

Chapter 56

Some math stuff

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56.1. Some useful estimates

Lemma 56.1.1. *For any $n \geq 2$, and $m \geq 1$, we have that $(1 - 1/n)^m \geq 1 - m/n$.*

Proof: Follows by induction. Indeed, for $m = 1$ the claim is immediate. For $m \geq 2$, we have

$$\left(1 - \frac{1}{n}\right)^m = \left(1 - \frac{1}{n}\right)\left(1 - \frac{1}{n}\right)^{m-1} \geq \left(1 - \frac{1}{n}\right)\left(1 - \frac{m-1}{n}\right) \geq 1 - \frac{m}{n}. \quad \blacksquare$$

This implies the following.

Lemma 56.1.2. *For any $m \leq n$, we have that $1 - m/n \leq (1 - 1/n)^m \leq \exp(-m/n)$.*

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