty} \Rightarrow \mathit{new\_ty}$$
$$\texttt{Rep\_}\mathit{nty} :: \mathit{new\_ty} \Rightarrow \mathit{ty}$$

where *modeling_set*::*ty*

Properties of a Defined Type

Theorems automatically provided:

Rep_*nty*: $\texttt{Rep\_}nty\ x \in modeling\_set$

Abs_*nty*_inverse: $\texttt{Abs\_}nty(\texttt{Rep\_}nty\ x) = x$

Rep_*nty*_inverse:

$y \in modeling\_set \Longrightarrow \texttt{Rep\_}nty(\texttt{Abs\_}nty\ y) = y$

Abs_*nty*_inject:

$[|x \in modeling\_set : y \in modeling\_set|] \Longrightarrow$

$(\texttt{Abs\_}nty\ x = \texttt{Abs\_}nty\ y) = (x = y)$

Rep_*nty*_inject: $(\texttt{Rep\_}nty\ x = \texttt{Rep\_}nty\ y) = (x = y)$